THE DYNAMO

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HAWKINS AND WALLIS

THE DYNAMO

THE DYNAMO

ITS THEORY, DESIGN, AND MANUFACTURE

BY

harles aesar

C. C. HAWKINS,

M.A., M.I.E.E., ASSOC.AMER.I.E.E.

AND

F. WALLIS,

B.A., M.I.E E.

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THE DYNAMO

CHAPTER XVII

ARMATURE REACTION IN CONTINUOUS-CURRENT DYNAMOS

§ 1. **Diameter of commutation and brush-position.**—In the continuous-current closed-coil dynamo the winding is divided by the brushes into as many current-sheets as there are poles. Thus in the case of an armature whether ring or drum-wound, in a 2-pole field, two sheets of current as shown by the dotted and crossed circles of Fig. 312 flow along the external surface from end to end, and the direction of

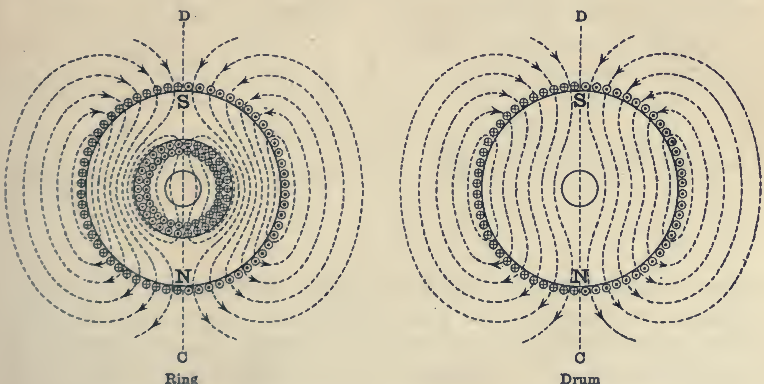


FIG. 312.—Magnetic field of 2-pole armature without field-magnet.

the currents in the active wires changes at the diameter of commutation (DC) corresponding to the position of the brushes, unless the armature is chord-wound, when the several current-sheets partially overlap. It may here be recalled that the relative position of the line of the brushes on the commutator and the line passing through the coils undergoing commutation on the armature core depends upon the way in which they are connected to the commutator sectors (see Chap. XI. § 17). In the slotted drum machine the connection is usually made at the centre of the end-connectors, so that the brush position is shifted $\frac{90^\circ}{p}$ away from the true line of commutation. In every case, then, the line or *diameter of commutation* (a term extended to multipolars by analogy from the 2-pole dynamo) must be understood to refer to the actual position of the coils undergoing commutation rather than to the position of the brushes corresponding thereto.

§ 2. **Distortion of the resultant field.**—Considering first an armature carrying current, but in which the brushes are so set that the diameter of commutation is coincident with the line of symmetry (VL) bisecting the interpolar gap, let a system of lines be drawn through the armature similar to the symmetrical distribution which holds when the armature is run on open circuit (Fig. 313). When the wires are paired to form current-turns, each line is found to pass through a certain number of armature ampere-turns; and of these turns the one half acts in the same direction as the exciting M.M.F., and the other half against it. The relation of the armature-turns to the direction of the flux on the two sides of the line of symmetry is reversed in opposite halves of the armature, but the forward and back ampere-turns are exactly balanced, so that they neutralise one another. Such a distribution is, however,

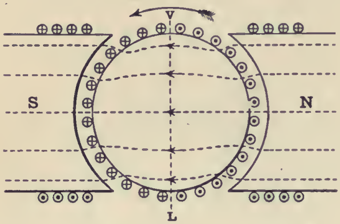


FIG. 313.

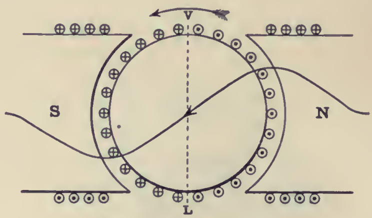


FIG. 314.

not the one that holds in reality; in nature the total flux is always the maximum possible with the given number of ampere-turns that are present, and is always linked with the maximum possible number of forward ampere-turns, or conversely passes through a minimum number of back ampere-turns; due regard being had to the reluctivity or permeability of the material of the circuit which may alter as the flux density at different points is varied with different distributions. It follows that the lines tend to become bent so as to pass through as many forward armature turns as possible; in other words, the field is distorted, the general tendency being for the lines to enter (*e.g.*, with the polarities shown) in the upper right quadrant, and to emerge at the lower left quadrant. All the lines cannot flow along the path indicated (Fig. 314), since the additional forward ampere turns through which the lines when thus concentrated and crowded together pass would not suffice to meet the combined effect of the two restraining causes, namely, the greater length of the path and the greater fall of magnetic potential that must result from the increased density of the flux even if the permeability of the material remained constant. The lines must therefore be more or less spread out, the total flux being retained at its highest possible value and linked with as many ampere-turns as possible on the armature, so that at one and the same time the energy stored in relation to both the exciting coils and the armature turns is a maximum. When the lines are thus more or less distorted and concentrated, in order to predict their distribution, it would be necessary to decide the question how far the increased number of forward armature turns through which they pass will suffice to counterbalance the two checking causes that have been mentioned. The average length of the distorted lines could, however, only be approximated by guess-work, and the actual number of forward and back ampere-turns traversed by any group of lines would be uncertain. A more detailed analysis is therefore necessary for the dynamo designer, the basis upon which it may be carried out being the examination of the effect of superposing on the initial field due to the field ampere-turns a second field due to the armature ampere-turns.

§ 3. **The magnetic field of the armature alone.** — In the first place, then, what is the nature of the field due to the armature ampere-turns considered by themselves? In the bipolar armatures of Fig. 312 the general effect of the two current-sheets is to convert the iron armature into a cylindrical electro-magnet with two polar surfaces at opposite ends of the diameter of commutation, and stretching along the entire length of the core. If the armature be ring-wound, with the turns of wire passing round and inside the core, the analogy to a pair of magnets of semicircular shape, each wound with a magnetising coil and placed so that their like poles abut on each other to form a common N. and a common S. pole, is at once apparent. If the armature be drum-wound, this effect is not immediately so apparent, but we may for simplicity neglect the actual end-connections of the wires and imagine each active wire connected across, in a plane at right angles to the diameter of commutation to a corresponding active wire on the opposite side of this diameter. A number of loops are thus obtained, each carrying a current equal to half the total armature current, and magnetising the core in a general vertical direction. Hence, whether the armature be ring or drum-wound, lines of flux due to the armature current would flow through the two halves of the core on either side of the diameter of commutation, issue from the lower surface and circle round to enter again at the upper surface; or, as it is otherwise expressed, poles are formed on the armature at opposite ends of the diameter of commutation.*

* In the case of the ring armature some lines will cross the interior of the ring, passing through the shaft and supporting hub (or spider), as indicated in Fig. 312.

Apart from the field-magnet, these lines of flux will be comparatively few in number (as shown in Fig. 312), but when the armature is surrounded by the iron pole-pieces their length of path in the air is enormously reduced, and a strong flow will arise, passing through the two pole-pieces and traversing the short air-gaps of the dynamo. Fig. 315 serves to show the course of the increased number of lines due to the currents in the active wires when the armature is placed within the embrace of the pole-pieces with its diameter of commutation coinciding with the interpolar line of symmetry, the field being otherwise entirely unmagnetised. When the armature is rotated the active wires pass successively from one half of the armature into the other, but the current sheets remain fixed in position on either side of the diameter of commutation, and therefore the field of lines which they produce will likewise remain stationary and unchanging. The magnetomotive force of the armature ampere-turns rises as the closed path round which it acts is expanded from the central horizontal line towards the top and bottom of the armature core. Taking any angle, *roq*, the M.M.F. between the limits r and q is proportional to the number of ampere-turns which they include; thus if τ = the total number of active wires evenly spaced round the armature, and the current flowing in each be J, the magnetomotive forces of the ampere-turns summed up between r, $q = 1{\cdot}257 J\tau \frac{\theta}{360^\circ}$, where θ = the angle *roq* expressed in degrees. The greatest magnetomotive force is thus obtained between the extreme top and bottom of the core, and is there equal to $1{\cdot}257 \frac{J \cdot \tau}{2}$, or when expressed more generally so as to include multipolar machines $= 1{\cdot}257 \frac{J\tau}{2p}$ where p is the number of pairs of poles.

Taking any closed path, as between *rq*, the flux which traverses it is equal to the magnetomotive force of the armature ampere-turns within those points, divided by the reluctance of the double air-gap and of the iron through which the lines pass. At the centre of the pole-face there is no M.M.F. and no flux; thence the induction rises to a maximum at the pole-tips. Outside the limits of the polar arc the armature M.M.F. continues to increase up to its maximum on the line of symmetry, but for l_g must now be substituted the increased length of air-path, namely $(l_g + \xi x)$, as explained in Chapter XV. § 6, so that the density decreases to a second minimum but not to zero on the line of symmetry. If the armature be imagined to be cut across and opened out, and the values of the density in the air-gap of the lines due to the armature alone be plotted for each point on its surface, the curve of flux-density so obtained will have the shape shown by the dotted line of Fig. 316.

§ 4. **Superposition of armature field on main field.**—Let us suppose that the sheets of armature current shown in Fig. 315 are due to rotation of the armature as a dynamo, say, in a counter-clockwise direction; then, in order to produce armature currents in the direction shown, there must be a N. pole on the right and a S. pole on the left hand, *i.e.* such a field as that shown in Fig. 59. Let that edge or corner of the pole under which an active wire first enters after passing through the interpolar gap be termed the "leading" edge as opposed to the "trailing" edge from under which it emerges into the interpolar gap. If we now compare together the general direction of the two sets of lines, namely, those of Fig. 59, that would be due to the field-magnet by itself, and those of Fig. 315, due to the armature ampere-turns alone, it will be found that the direction of the lines due solely to the armature current is immediately opposed to the direction of the field-magnet

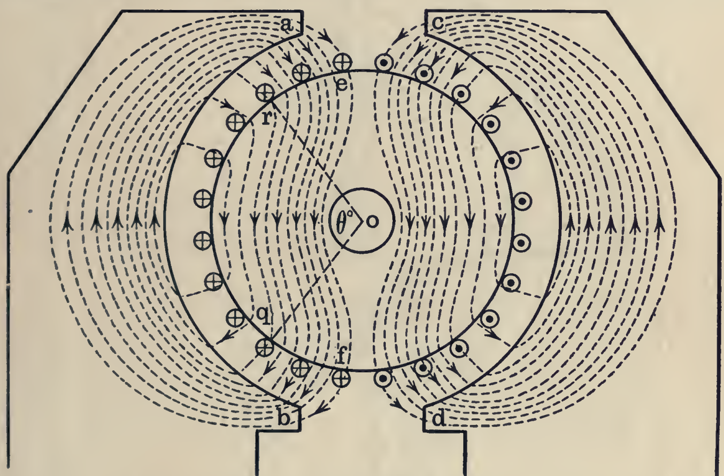


FIG. 315.—Magnetic field of armature alone when placed within pole-pieces.

lines in the air-gap under the leading half of each pole-face as, *e.g.*, under the two leading pole-tips (*a*, *d*), while they are in the same direction in the trailing half of each air-gap as, *e.g.*, under the two trailing pole-tips (*b*, *c*). But when the dynamo is at work and supplying current, the magnetomotive forces of both the field ampere-turns and the armature ampere-turns are simultaneously present, and in nature there is only one resultant distribution of field due to the combined effect of the two acting together. The resultant field which satisfies all the conditions may be arrived at by an imaginary superposition of the two sets of lines, the one on the other, followed by a process of compounding the two vectorially, with due regard to their directions, sense, and strength. Since over the two air-gaps the direction, although not necessarily the sense, of the lines due to the current-turns of the armature exactly coincides with that of the main field lines, they may be directly added together, account being taken of their respective algebraic signs; but it must then be borne in mind that the reluctance of the iron at any part of the circuit must be that which corresponds to the resultant density, so that owing to the change of the reluctivity of the iron the value of each of the component sets may be different from that which it would have even with the same M.M.F. if only one set were

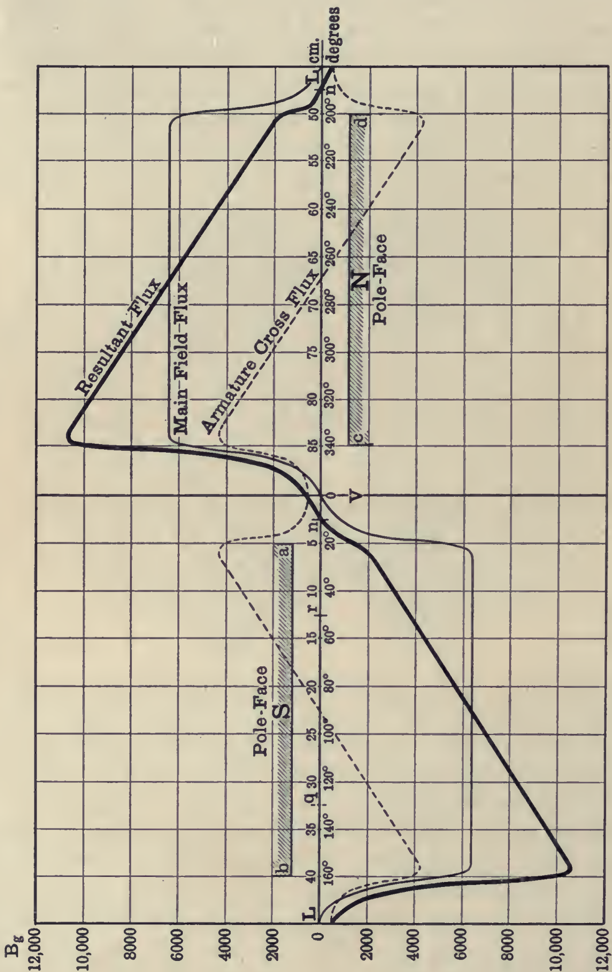


FIG. 316.—Flux distribution in smooth-core armature under load.

present. Assuming, then, that the dotted curve of Fig. 316 gives the flux-density in the air-gap due to the armature alone acting on the iron circuit in its resultant state, and similarly that the thin-line curve shows the density in the air-gap of the symmetrical flux due to the field-magnet turns acting alone on the circuit in its resultant state, the algebraic addition of the two fluxes yields the curve of resultant air-gap induction shown by the thick line of Fig. 316. At the centre of the pole-face the induction has its normal value $B_g = \frac{1{\cdot}257 X_g}{2 l_g}$, and on the one side it rises to a maximum B_g'' at the trailing pole-tip *b*, while on the other side it decreases to B_g', the slope within the limits of the pole being uniform as shown by the inclined straight line. In the other half of the armature the distribution of the field in the air-gap is altered in exactly the same way, but now the leading pole-corner where the induction is below the average is at *d*, while the trailing half where the induction is above the average is at *c*.

Since the system of armature lines passes through the core and pole-piece in a direction across that of the main field, it is termed the

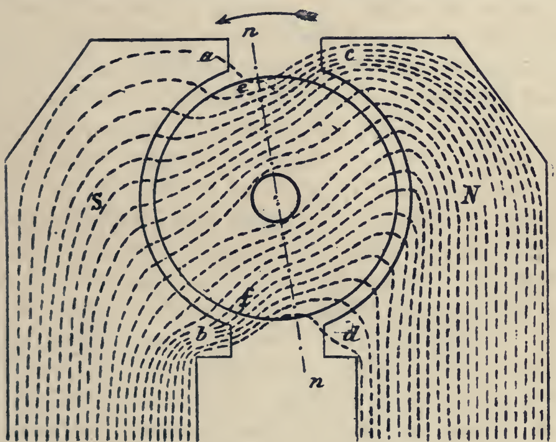


FIG. 317.—Magnetic field of dynamo reacted on by armature current.

cross-flux, and the ampere-turns which produce it are known as *the cross ampere-turns of the armature.*

The general result of the reaction of the armature currents on the field is shown in Fig. 317, from which it will be seen that the flux is, as it were, displaced and twisted round in the direction of rotation. The distribution of the field or the induction in the air-gap on either side of the armature, instead of being uniform over the greater part of the bored face of the pole-piece, varies continuously from a minimum value at *a e* to a maximum at *b f*; the densest part of the field is shifted up to and beyond the trailing pole-tip, while the field under the leading pole-corner is weakened. The distortion of the resultant field now corresponds to the inductance of the armature current-turns. If the armature current were reduced in strength, the lines would straighten themselves and cut the active wires, giving a forward E.M.F., and tending to keep up the previous value of the current. In so doing the stored energy of the armature current-turns, due to their reaction on the magnetic field, would reappear. The greater the current passed through the armature, the greater will be the forward displacement or twisting round of the field, and any such displacement implies a corresponding displacement of the resultant neutral line *nn* where the lines change their direction in relation to the armature surface, as is evident from Figs. 316 and 317, where *nn* is seen to be shifted forwards in advance of the vertical line of symmetry midway between the poles.

§ 5. **Determination of curves of main, cross, and resultant flux with iron assumed of constant permeability.**—The quantitative values have still to be considered in closer detail, and the method by which the curves may be plotted. Provided that the iron portion of the cross circuit is of constant permeability, and so is not affected by the actual value of the resultant density, the M.M.F. of the cross ampere-turns within any angle *roq* is divided equally between the two halves of the cross path, and the distribution of the cross flux on either side of the pole centre is symmetrical. Even if the permeability is variable, the same theoretical conditions—namely, equal division of the M.M.F. into two halves and symmetry of the cross flux on either side of the pole-centre—are practically attained when the permeability of the iron is initially high, *i.e.* when the armature is far from saturation and the area across the narrowest parts of the pole-pieces is considerable, so that there is no throttling of the lines, either across the neck or at the pole-corners. Almost the whole of the cross magnetomotive force may then be regarded as expended in producing the cross flux in the two air-gaps traversed by each set of lines on either side of the armature, the influence of the iron portion of the path becoming negligible. Under these circumstances the density of each component flux at any point under the pole becomes simply equal to its appropriate M.M.F. divided by the air-gap. Or correspondingly, in order to determine the resultant induction, it becomes permissible to *add together algebraically* the two M.M.F.'s and divide by the length of the path in air.

The magnetomotive force impressed by the field-winding on each air-gap between the surfaces of a pole and the armature core is $1{\cdot}257\,\frac{X_g}{2}$.

The magnetomotive force due to the cross turns acting on the single air-gap at any point r is $1{\cdot}257\,\frac{J\tau}{2}\cdot\frac{\theta}{360^\circ}$. The resultant induction at the point is therefore

$$1{\cdot}257\left(\frac{X_g}{2}\pm\frac{J\tau}{2}\cdot\frac{\theta}{360^\circ}\right)\div l_g$$

The most important pair of symmetrical points are those at the leading and trailing pole-tips, or approximately between *a,e* and *f,b*. Here the polar angle ϕ may be substituted for θ, and the resultant induction at the leading pole-tip is very closely

$$B_g' = \frac{1{\cdot}257\left(\frac{X_g}{2} - \frac{J\tau}{2} \cdot \frac{\phi}{360^\circ}\right)}{l_g} \quad . \quad . \quad . \quad . \quad (85)$$

and at the trailing pole-tip, where the cross M.M.F. assists that of the field excitation,

$$B_g'' = \frac{1{\cdot}257\left(\frac{X_g}{2} + \frac{J\tau}{2} \cdot \frac{\phi}{360^\circ}\right)}{l_g} \quad . \quad . \quad . \quad . \quad (86)$$

The two values gradually approximate as we approach the centre of the pole-face, where the induction again has its normal value $B_g = \frac{1{\cdot}257 X_g}{2 l_g}$. Such is the case of the smooth-surface armature in which the air-gaps are of considerable length and their reluctance entirely preponderates over that of the iron. An example of a smooth-surface armature in which the influence of the iron can be neglected is presented in Fig. 316, of which the numerical data are as follows. The pole-pitch of the armature is 45 centimetres, and the pole-width is 0·78 of the pole-pitch, or 35 centimetres, so that, twice the pole-pitch being reckoned as equal to 360°, the polar angle is 140°. The normal induction in the air-gap is $B_g = 6400$, and $l_g = 0{\cdot}84$ centimetre, so that $X_g = 0{\cdot}8 \times 6400 \times 0{\cdot}84 \times 2 = 8550$. Let the cross ampere-wires within the pole-pitch reach the high value of 7950, so that the maximum cross M.M.F. of the armature is $1{\cdot}257 \times 7950$, and the cross M.M.F. acting between the pole-tips is $1{\cdot}257 \times 7950 \times 0{\cdot}78 = 1{\cdot}257 \times 6200$. The density of the cross flux under the pole for any point distant x centimetres from its centre is then $1{\cdot}257 \frac{6200}{2 l_g} \cdot \frac{2x}{35} = 1{\cdot}257 \frac{355x}{2 l_g}$, and the resultant density with main and cross flux superposed is $B = \frac{1{\cdot}257(8550 \mp 355x)}{1{\cdot}68}$.

While the formulæ of equations (85) and (86) closely approximate to the facts, they are not strictly correct owing to the slight reduction of the density at the extreme edges of the poles, which has been pointed out in Chapter XV. § 6, and which virtually has the same effect as if the value of l_g were at the edges slightly increased. In order to deal with this, and to complete satisfactorily the exploration of the flux-distribution in the smooth-surface armature throughout the interpolar gaps, which is of especial importance from its bearing on the problem of sparkless commutation, it is convenient to convert the curves of Fig. 264 into a corresponding set of reluctance curves by taking the reciprocal values of the ordinates; thus Fig. 318, being the reciprocal of the curve of Fig. 264 for $\frac{c}{l_g} = \infty$, gives the length of the air-path at any point in the interpolar region and at the extreme edges of the pole in terms of the normal air-gap; while at the top are added the corresponding approximate factors by which this ratio must be multiplied when the angle at which the pole-edge is inclined to the armature surface is other than a right angle. The use of this curve in relation to the cross flux is then as follows. In the present case $c = 5$ centimetres. Thence $\frac{c}{l_g} = \frac{5}{0{\cdot}84} = 5{\cdot}95$.

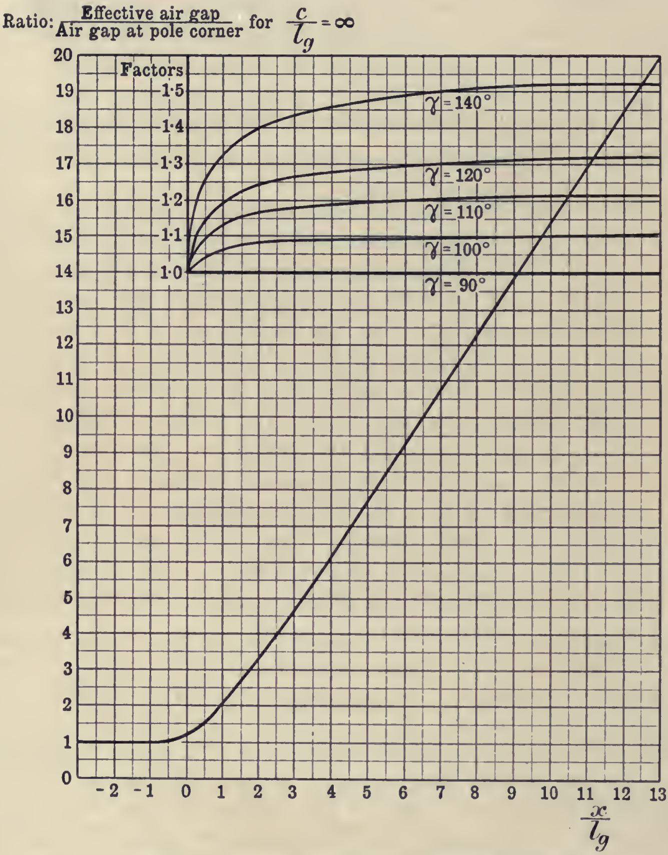


FIG. 318.—Curve of effective air-length of fringe.

For the armature cross flux the curve of Fig. 264 for $\frac{c}{l_g} = \infty$ is to be used up to the point where the abscissa $\frac{x}{l_g} = 5{\cdot}95$, after which the flux is as it were picked up by the neighbouring pole, and the flux-curve is repeated symmetrically up to the second pole-edge. Thus on the line of symmetry the reluctance of the air-path reaches its maximum of 10, this figure being obtained when the value 9·21 corresponding to $\frac{x}{l} = 5{\cdot}95$ is multiplied by the normal air-gap = 0·84 centimetre, and also by the factor 1·295 to allow for the receding pole-edge assumed to make an angle of 120° with the armature surface. Other points are similarly calculated, and the dotted curve of Fig. 319 is obtained which shows how the length of the air-path declines from its maximum of 10 to its

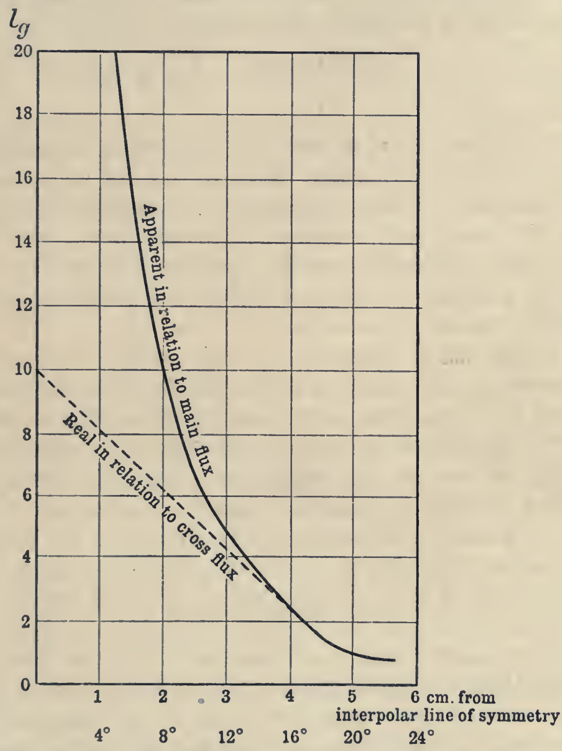


FIG. 319.—Real and apparent air-lengths of fringe.

normal value of 0·84 at a distance of about 0·84 centimetre within the pole-corner.

In the same way, from Fig. 264 is obtained the apparent length of the air-path for each point between the line of symmetry and the pole-corner in relation to the main field; a curve for $\frac{c}{l_g} = 5{\cdot}95$ must for this purpose be interpolated near to the curve for $\frac{c}{l_g} = 5$. The result is shown in the full-line curve of Fig. 319, from which it will be seen how the action of the neighbouring pole opposing the first pole causes the apparent length of path to approach infinity at the symmetrical centre, or to become asymptotic to the vertical axis.

More accurately, therefore, the resultant induction at each spot within the leading interpolar region is

$$B_g' = \frac{\text{Field M.M.F.}}{2l_{gx}(\text{apparent})} - \frac{\text{cross M.M.F.}}{2l_{gx}(\text{real})}$$

or if the apparent length of path in relation to the main field is q times the real length in relation to the cross flux,

$$B_g' = \frac{1}{2l_{gx}(\text{real})}\left\{\frac{\text{Field M.M.F.}}{q} - \text{cross M.M.F.}\right\} \quad . \quad . \quad (85a)$$

With this correction the neutral line is found to be displaced forwards about 10° or 2·5 centimetres ahead of the line of symmetry, *i.e.* to a position midway between the centre of the interpolar gap and the pole-edge. The distortion is very marked, the maximum value of the air-gap density close within the trailing pole-edge being $B_g'' = 6400 + 4370 = 10{,}770$, and the corresponding value close within the leading pole-tip being $B_g' = 6400 - 4370 = 2030$.

If in addition to the double air-gap we were to consider a depth of say 4 centimetres of iron both on the pole-face and on the armature core on either side, somewhat as indicated by the shaded bands of Fig. 320, and if the assumption be made that the iron is of constant permeability, he bands of iron are put on the same footing as the air-gaps. The difference of magnetic potential between the extremes of the shaded regions in relation to the main flux would exceed that over the air-gaps alone by say $f(B) \times 8 \times 2 = 2{\cdot}62 \times 16 = 42$ C.G.S. units. The permeability of the iron is here implicitly given as $\mu = \frac{6400}{2{\cdot}62} = 2440$. Then, since this is assumed to be constant, and if for the sake of simplicity the small M.M.F. required to pass the cross flux upwards or downwards from the one half of the pole or armature to the other half be neglected, the maximum density of the cross flux one centimetre from the extreme edge of the pole-tip becomes $B = \dfrac{1{\cdot}257 \times 355 \times 16{\cdot}5}{1{\cdot}68 + \dfrac{16}{2440}} = \dfrac{7350}{1{\cdot}68 + 0{\cdot}0065}$ $= 4355$, as compared with 4370 in the preceding paragraph, from which the small influence of the iron portion of the path under these conditions becomes evident.

§ 6. **No diminution of total flux from cross turns with iron assumed of constant permeability.**—So far it has been assumed that the brushes are given no forward lead, *i.e.* that the diameter of commutation coincides with the vertical line of symmetry, and the materials of the circuit have been supposed to have a constant permeability. Under these conditions it is now evident that the *average* induction in the air-gap, or, taking into account the length of the armature, the total number of lines entering into it on the one side and leaving on the other, is entirely unaffected by the current-turns of the armature, since the induction is as much strengthened over the one half of each polar face as it is weakened over the other half, and the area of the resultant field curve is equal to the area of the undistorted field-curve. At any two points, such as r, q (Figs. 315 and 316), equidistant on either side of the line through the centre of a pole, the cross density has the same numerical value, but in the one case must be deducted from and in the other case added to the density of the main flux.

An entirely distinct question still remains, namely, whether owing to the distortion of the field there is any reduction of the E.M.F. of the armature even though the total flux may remain the same. The brushes being assumed to be on the interpolar line of symmetry, all the active wires within the angle by which the neutral line has been shifted ahead of the line of symmetry are positively harmful, so that on this account there would be a loss of E.M.F. But on the other hand, this back E.M.F. is balanced by an equal increase in the forward E.M.F. of the active wires within the same angle behind the symmetrical line. There is in this case, therefore, no net loss of E.M.F. in spite of the displacement of the resultant field,—a result which may also easily be arrived at by the following consideration. Since the E.M.F. of a drum armature as a whole is proportional to the algebraic sum of the resultant flux embraced within a loop situated under the brushes, if this resultant flux is reproduced by the superposition of two separate flux-distribution curves it is evident that the E.M.F. of the armature may equally well be calculated by considering the E.M.F. given separately by each of the two component flux-curves, each being summed up algebraically within the embrace of the short-circuited loop. The cross flux being symmetrical with respect to the diameter of commutation, there is the same number of cross lines between the pole-centre and one side of the loop as between the pole-centre and the other side of the loop, but their direction is opposite, so that they cancel out. There is therefore no net effect from the cross flux, and the initial symmetrical main field remaining unaltered, there is no reduction of the E.M.F. in spite of the fact that the diameter of commutation does not correspond with the position of the resultant neutral line.

There is, however, a disturbing cause present even in the smooth-core armature, since the trailing pole-tip and the armature iron beneath it become highly saturated, while at the leading pole-tip the flux-density is weakened. The same cause affects the toothed armature to a much greater degree, owing to its shorter air-gaps and the greater distortion of the flux; the problem is also further complicated by the teeth, which are initially worked with a high value for their normal induction. The effect of the change of permeability of the iron needs then to be examined, and first let the smooth armature be considered. It is clear that with the brushes on the line of symmetry the total flux cannot be increased by the armature current, since, when the circuit is closed and the current arises, the process is one of progressive distortion of the resultant flux, and no separate system of lines, linked solely with the armature turns, is actually called into being (save around the end-connectors, which are not here under consideration). But as the displacement grows with a rising armature current, the permeabilities of the two corners alter, and even when the one falls and the other rises the change may take place to different degrees in the two. Thus Fig. 320 may be taken to represent the case of an increasing reluctivity as the trailing pole-corners are approached; the pole face and armature surface are to be regarded as made up of a number of strips of different materials gradually increasing in reluctivity as indicated by the increasing depth of the shading when we pass from one quarter to another. The possibility, therefore, of a reduction in the total flux due to the permeability of the one corner being reduced by more than the amount of the rise in the other must be investigated. Or again, the permeability of both corners might become reduced to the same or different degrees.

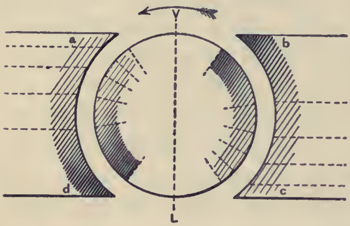


FIG. 320.

At any moment the actual resultant flux must contain combined within itself both the flux that would be produced by the field winding if acting alone on the circuit with its permeabilities in their resultant state as due to the resultant distribution, and likewise the cross flux that would be due to the cross armature turns if they also were acting alone on the circuit in its given resultant state. The distribution can therefore only be traced by assuming a certain change in the permeabilities, determining each component flux separately and subsequently combining them. By a series of approximations one could thus finally obtain such a resultant distribution as agrees with the resultant permeabilities that have been assumed.

§ 7. **Reduction of field in smooth armature with iron of varying permeability negligible.**—In the smooth-core armature a peculiarity is rendered evident from Fig. 133. When the normal density in the air-gap and in the pole-face or armature surface (each being of practically equal area with the air-gap) is in the neighbourhood of $B = 6000$, it is not far from corresponding with the point of maximum permeability of the iron or steel. Hence when distortion of the resultant field takes place, the new values of μ at the two pole-corners are in both cases lower than the original values for the undistorted field, and Fig. 320 does not represent the true facts of such a case. Further, for considerable changes on either side of the maximum point there is no great divergence between the reduced values. Roughly speaking then, under the approximate assumption that the value of μ at each pole-corner is alike, both the main and the cross field remain symmetrical about the centre of a pole, but both are slightly reduced. A few trials show that for the same cross M.M.F. of 7350, and the same main M.M.F. of 10,792 over the air-gaps and shaded iron bands, the conditions are satisfied if the permeabilities at the two corners are reduced to 1700. At the extreme edges the density of the main flux is now 6390, and from the new equation $B\left(1{\cdot}68 + \frac{16}{1700}\right) = B(1{\cdot}68 + 0{\cdot}0094) = 7350$, the maximum density of the cross flux is $B = 4350$. The resultant inductions at the two corners thus become $B_g' = 6390 - 4350 = 2040$, and $B_g'' = 6390 + 4350 = 10{,}740$. The shape of the main component curve of Fig. 316 now becomes slightly bowed, but owing to the preponderating influence of the two air-gaps the reduction of the total flux is practically negligible, since even at the two corners it is only reduced from 6400 to 6390. There is thus a very small reduction of the E.M.F. even with the brushes on the symmetrical line, owing to the main flux being slightly reduced. But as the component curve of the cross flux still remains symmetrical on either side of the pole-centre, and so is symmetrical in relation to the diameter of commutation, there is no *further* loss of E.M.F. even though the diameter of commutation does not coincide with the resultant neutral line. In short, with the brushes retained on the line of symmetry, the cross flux can only affect the E.M.F. by altering the magnetic state of the iron, and so influencing the main symmetrical lines.

§ 8. **Effect of forward lead of the diameter of commutation.**—The effect of advancing the diameter of commutation forwards beyond the line of symmetry has now to be considered, and will be found to introduce an entirely new result. So long as the two coincide, the armature ampere-turns all tended to produce a cross-magnetisation, the general direction of which in the armature was at right angles to the direction of magnetisation due to the field alone. But now, when the brushes are given a forward lead, if the directions of the currents in the active wires on the armature be compared with the directions of the currents in the magnetising coils of the field, as shown by dotted and crossed circles in Fig. 321, it will be seen that in all the active wires lying between the two vertical lines *k l*, *m u* the direction of the current is opposed to the direction of the current in those field-magnet wires which are on the same side of the lines of the field; and further, that the loops between *km* embrace practically the entire magnetic circuit. The active wires may now be regarded as connected together across the ends of the armature in two sets of turns, one forming a coil in a vertical and the other a coil in a horizontal plane. These are seen in Fig. 322, the imaginary end-

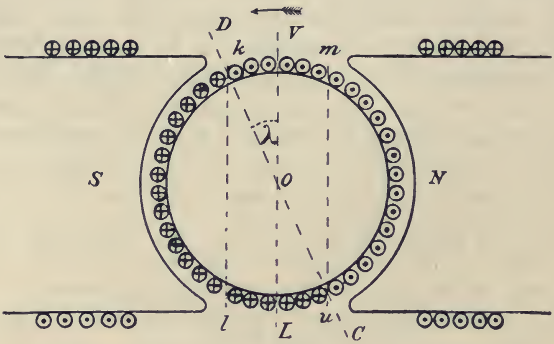


FIG. 321.—Ampere-turns of field and armature.

connections of the vertical coil being shown by full lines, and those of the horizontal coil by dotted lines. Thus wires 1 and 2 are equivalent to one turn or loop, round which a current of $\frac{C_a}{2}$ amperes flows, and similarly wires 3 and 4 are equivalent to another turn, but in a plane at right angles to the former. The entire number of armature ampere-turns is divided into two sets, and we are justified in so dividing them, since the effects of the two are largely distinct. The two vertical lines of Fig. 321 are fixed in position by the *angle of lead*, the angle *kom* or *lou* being equal to twice the angle of lead λ of the diameter of commutation, reckoned from the vertical diameter or line of symmetry. The belt of ampere-turns included between the lines *kl*, *mu* are in direct opposition to the ampere-turns of the field, and tend to demagnetise the armature and field; they are therefore known as the *back ampere-turns* of the armature. On the other hand, the belt of ampere-turns lying horizontally between the points *kl* or *mu* have the same effect as was previously considered when the diameter of commutation coincided with the line of symmetry; that is, they have a cross-magnetising tendency, which distorts the field round in the direction of rotation, weakening it at the leading pole-corners and strengthening

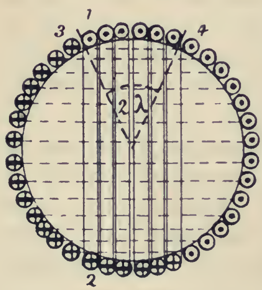


FIG. 322.—Back and cross ampere-turns of armature.

it at the trailing pole-corners, so that they are still "cross ampere-turns." Every conductor not situated on the neutral line or at right angles to it has both a cross and a direct magnetising tendency, but of those situated under one pole-piece the forward tendency of one half is balanced by the back effect on the other half, leaving them pure cross loops, while of those between the pole-tips the cross tendency of the one half is balanced by the other half, which tend to lessen the distortion, so that only their back effect is summed up.

Fig. 323 shows in the case of a multipolar dynamo the analogous division of the armature ampere-turns into the two belts of back and cross turns. The back ampere-turns acting on the magnetic circuit of a 2-pole dynamo or on each magnetic circuit in a multi-

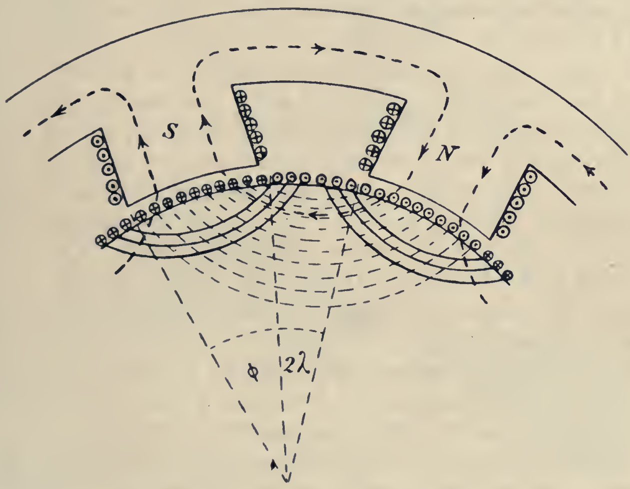


FIG. 323.—Back and cross ampere-turns of multipolar armature.

polar machine are equal to the product of the current in any one active wire multiplied by the number of active wires inclosed within twice the angle of lead, or if λ be expressed in degrees,

$$X_b = J\tau \cdot \frac{2\lambda}{360^\circ}. \quad \quad (87)$$

With the given angle of lead the maximum value of the cross ampere-turns is conversely

$$J\tau \cdot \frac{\theta'}{360^\circ} \quad \quad (88)$$

where θ' is also expressed in degrees and is $= \frac{360^\circ}{2p} - 2\lambda$. The relative proportion of these two sets of ampere-turns will depend entirely upon the angle of lead. Together they form a total number of magnetising turns $\frac{J\tau}{2p}$, or the current-sheet corresponding to the one pole, and the number of such sheets is of course equal to the number of poles.

In order to determine the resultant distribution of the field round a smooth-surface armature when the brushes are given an angle of lead, the same principles as in § 5 are applicable, but now in the first place must be plotted the symmetrical main field *as weakened by the back ampere-turns of the armature* within 2λ, and with this must subsequently be combined the cross flux due to the cross ampere-turns within the angle θ'. At each spot the component flux-density is equal to the M.M.F. divided by the appropriate length of air-path, real or apparent, as explained in § 5. But as opposed to the case of § 5 when the brushes were on the line of symmetry, the net M.M.F. which causes the symmetrical main flux and which is $1{\cdot}257X_g$ over the pole-faces within the arc *lk*, must now, owing to the lead of the brushes, be made to increase gradually within the arc *k*V; as soon as the diameter of commutation is passed and we approach the line of symmetry, in proportion as the back ampere-turns are left behind the symmetrical M.M.F. rises until it reaches a maximum value of $1{\cdot}257\,(X_g+X_b)$ for a point midway between the poles. On the other hand, the M.M.F. of the cross ampere-turns of the armature rises to a maximum at the diameter of commutation as fixed by the brush position, and then remains constant up to the line of symmetry. If, in the same armature as before considered in § 5, it be assumed that the back ampere-turns are compensated by compound winding or by increase of the shunt exciting current, and if the brushes are given an angle of lead of 10°, corresponding to 2·5 centimetres on the armature core, or half-way between the line of symmetry and the pole-corner, the main field M.M.F. rises from 10,750 at a point 2·5 centimetres ahead of the line of symmetry up to a maximum of $10{,}750+1110=11{,}860$ on the line of symmetry. The cross M.M.F. rises from 7800 between the pole-edges up to a maximum of 8900 at the diameter of commutation, and thence remains constant. Since we have assumed the back ampere-turns of the armature to be compensated by an equal increase in the field ampere-turns, the resultant curve under the poles within the arc *lk* or *mu* is entirely unaffected, and the only change from Fig. 316 would be a small difference within *km* or *lu*. There is, however, a further difference of some importance as affecting the value of the armature E.M.F. When the brushes were retained upon the line of symmetry, the advance of the neutral line of zero field ahead did not affect the armature E.M.F. But now that the brushes are given an angle of lead, even if the field excitation is increased so as to keep the total flux unaltered, there is only one position of the brushes which will yield the same E.M.F., namely, when they again overtake the neutral line. In any other position some portion of the flux embraced within a loop in the position of short-circuit is cancelled out by an equal portion having the opposite direction in regard to the loop.

This effect of the lead of the brushes upon the E.M.F. is best followed in detail by again analysing the resultant flux after the method of § 6, but into its three components and tracing the separate E.M.F.'s which result from their relation to a loop when in the position of short-circuit, wherever this happens to be fixed by the diameter of commutation (Fig. 324). In addition to the initial symmetrical field which

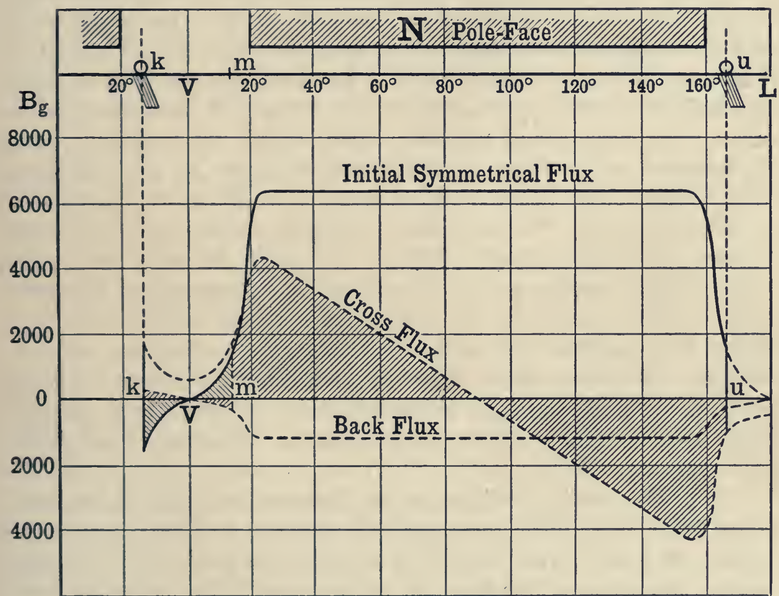


FIG. 324.—Components of the resultant E.M.F. curve of smooth armature with distorted field.

would be due to the exciting field turns if acting alone, there is also present another symmetrical field of opposite direction due to the back ampere-turns of the armature. Although both are symmetrical with respect to a pole-face, they are no longer so with respect to the diameter of commutation. With the same field excitation, therefore, not only is there a back E.M.F. from the net amount of the back flux embraced within the short-circuited loop, but there is also a reduction of the E.M.F. from the original symmetrical field due to the fact that the lines within the angle of lead *k*V (Figs. 324 and 320) ahead of the line of symmetry cancel out the same number of lines within an equal angle V*m* behind the line of symmetry. The cross flux is also not symmetrical with respect to the diameter of commutation, although symmetrical with respect to the poles; but as opposed to the main field, the net amount of cross flux embraced within the loop is equal to the number of lines within twice the angle of lead, *i.e.* to the number within the arc kV *plus* the same number within an equal arc Vm. Further, the E.M.F. which would result from this cross flux is a forward E.M.F. The net effect from the three components is therefore simply a question of the relative amount of the loss from the main field and the gain from the cross flux. If the polar arc were nearly equal to the pole-pitch (leakage of the lines immediately across from one pole-edge to the other being imagined to be absent), so that the length of the air-gap was uniform round the entire armature, it will be found that whatever the angle of lead, the net amount of flux from the back ampere-turns embraced within the short-circuited loop would be precisely equal and opposite to the net amount of cross flux from the cross ampere-turns, and the two would cancel one another. There is, however, still left the reduction of E.M.F. due to the position of the brushes not being symmetrical in relation to the original symmetrical field. The reduction of E.M.F. is thus under the above assumption simply that which is due to the angle of lead of the brushes in the original undistorted field, and could be compensated by increasing the field excitation by the amount of the back turns so as to again bring the resultant neutral line exactly into coincidence with the diameter of commutation. In short, the distortion does not in itself affect the E.M.F.,—a result which might be expected, since otherwise conductors carrying a steady current would in effect have been cutting their own lines.

In practice, owing to the wide gap between the pole-tips, the length of air-path presented to the cross flux within the arc km is many times greater than the normal length of air-path presented to the back flux under the pole-faces. Hence the net amount of the cross flux within the short-circuited loop, *i.e.* the weak fringe of the cross field outside the pole-faces, is practically negligible as compared with the net amount of the back flux under the pole-faces, and equality between the two would only begin to hold in the extreme case of an angle of lead of 45° or more, when both the cross flux between km and the back flux between mu would pass through circuits of approximately the same area and of reluctance determined chiefly by the length of the double air-gap, $2l_g$. Apart from such extreme cases which are not of practical occurrence, the effect of the cross flux may be neglected, and we simply have to deduct the back flux from the original symmetrical field, and so obtain a symmetrical field as weakened by the back ampere-turns of the armature. Or to put the same in other words, if we consider the three superposed fields as acting upon the wires in their several instantaneous positions, the magnetic effect of the back ampere-wires preponderates over their electrical effect as cutting the cross flux. It thus suffices in practice to return simply to the original symmetrical field as weakened by the back ampere-turns, and to deduce the resultant armature E.M.F. from this field, any gain or loss of E.M.F. owing to the brush position not being symmetrical with it being neglected unless the angle of lead is abnormally great.

§ 9. **Effect of backward trail of the diameter of commutation.**—If in a dynamo the brushes be shifted in the reverse direction, and be given an *angle of trail* behind the line of symmetry, the effect of the direct magnetising turns on the armature is also reversed; instead of being back ampere-turns, they now become *forward ampere-turns*, assisting in the magnetisation of the armature.

There is therefore a forward E.M.F. from the amount of the flux added to the initial symmetrical field within the embrace of the short-circuited loop. So, too, the effect of the cross flux within twice the angle of trail is reversed, and the E.M.F. which thence results is a back E.M.F. On the other hand, the reduction in the E.M.F. from the initial symmetrical field, due to the flux within the arc between the lines of symmetry and commutation cancelling out an equal amount of flux on the other side of the line of symmetry, still holds good. Hence in the imaginary case of a polar arc equal to the pole-pitch, this latter reduction of E.M.F. would alone remain. But in practice, for the reason explained in § 9, the forward flux would for a considerable angle of trail greatly preponderate over the weak fringe of the cross field within the interpolar gap. The total E.M.F. is therefore simply a question of the difference between the gain from the forward flux and the loss from the unsymmetrical position of the brushes in relation to the initial symmetrical field. For small movement of the brushes away from the symmetrical line there is a net gain, but eventually with larger angles of trail a balance is reached after which the E.M.F. diminishes. With small angles of trail there is also the additional advantage that the leakage is reduced for the same total value of the armature flux; the magnetising turns are wound more uniformly over the magnetic circuit, since in effect some of the magnetising turns now embrace the rotating armature instead of being all concentrated upon the magnet, and this reduces the leakage. Unless, however, special devices are used in connection with the commutation,* a backward trail of the brushes in a dynamo is always accompanied by such excessive sparking that it cannot be used in practice. It should be especially noted that the conductors which assist in increasing the total flux themselves induce a back E.M.F., although it may be of but small amount. And this is in consonance with the principle that the active wires of an armature cannot themselves increase its E.M.F. when the current which

* As in Sayers' regenerative dynamos, Brit. Pat. Nos. 16,572 (1891) and 10,298 (1893).

they are carrying is increased. It is quite true that if the brushes of a dynamo are shifted backwards, the armature may be made to excite its own field without the presence of any field-winding proper; the action is started by residual magnetism, and magnified by the exciting turns which are obtained on the armature, but these same turns cannot also actively generate a useful E.M.F. It is only the case that their magnetic effect may be more potent than their electrical effect.

§ 10. **Effect of varying permeability of teeth in a slotted armature.**—Returning to the case of a toothed armature with the brushes on the line of symmetry, when current flows through the winding, and distortion commences, the density in the trailing teeth increases and their permeability falls, while the reverse takes place in the leading teeth. If the density of either the main field or of the cross field for an equal width of arc at each end of the pole-face were alike, the new values of the resultant density for any two symmetrical points could be easily calculated. But since in reality (as contrasted with the case of § 7) the change of permeability of the teeth results in values differing from one another at the two corners, or at any two symmetrical points, the one being reduced more than the other is increased, the effect upon either the main field or the cross field considered separately is to concentrate them towards the leading corner where the permeability is higher, and to lessen the density on the trailing side where the permeability is lower. The lines of the main flux are, as it were, drawn towards the leading corner and are thinned out towards the trailing corner, so that the curve of

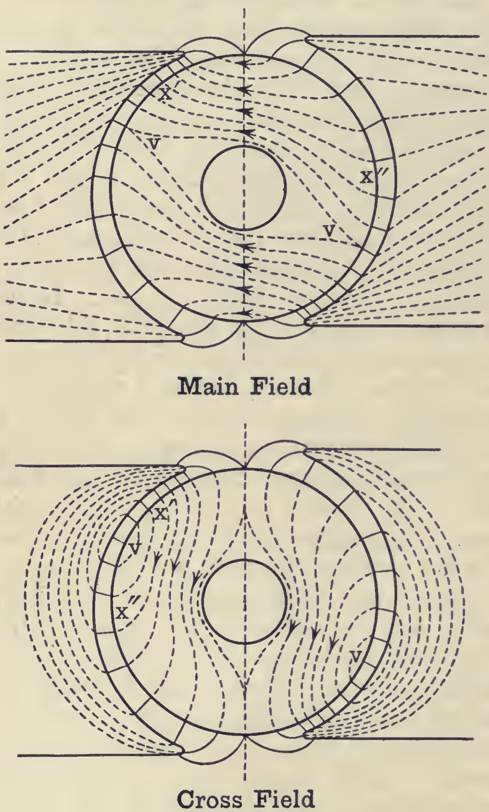


FIG. 325.—Main and cross fields with eccentric air-gaps.

induction in the air-gap is no longer symmetrical about the centre of the pole-face. The cross flux is also displaced in an analogous manner, and its central point on its own neutral line is shifted backwards against the direction of rotation. The effect on either of the two component fluxes is exactly analogous to that which would be produced if the air-gap was progressively shortened towards each leading pole-corner and lengthened towards each trailing corner; and this method of illustration has been adopted in Fig. 325, the longer air-gap representing the equivalent of the higher reluctance of the teeth on the trailing side.

The surface of the armature may in such a case be considered as divided into a number of compartments each of which carries the same flux; thus in Fig. 320 the two compartments of different widths opposite to *a* and *b* are in series in respect to the main flux, while the two compartments opposite to *a* and *d* are in series with respect to the cross flux, and further, *b* and *d* are precisely alike. Considering any tube of force, it is evident that it crosses a gap of air and iron teeth of high reluctance, and then a gap of lower reluctance in series. Under these conditions, given a certain excitation $X_g + X_t$ available between the limits of the pole faces and roots of the teeth, it will be found that as opposed to the case of § 7 where the reluctance was symmetrical about the centre of the pole-face, the total difference of magnetic potential must be *unequally divided* between any two corresponding air-gaps which are in series in order that the total flux may be the maximum possible. Thus let the difference of magnetic potential impressed upon the armature between opposing pole-faces (the reluctance of the armature core being neglected) be $1{\cdot}257\ (X_g + X_t) = 15{,}720$, and let the reluctance per square centimetre of the path traversed by any one tube on the one side at x' be $\mathfrak{R}_{x'} = 0{\cdot}895$, and on the other side at x'' be $\mathfrak{R}_{x''} = 1{\cdot}131$. If we start with an equal loss of potential on either side, the flux-density of the tube at x' will be $\frac{7860}{0{\cdot}895} = 8790$. Only the same difference of potential remains available on the other side, so that, in order to pass the same amount of flux over the higher reluctance, the density must be reduced to $\frac{7860}{1{\cdot}131} = 6950$; in other words, the cross-sectional area of the tube must be increased in the proportion of $\frac{8790}{6950}$, by increasing its width to $1{\cdot}263$ centimetres. Since the same distribution must also hold in the second half of the same pole, we have thus in effect used up a width of $2{\cdot}263$ centimetres to pass a flux of 8790×2 lines, giving an average density of 7750. If the result of reserving some portion of the M.M.F. $\frac{1{\cdot}257\,(X_g + X_t)}{2}$ from the side of lower reluctance, and expending it upon the side of higher reluctance is tried in the same way, the average density is raised, until a maximum is reached, when the difference of potential is divided over $\mathfrak{R}_{x'}$ and $\mathfrak{R}_{x''}$ in the proportion of $\sqrt{\mathfrak{R}_{x'}} : \sqrt{\mathfrak{R}_{x''}}$, or as $1 : \sqrt{m'}$, where m' is the ratio $\frac{\mathfrak{R}_{x''}}{\mathfrak{R}_{x'}}$ of the two reluctances per square centimetre of path; *i.e.* the potentials expended over the two portions of the circuit at x' and x'' must be $\frac{1{\cdot}257\,(X_g+X_t)\,.\,\sqrt{\mathfrak{R}_{x'}}}{(\sqrt{\mathfrak{R}_{x'}}+\sqrt{\mathfrak{R}_{x''}})}$ and $\frac{1{\cdot}257\,(X_g+X_t)\,.\,\sqrt{\mathfrak{R}_{x''}}}{(\sqrt{\mathfrak{R}_{x'}}+\sqrt{\mathfrak{R}_{x''}})}$, or in the above case 7400 and 8320. The densities on the two sides are then as $1 : \frac{1}{\sqrt{m'}}$, and the area of the tube at x'' is $\sqrt{m'}$ times that at x'.

This general law follows at once from mathematical considerations. If the cross-section of the tube on the one side is y times that on the other, the two reluctances in series are $\mathfrak{R}_{x'} + \mathfrak{R}_{x'} \,.\, \frac{m'}{y}$, and the flux through the considered tube with a given X is $\frac{1{\cdot}257\,X}{\mathfrak{R}_{x'} + \mathfrak{R}_{x'} \,.\, \frac{m'}{y}}$. The average width of the two strips is $\frac{1+y}{2}$, so that the average density is $\frac{1{\cdot}257\,X\,.\,2y}{\mathfrak{R}_{x'}(y+m')(1+y)}$. Since X is a constant, and $\mathfrak{R}_{x'}$ and m' are given, this average density becomes a maximum when $\frac{y^2+y\,(m'+1)+m'}{y}$ is a minimum, and this occurs when $y = \sqrt{m'}$.

In the above a basis of 1 square centimetre of area on the side of lower reluctance was taken for convenience, but it is evident that this may be indefinitely reduced without affecting the ratios.

Thus with reluctance varying continuously from one edge to the other of the pole-face, the tubes have in general a different cross-section in the two stages of their path. There is, however, one tube of the main field which has the same width on the two sides, and the position of this tube at v (Fig. 325) divides the total flux into two equal parts; at the same time it also marks the spot where the cross flux is zero, and where the resultant density of the field has the normal value which it had before distortion by the cross flux.

§ 11. **Approximate analysis of distribution in a toothed armature.**—In order to estimate the effect of armature reaction upon the total main flux in a toothed armature, a case should be taken in which the teeth are long and highly saturated and vary but little in width; *i.e.* the armature should have deep slots and be of fairly large diameter. Let the armature be 150 cm. in diameter, 8-pole and with 220 slots each 4 cm. deep and 1·07 cm. wide. Let the ratio of the polar arc to the pole-pitch be $\beta = 0\cdot75$, so that the polar angle is $33^\circ\cdot75$, or measured on the armature core the arc is 44·2 cm., the pole-pitch being 59 cm. Let the single air-gap be $\frac{5}{16}'' = 0\cdot795$ cm.; the ratios $\frac{w_{t1}}{w_s}$ and $\frac{w_s}{l_g}$ are thus $\frac{1\cdot07}{1\cdot07} = 1$, and $\frac{1\cdot07}{0\cdot795} = 1\cdot346$, whence from Fig. 273 the coefficient m is 1·11, and the equivalent length of the air-gap is $m \,.\, l_g = 1\cdot11 \times 0\cdot795 = 0\cdot882$ cm. The virtual breadth of the air-gap area is $A' + K_2 \,.\, l_g = 46\cdot23$, or 46 cm. measured on the armature core, and in this are embraced 21·45 teeth. The number of lines corresponding to one tooth and per 1 cm. of length across the core is therefore $z = B_g \,.\, \frac{46}{21\cdot45} = 2\cdot14\, B_g$. The widths of the tooth are 1·07 cm. at the tip, 1·015 cm. at the centre, and 0·96 cm. at the root. Allowing $12\frac{1}{2}$ per cent. for the insulation between the discs, the area of iron per tooth and per 1 cm. of axial length is 0·875 time the above widths. Up to air-gap densities of about $B_g = 7000$, *i.e.* until the teeth begin to become saturated, the number of lines passing through the iron teeth will be about 77 per cent. of the total flux at the top and 93 per cent. at the centre; beyond this value of B_g the real density will diverge from the apparent, while for the root of the tooth the real density must in all cases be calculated from the apparent density when all the lines are assumed to pass through the iron. In the present case

$$\text{at the tip} \quad \frac{2\cdot14\, B_g \times 0\cdot77}{0\cdot875 \times 1\cdot07} = 1\cdot76\, B_g$$

$$\text{at the centre} \quad \frac{2\cdot14\, B_g \times 0\cdot93}{0\cdot875 \times 1\cdot015} = 2\cdot24\, B_g$$

$$\text{at the root} \quad \frac{2\cdot14\, B_g}{0\cdot875 \times 0\cdot96} = 2\cdot54\, B_g$$

and since the normal air-gap density is as high as $B_g = 8250$, the above values may be taken as the apparent densities in the iron tooth at tip, centre, and root, namely, 14,500, 18,480, and 20,950. The real densities at centre and root for the particular values of $\frac{w_t}{w_s} = \frac{1\cdot015}{1\cdot07} = 0\cdot95$, and $\frac{w_{t2}}{w_s} = \frac{0\cdot96}{1\cdot07} = 0\cdot896$ are by Fig. 277 reduced to 18,350 and 20,400. The average M.M.F. per centimetre length of tooth by equation (69) and from Fig. 132 is thus $\frac{20 + 140 \times 4 + 300}{6} = 147$, and the total over $l_t = 4$ cm. is 588. The difference of magnetic potential expended over the single air-gap is $B_g \,.\, ml_g = 8250 \times 0\cdot882 = 7270$, so that the total M.M.F. over gap and teeth on one side is normally $7270 + 588 = 7858$, or say 15,720 over the air-gaps and teeth on the two sides.

For the purpose of comparison at different densities, the reluctance of the teeth is conveniently reduced to its equivalent per square centimetre of cross-section of the path; the reluctance of the single air-gap per square centimetre is of course equal to its virtual length ml_g and is a constant $=0{\cdot}882$, but the reluctance of the teeth per square centimetre of path is for each density $\frac{\text{mean H}\times l_t}{B_g}$ on each side of the armature; *e.g.* with the normal density it is $\frac{147\times 4}{8250}=0{\cdot}0715$, so that the reluctance of the air-gap and teeth in series on one side is then $0{\cdot}882+0{\cdot}0715=0{\cdot}9535$ per square centimetre. In a similar way can be worked out a series of values for the M.M.F. expended over the teeth, and the reluctance per square centimetre of path through air-gap and teeth for different densities. These are plotted in Fig. 326.

A number of correlative values for the leading and trailing densities

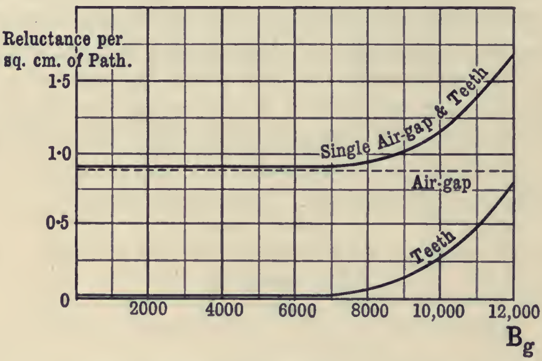


FIG. 326.—Reluctance per square centimetre of air-gap and teeth.

in the air-gap can now be calculated, by the principles described in § 10; each pair of values corresponds to two points x' and x'', the one forming the entrance point and the other the exit point of the same tube of flux. Certain resultant densities are in the first place assumed, and the two component fluxes are calculated; the value of one of the former must then be corrected until the resultants of the components agree with the assumed densities and reluctances, these latter being always altered to agree with any alteration in the value assumed for the resultant density.

Let it be assumed that at some point x'' on the trailing side of the pole-face the resultant density in the air-gap is $B_g''=10{,}000$, and that at the correlative point x' on the leading side $B_g'=6200$. The ratio of the two reluctances per square centimetre for these two densities is then by Fig. 326 $m'=\frac{\mathcal{R}_{x''}}{\mathcal{R}_{x'}}=\frac{1{\cdot}131}{0{\cdot}895}=1{\cdot}266$, and $\sqrt{m'}=1{\cdot}126$. The total reluctance of two corresponding compartments in series is $0{\cdot}895$ $(1+1{\cdot}126)=1{\cdot}9$. The main flux produced by $1{\cdot}257\ (X_g+X_t)=15{,}720$ is $\frac{15720}{1{\cdot}9}=8260$ distributed over a strip 1 centimetre wide on the leading side, and $1{\cdot}126$ centimetre wide on the trailing side, so that the two densities are 8260 on the leading and $\frac{8260}{1{\cdot}126}=7350$ on the trailing side, the average being always less than the normal. At the trailing side, in order that the resultant density in the air-gap may there have the value which has been assumed for it, the density of the component cross flux must be $10{,}000-7350=2650$, and its total amount in the wider compartment on this trailing side will be $2650\times1{\cdot}126=2980$. Since these lines are concentrated within one centimetre on the leading side, their density is then 2980, and the resultant density, being the difference between the main and cross components, would be $8260-2980=5280$. But this does not agree with the assumed value of $B_g{}'=6200$, so that the two assumed resultant densities are not possible corresponding values, and one or other with its reluctance must be altered. The permeability of the teeth on the leading side is so high that they have but little effect on the reluctance, and the calculated value of $B_g{}'$ will be but little raised when a new lower value of $\mathfrak{R}_{x'}$ is inserted. Hence if we assume $B_g{}'=5315$, and $\mathfrak{R}_{x'}=0{\cdot}89$, $m'=\frac{1{\cdot}131}{0{\cdot}89}$ $=1{\cdot}273$, and $\sqrt{m'}=1{\cdot}129$. The total reluctance is now $0{\cdot}89\ (1+1{\cdot}129)$ $=1{\cdot}892$; the main flux is thereby increased to 8300 with a component density $\frac{8300}{1{\cdot}129}=7355$ on the trailing side. The density of the component cross flux now becomes $10{,}000-7355=2645$ on the trailing side, and its total amount $2645\times1{\cdot}129=2985$, whence the resultant density on the leading side is $B_g{}'=8300-2985=5315$. Thus two corresponding values of $B_g{}'$ and $B_g{}''$ have been found, namely, 10,000 and 5315, and their distance apart must be such that the cross M.M.F. between them is $2985\times1{\cdot}892=5660$ C.G.S. units.

With a larger amount of distortion and a greater resultant density in the air-gap at the trailing side it is evident that the reluctance of air-gap and teeth per square centimetre on the leading side approximates more and more closely to that of the air-gap alone, or to $0{\cdot}882$, so that it becomes almost unnecessary to calculate it. Thus let the resultant density on the trailing side be $B_g{}''=11{,}080$, for which $\mathfrak{R}_{x''}=1{\cdot}44$. $\mathfrak{R}_{x'}$ on the leading side may then be assumed as $0{\cdot}885$, since $B_g{}'$ will in any case be less than 5000. Thence $m'=\frac{1{\cdot}44}{0{\cdot}885}=1{\cdot}628$, and $\sqrt{m'}=1{\cdot}276$. The joint reluctance on the two sides is now $0{\cdot}885$ $(1+1{\cdot}276)=2{\cdot}012$, and the flux of the main field is $\frac{15720}{2{\cdot}012}=7820$ with component densities of $\frac{7820}{1\cdot276}=6130$ on the trailing side, and of 7820 on the leading side. The density of the cross flux on the trailing side must therefore be $11,080-6130=4950$, and its total amount $4950\times1\cdot276=6310$. When concentrated in one centimetre width on the leading side, this gives for the resultant density $B_g'=7820-6310=1510$. Two more corresponding values of the resultant densities in the air-gap have thus been found, and their distance apart must be such that the cross M.M.F. between the points is $6310\times2\cdot012=12,700$. The resultant induction B_g' on the leading side, namely, 1510, is as low as should be allowed close up to the leading pole-tip, say at a distance of $l_g=0\cdot795$ centimetre within the tip, from which point the virtual length of the air-path rapidly increases and the induction falls; it is lower than that which was previously found for the smooth-core armature of § 7, and it may be taken as approximating to the minimum value permissible in the practical case of a toothed dynamo armature with carbon brushes. For reasons to be explained in the next chapter, it is advisable that the resultant B_g' at the leading pole-tip should not fall below a certain minimum, and although by the use of carbon brushes this minimum may be lower than with copper gauze brushes, yet the induction at the leading pole-tip should not be reversed or even reduced to zero (see Chap. XVIII. § 34). The distance between the last two corresponding points may therefore be taken as equal to the polar arc less a width equal to the air-gap at either end, *i.e.* $44\cdot2-0\cdot795\times2=42\cdot6$ centimetres. The total cross M.M.F. of the armature corresponding to the pole-pitch would therefore be $12,700\times\frac{59}{42\cdot6}=17,550$, given by $\frac{17,550}{1\cdot257}=14,000$ ampere-turns, or ampere-wires within the pole-pitch, *i.e.* 237 ampere-wires per centimetre of periphery of the armature core, and this will form the maximum load which armature reaction permits it to carry.

A series of corresponding densities with their distances apart may thus be approximately calculated, but the complete curve of air-gap induction over the pole-pitch cannot yet be plotted, since their exact location is not known. Indeed, it has been assumed without proof that if the density close within the leading pole-tip is 1510, the corresponding point would fall similarly close within the trailing pole-tip. A method of relating together the pairs of corresponding points is required, and also a starting-point from which to reckon the distances.

§ 12. **Curves of flux-distribution in air-gap of toothed armature.**—At any two corresponding points the flux-densities as calculated above are given by the expressions $\frac{1\cdot257\,X}{\sqrt{\mathfrak{R}_{x'}}\cdot(\sqrt{\mathfrak{R}_{x'}}+\sqrt{\mathfrak{R}_{x''}})}$ and $\frac{1\cdot257\,X}{\sqrt{\mathfrak{R}_{x''}}\cdot(\sqrt{\mathfrak{R}_{x'}}\times\sqrt{\mathfrak{R}_{x''}})}$, where X is for the case of the main field the constant excitation which is available for the air-gaps and teeth, and in the case of the cross flux is the ampere-turns between the two points in question. At some one point in the interpolar gap, or near thereto, the lines of the component fields change their direction, those of the main field just dipping into the core, and those of the cross field bifurcating into either one of the two neighbouring pole-corners. Starting from this point, let x_1 and x_2 be any two distances on the armature core in centimetres, the first in the direction of the leading side and the second in the direction of the trailing side. Then if these two points are so chosen as to be a pair of correlative points x' and x'' such that the lines which, *e.g.*, enter in at x' issue forth from x'', the integral of the component flux, whether main or cross, up to x', must be equal to the integral of the flux up to x'', the two groups of lines finally meeting at some point v which forms the virtual centre dividing them into two halves. The integrals of the fluxes up to any pair of correlative points are then

$$\int_{x_1=o}^{x_1=x'} \frac{1{\cdot}257\ \mathrm{X}}{\sqrt{\mathfrak{R}_{x'}}\cdot(\sqrt{\mathfrak{R}_{x'}}+\sqrt{\mathfrak{R}_{x''}})}\cdot dx_1$$
$$=\int_{x_2=o}^{x_2=x''} \frac{1{\cdot}257\ \mathrm{X}}{\sqrt{\mathfrak{R}_{x''}}\cdot(\sqrt{\mathfrak{R}_{x'}}+\sqrt{\mathfrak{R}_{x''}})}\cdot dx_2$$

or since for every corresponding step on either side $\sqrt{\mathfrak{R}_{x'}}+\sqrt{\mathfrak{R}_{x''}}$ although gradually increasing is equal on each side of the equation, and X is a constant, we have $\int_o^{x'} \frac{dx_1}{\sqrt{\mathfrak{R}_{x'}}} = \int_o^{x''} \frac{dx_2}{\sqrt{\mathfrak{R}_{x''}}}$

If therefore a curve of $\mathfrak{R}_x$ is plotted, and from this is derived a second curve of the reciprocal of $\sqrt{\mathfrak{R}_x}$ as in Fig. 327, any number of pairs of correlative points may be obtained by taking the area included within the curve up to a distance x' from the starting-point on the leading side and finding a distance x'' from the opposite end such that an equal area is included. Thus in Fig. 328 the area included within the $\frac{1}{\sqrt{\mathfrak{R}_x}}$ curve of Fig. 327 for any given distance from the starting-point at either end has been plotted in arbitrary units so as to give two derived curves of area. If any pair of points which lie on the same horizontal line, and so have the same ordinate, are taken, their abscissæ give corresponding distances x' and x'' from the point where the component fluxes change their direction, the two finally meeting at some point v which is not the centre of the pole.

The position of the starting-point is still needed, and this may be determined by the consideration that outside the pole-edges the value of the length of the air-path, whether real in relation to the cross flux or apparent in relation to the main flux, so quickly rises that the effect of the saturation of the teeth becomes insignificant by comparison.

If the curves, *e.g.*, of the real reluctance in relation to the cross flux are plotted for each of the pole-corners as in Fig. 319, and are superposed on one another, although divergent at the actual pole-edges, they quickly draw together as the pole-corners are left behind and coincide at a short distance therefrom; not only is the reluctance of the teeth a smaller percentage of the whole, but its actual amount at the trailing side decreases rapidly. Still more is this the case with the apparent reluctance in relation to the main field. In the analogous case of eccentric pole-pieces, where only air reluctances come into question and the iron effect is negligible, it can be shown that the

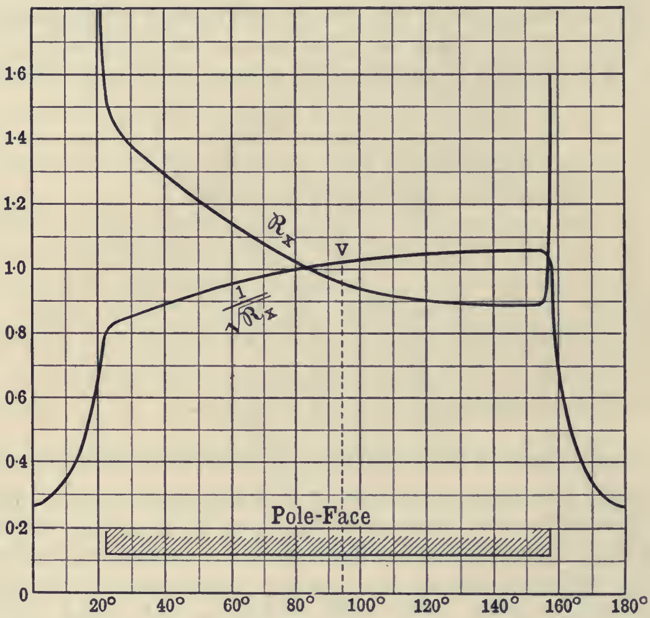


FIG. 327.

neutral line will shift towards the pole-edge having the smaller air-gap and higher flux-density, and this is borne out by experiment.* But in toothed armatures the exact location of the neutral line of the component fields is more difficult of prediction, since, as the resultant density decreases, the reluctance of the teeth will again begin to rise; on the resultant neutral line where the actual density is zero, the reluctance of the teeth per square centimetre may well be higher than in a corresponding portion on the trailing side where the resultant

* As in Fig. 24 of Dr. W. M. Thornton's paper on "The Distribution of Magnetic Induction and Hysteresis Loss in Armatures," *Journal Inst. Electr. Eng.*, vol. xxxvii. p. 125.

density is much greater, for the permeability will increase with the higher density. Yet any such effect is almost negligible, and with considerable accuracy the neutral line on which the cross flux divides and the main flux falls to zero may practically be identified with the line of symmetry between the poles. At this point the loss of potential over the path on the trailing side which has been decreasing after the trailing pole-corner is passed once again becomes equal to the loss of potential over the path on the leading side which has been increasing.

Starting, then, from the symmetrical line at either end of the pole-pitch, let the reluctance per sq. cm. for either flux calculated as in § 5 be plotted in relation to distance along the armature core. Up to a short

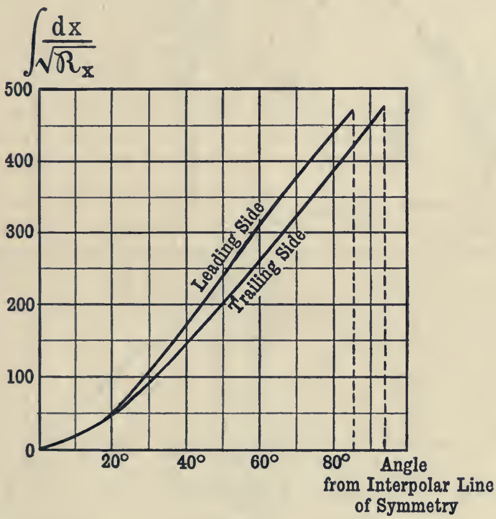


FIG 328.

distance from the pole-corners $\mathfrak{R}_{x_1}$ and $\mathfrak{R}_{x_2}$ will be practically identical, divergence only beginning close to the pole-tips. We are justified, therefore, in placing the two values which were last calculated numerically for $\mathfrak{R}_{x'}$ and $\mathfrak{R}_{x''}$ at a distance of l_g centimetres within each pole-corner, before the fringing outwards of the field has begun to affect the proper action of the different degree of saturation of the teeth at either end. The curve of $\mathfrak{R}_x$ can then be completed within the polar arc with close accuracy, the normal reluctance of air-gap and teeth being placed a short distance away from the centre of the pole towards the leading side. From the curve of $\frac{1}{\sqrt{\mathfrak{R}_x}}$ is obtained the two derived curves of Fig. 328, and thence the position of the corresponding points. Thus a pair of corresponding points are situated at 64° from the centre of the interpolar gap on the trailing side and 58° on the leading side, so that the distance between them is 122° − 64° = 58°, corresponding to the first pair of densities which were above calculated. The complete curves of Fig. 329 can then be plotted for either the main or cross field, and finally the curve of the resultant induction in the air-gap. It only remains to see that as we proceed the resultant

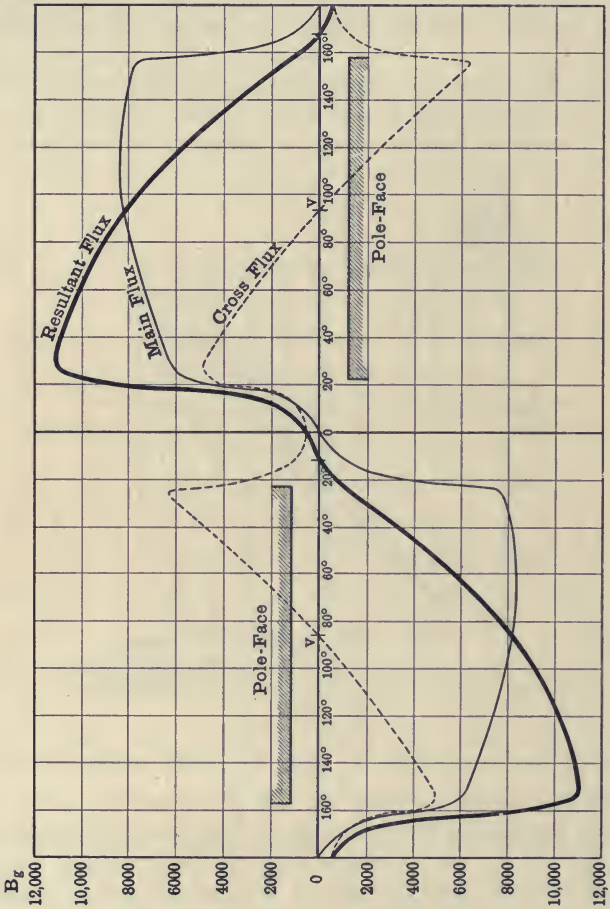


FIG. 329.—Distribution of flux in toothed armature under load.

densities agree with those assumed in Fig. 327, and to readjust these latter if need be.

The curves of Fig. 329 are thus plotted with each pole-pitch as a base line divided into 180° for purpose of comparison with Fig. 316. The virtual centre of the two component fields at *v*, where the cross flux is zero and the main field has its initial value before distortion, is seen to be shifted approximately 4° or 1·31 centimetre towards the leading side, the reluctance at this point having its normal value of 0·9535. The actual resultant density at the extreme corners of the poles is obtained from $m' = \frac{1{\cdot}52}{1{\cdot}2}$, or $\sqrt{m'} = 1{\cdot}125$. The reluctances in series are $1{\cdot}2\,(1 + 1{\cdot}125) = 2{\cdot}55$, whence the density of the main field at the trailing and leading corners respectively is 5485 and 6165, that of the cross flux is 4575 and 5150, and the resultant densities are $5485 + 4575 = 10{,}060$ and $6165 - 5150 = 1015$. The cross M.M.F. between the same points is $5150 \times 2{\cdot}55 = 13{,}150$, and the value of B_g' as above stated approaches the minimum that is permissible.

Although the ordinates of the two component curves taken separately are displaced backwards, yet their algebraic sum yields a resultant field which is throughout displaced forwards; as the armature current is increased, the virtual centre of the fields shifts backwards, yet this action only serves as a check upon the amount of distortion which would otherwise hold, and the resultant field is always shifted forwards. The density of the main flux on the leading side may exceed the initial density before distortion, but the average density of any two corresponding compartments is always slightly less than the initial, although it may be to a very small extent.

§ 13. **Amount of reduction of flux and of E.M.F.** The question of the amount of the reduction in the total value of the main flux bears directly upon the armature E.M.F. Since the brushes are assumed to be so far on the symmetrical line, the cross flux neither adds to nor subtracts from the E.M.F., and there are no back ampere-turns in question. Yet just as in the case of the smooth-core armature, the presence of the armature cross ampere-turns may affect the magnetic properties of the circuit through the saturation of one part of it. The E.M.F. of the toothed armature when carrying current as compared with that on open circuit for the same excitation is thus with the brushes on VL dependent solely upon the value of the total flux due to the field winding as acting upon the altered permeances of the circuit, and the only differences from the case of the smooth-core armature of § 7 are that the change of permeability is more considerable in the toothed armature, and that it causes the two component curves to become unsymmetrical with respect to the centre of the pole-face. Yet even in the toothed armature the reduction in the total value of the main flux is not very large.

In the case treated above, the reduction of the main flux for a constant interpolar excitation amounts to about $5\frac{1}{2}$ per cent. The assumption of a constant loss of potential over the teeth and air-gaps does not, however, strictly represent the actual facts in practice. It is the total excitation that would really be maintained constant in such a case, and if the main flux were reduced by some $5\frac{1}{2}$ per cent. the reduced loss of potential over the magnet core and yoke would set free probably as much as 700 additional ampere-turns to be expended between the poles. Actually, therefore, there would be an increase in the ampere-turns available for the air-gaps and teeth, amounting, say, to 400 AT., or 500 units of M.M.F., which will assist in maintaining the initial average density and total flux. Such a reduction as 3 or 4 per cent. probably represents the utmost that may be anticipated in a fully loaded and saturated armature, since the case considered was an extreme one, in so far that the teeth were well saturated over their whole length. This amount would therefore have to be allowed for in the E.M.F. equation by taking, say, 0·97 of the no-load Z_a.

§ 14. **Back ampere-turns of smooth and toothed armatures.**—In the case of smooth-surface armatures which necessarily have comparatively long air-gaps it has been shown that the reduction of the total flux even with maximum distortion of the field is practically negligible. In the equation, then, for the ampere-turns required on the field it is only necessary to introduce in the fourth place the value X_b as explained in Chapter XV. § 4. The correctness of the method is, however, limited to the ordinary cases of practice in which the brushes are seldom advanced farther than up to the pole-corner: under such conditions all the ampere-turns within twice the angle of lead so nearly embrace the entire magnetic circuit through the armature that they may be allowed their full weight, and must be counterbalanced by an equal number of forward ampere-turns on the field, $X_b = J\tau \cdot \frac{2\lambda}{360°}$. But if the brushes are advanced beyond the pole-edge and the lead becomes abnormal, some allowance must be made for the fact that the back ampere-turns are not all of equal efficacy, and for X_b must be substituted a proportion only, such as $J \cdot \tau \cdot \frac{2\lambda}{360°} \cdot \cos \lambda$. With the symmetrical distribution of the component fluxes that holds in smooth armatures, any angle of advance of the diameter of commutation implies a reduction of the E.M.F. proportional to the number of lines of the main field that is embraced between the two lines of commutation and of symmetry. When as usual the angle of lead does not extend beyond the pole-corner, only the symmetrical fringe from the pole-edge is lost, and this is so small a proportion that its effect is in most cases nearly inappreciable. But with a very large angle of lead the proportion of the active wires which give a back E.M.F. due to cutting the lines of the symmetrical main field in the wrong direction quickly increases; even if the magnet be separately excited and the value of the symmetrical field be maintained against the back ampere-turns, the net E.M.F. diminishes until with the brushes under the centre of the pole it is reduced to zero.

In the case of a toothed armature when the brushes are retained on the line of symmetry with a large armature current there may, as explained in § 13, be some reduction of the E.M.F. for the same excitation due to a reduction of the total main flux. It follows that, when there is some actual angle of lead, the X_b deduced from its value does not express the full effect of the armature reaction in decreasing the E.M.F., although the apparent increase of the back ampere-turns should rather be attributed to its proper cause, namely, the lesser value of Z_a as compared with its no-load value for the same net excitation.

For taking into account the whole effect of armature reaction in determining the field-winding, other methods have been proposed, mostly based on the composition of magnetomotive forces by a parallelogram law. They can, however, only be regarded at best as approximations to the actual facts. In default of time to complete during the process of design a detailed analysis, such as in § 12, it is as satisfactory and accurate to allow for the effect of distortion by adding a purely empirical amount to the excitation over air-gaps, teeth, and core which would suffice for a certain Z_a at no-load, as to adopt any of the approximate constructions that have been proposed. To determine the excitation at full-load, the course recommended in the case of high initial saturations of teeth or pole-tips would therefore be to calculate $X_g + X_t + X_a$ for the required full-load Z_a, but under the supposition of open circuit, to then increase X_t by 50 to 100 per cent. to allow for the effect of distortion in reducing the flux, and finally, by adding the calculated value of X_b, to obtain X_P acting between the poles and determining the leakage.

Both in the smooth and toothed armatures it must also be borne in mind that any eddy-currents in the armature exert a demagnetising effect on the field exactly analogous to that of the actual back ampere-turns. In the toothed armature, if the slots are wide and there are large eddy-currents in the pole-pieces, these again may require to be counterbalanced by an increase * which must be added to the item X_b in reckoning out the field-winding.

* See Niethammer, *E.T.Z.*, 1899, p. 768.

CHAPTER XVIII

COMMUTATION AND SPARKING AT THE BRUSHES

§ 1. **Sparking at the brushes.**—All who have had practical experience of the working of dynamos giving a continuous current will know that in most cases the brushes by which the current is collected are so mounted as to permit of their being shifted round the cylindrical surface of the commutator at least through some small angle. They will also be aware that if the position of the tips of the brushes, as they press on the commutator, be not properly adjusted, the result will be *sparking at the brushes.* The nature of these sparks is similar to the sparking which occurs on breaking an inductive circuit at a switch. As the edge of the blade leaves the jaws of the switch and their surface of contact diminishes, the current-density progressively rises; further the energy stored in the circuit in virtue of its inductance tends to keep up the value of the falling current; hence the local heat developed at the vanishing contact eventually raises the temperature of the metal to such a point that at the moment of rupture it becomes incandescent, and is volatilised, with the consequent formation of an arc in which the greater part of the stored energy of the circuit is finally expended. In the same way, sparking between the brushes and commutator sectors has its origin in an excessive current-density which continually recurs at their point of contact; and with the more or less sudden opening of the closed path in which current is still flowing, this culminates in a succession of small arcs between the moving sectors and the stationary brush. The waste of energy involved in such sparking is but small, but its presence always tends to shorten the life of the commutator and brushes, so that its suppression, so far as possible, is in every way desirable. The presence or possibility of sparking at the brushes is, in truth, the peculiar bane of continuous-current bi- or multi-polar dynamos, as contrasted with alternators; the current of the latter may require to be collected by brushes or rubbing-contacts such as have been shown in many previous diagrams; but the nicety of adjustment required by the brushes of hetero-polar continuous-current machines, if sparking is to be minimised or entirely avoided, is a disagreeable characteristic of their whole class, and is entirely due to the presence of the commutator as opposed to the simple collecting rings of the alternator. The requirement that the brushes of the continuous-current dynamo should be properly adjusted by no means expresses the full extent of the disadvantage; more than this, the exact position which the tips of the brushes should have on the commutator will in general depend on the amount of the current or load on the armature, and hence their setting may require to be varied to meet changes of the load. If the load on a dynamo were never altered the position of the brushes might be accurately adjusted once and for all, and in that position they might be fixed; but as the conditions of ordinary working are by no means those of constant load, a fixed position must be more or less of a compromise.

Unless, then, an approximately correct adjustment of the brush position can be obtained, a row of sparks will appear, leaping across between the commutator and the tips of the brushes. These sparks may be small, bluish-white in colour, and comparatively harmless; or if the inexactness of the adjustment be considerable, they may be of a reddish colour and extremely violent. But in either case, if allowed to continue, they will sooner or later pit the surface of the commutator sectors, destroy their smoothness and evenness, and heat the brushes. Once started, the effects are cumulative, and the mischief grows apace: the commutator becomes untrue and worn into deep and rugged grooves; increased sparking is caused by the "jumping" of the brushes as they pass from sector to sector, and perhaps the tips of the brushes become partially fused; thus the commutator is gradually eaten away until its state is past all remedy. To check the evil it is necessary to continually trim the brush-tips and "true up" the surface of the commutator by turning it in a lathe or by grinding it with an emery wheel, and these often-repeated processes result in a greatly reduced life of both commutator and brushes.

§ 2. **Means for adjustment of the brush position.**—Sparking in closed-coil armatures is therefore an unmitigated evil, to be overcome almost at any cost, and the possibility of moving the brushes so that the position of their tips may be adjusted to suit the normal working load is usually a necessity. It should further be possible to do this easily and steadily while the machine is running, and without in any way interrupting the passage of the current. In the 2-pole dynamo this is effected by mounting the brushes on two arms projecting horizontally from a cast-iron "rocking bar" or lever. This latter is made in two pieces, bolted together so as to fit into a groove turned in the surface of the bearing next to the commutator; it is then locked in position either by a set screw or more usually by a pinching screw, which clamps the two halves together and causes them to closely embrace the turned portion of the bearing. From opposite ends of the rocking lever there project two gun-metal spindles, entirely insulated from the iron by means of micanite or ambroin bushes, but rigidly held in position by means of nut and collar or by insulated screws; the brush boxes which hold the brushes are threaded on the spindles and firmly fixed in place. Fig. 330 shows the rocking-bar mounted in position on the bearing and carrying two sets of copper gauze brushes. Each brush box is fitted with a spring, by which the brushes are kept pressed down on the commutator, and also with a "hold off" catch, by which the brushes are held when their tips are raised off the commutator surface. When the pinching screw of the rocker is slightly loosened, the whole can be swung round

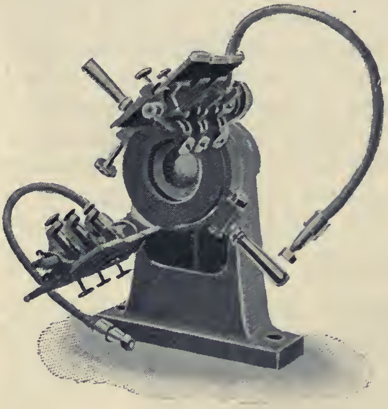


FIG. 330.—Two-pole brush gear with copper gauze brushes.

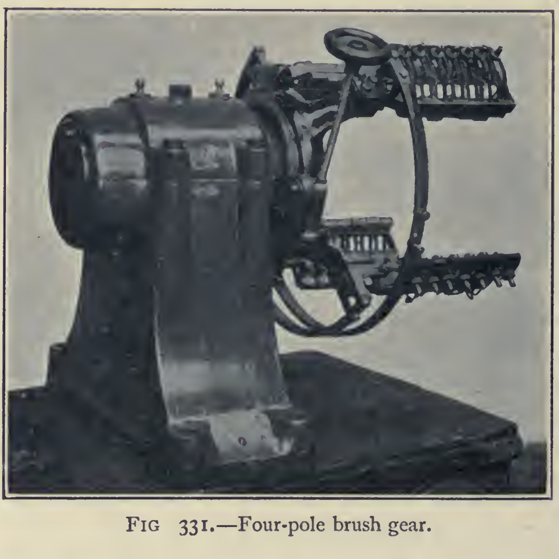


FIG 331.—Four-pole brush gear.

concentrically with the cylindrical commutator by means of a handle or handles on the lever.

In multipolar machines, using as many sets of brushes as there are poles, the rocker takes the more complicated shape of a star frame, on the projecting rays of which the insulated brush spindles are fixed, and it is then usual for the shifting to be effected by mechanical gearing. Figs. 331 and 332 show the brush gear of a 4-pole dynamo in place on the plummer-block, with four sets of carbon brushes, opposite pairs being connected together by copper rings; in the former the hand-

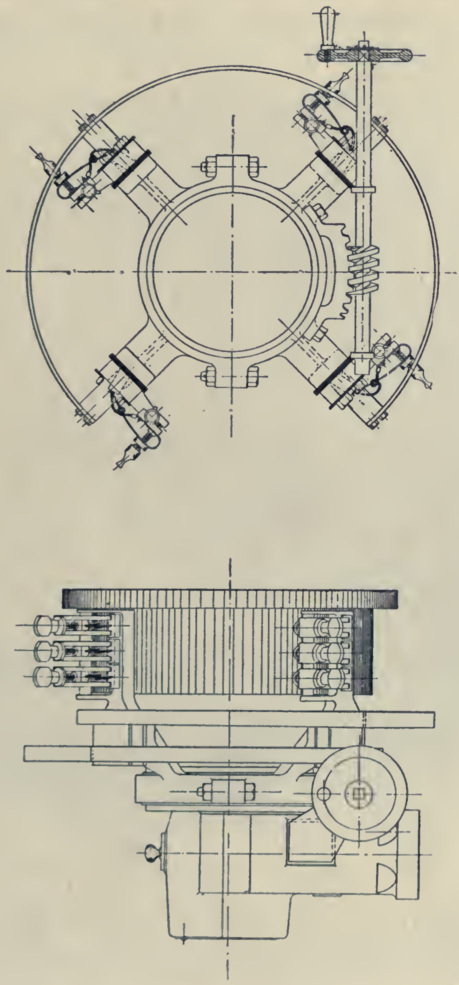


Fig. 332.—Four-pole brush gear.

wheel actuates a link connected to the rocker, while in the latter a worm gears into worm-wheel teeth on a sector fastened to the rocker. Fig. 333 shows the plummer-block for a 4-pole dynamo (manufactured by Messrs. Thomas Parker Ltd.), in which the brush-gear is moved by means of spur gearing. In large multipolar machines a cast-iron ring is frequently mounted on the face of the magnet yoke-ring, and rotated within the circular groove which forms its seating by similar mechanical gearing. From this ring project as many arms as there are poles, and to these are attached the brush spindles (cp. Figs. 334 and 335).

§ 3. **The process of short-circuiting a section.**—The exact nature of the process by which the current is commuted in a section of the winding of a closed-coil armature during the time when it is *short-circuited* has hitherto only been generally described. It has been shown

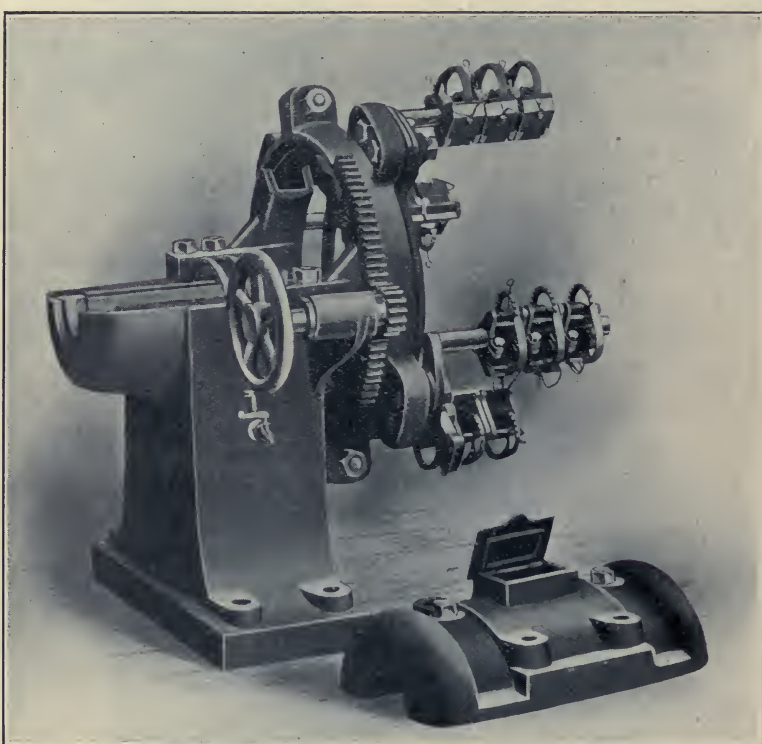


FIG. 333.—Four-pole brush gear. (Messrs. Thomas Parker Ltd.)

(Chap. XI. § 2) that the brushes must be placed so that they sum up the E.M.F.'s generated in the groups of sections forming the two or more parallels into which the winding is divided, and at the same time so that they short-circuit each separate section when it is passing approximately through the neutral gap between the poles, and has therefore little or no E.M.F. generated in it. Hence, if the field be bipolar their position will be at opposite ends of a diameter, corresponding roughly with a position of the short-circuited coils between the poles or on a line of symmetry at right angles to the general direction of the field. This preliminary description now requires to be further amplified.

Considering any one section of the armature winding (whether a single loop or a coil of many loops), terminated by connection to a commutator sector at either end, let us call that sector which first enters

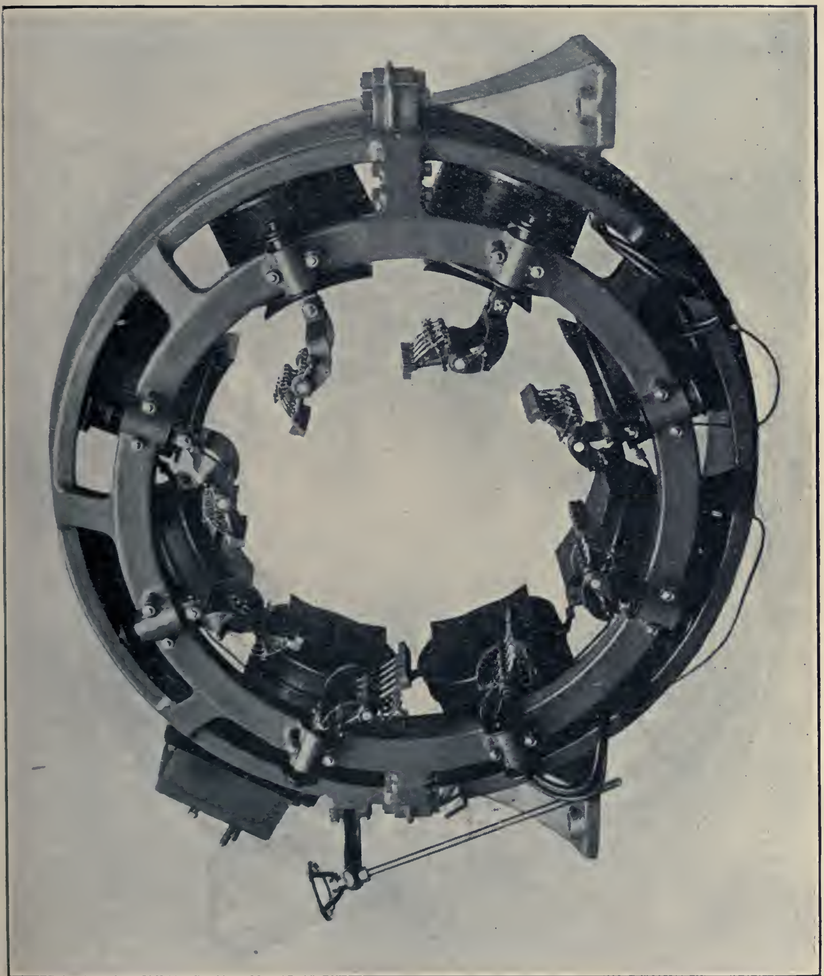


FIG. 334.—Eight-pole brush gear mounted on magnet frame.

under the edge of the stationary brush the "leading" sector of the coil, just as that edge or corner of a pole-piece under which a coil first enters after passing through the gap between two pole-pieces has already been called the "leading" edge, these being opposed respectively to the "trailing" sector and the "trailing" edge or corner.

Let the simplest case be taken in which the width of contact of the brushes on the circumference of the commutator is equal to or less than the width of a sector, so that there is never more than one section short-circuited at a time at each set of brushes. Then at any time during short-circuit, if i be the instantaneous value of the current in the

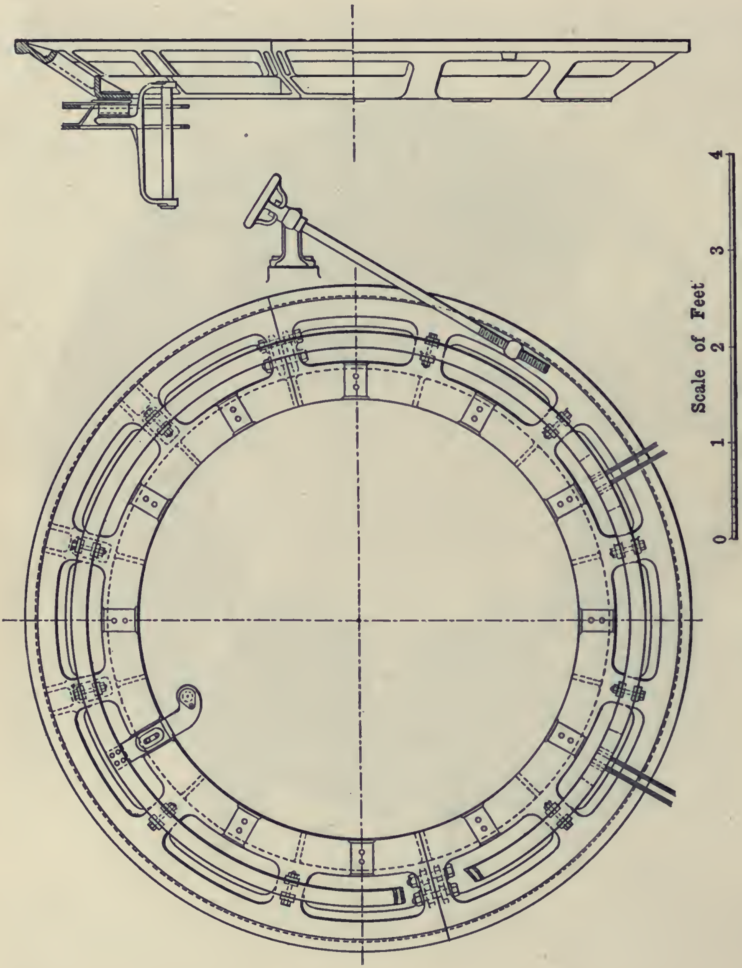


FIG. 335.—Twelve-pole brush carrier.

short-circuited coil, and J be the full current then flowing in any one branch of the armature $\left(=\frac{C_a}{2}\text{ or }\frac{C_a}{2p}\right.$, according as the armature is simple wave- or lap-wound or in general $\left.=\frac{C_a}{q}\right)$, the current in the leading sector 1 (Fig. 336) is $i_1 = J + i$, and in the trailing sector 2 is $i_2 = -J + i$, the direction of the current before commutation being taken as the positive direction round the short-circuit, and due regard being paid to the algebraic sign of i. Thus at the commencement of short-circuit, $i = J$, $i_1 = 2J$, and $i_2 = 0$. The total period of short-circuit, being the time that elapses between entrance of the trailing sector under the brush and emergence of the leading sector on the other side in the ordinary case of a simple lap-wound drum, is directly proportional to the thickness of the brushes less one strip of mica, and inversely proportional to the peripheral speed of the commutator; or in seconds,

$$T = \frac{(b_1 - m) \times 60}{\pi D_k N} \quad . \quad . \quad . \quad . \quad . \quad (89)$$

where b_1 = the width of the brush contact in the direction of rotation, m = the thickness of a mica strip, D_k = the diameter of the commutator,

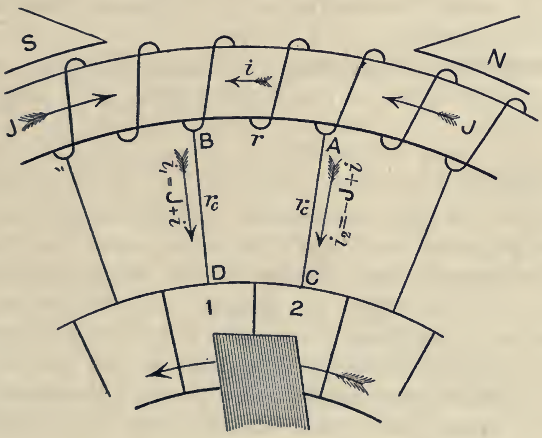


FIG. 336.—Commutation of current.

all expressed in the same units, and N = the number of revolutions per minute. It is usually but a small fraction of a second averaging from $\frac{1}{200}$th to $\frac{1}{1000}$th; *e.g.*, with carbon brushes $\frac{3}{4}''$ thick set so as to give practically the same width of contact (after deducting the thickness of a mica strip), on the circumference of a commutator $7\frac{1}{2}''$ in diameter and running at 900 revs. per minute, $T = \frac{0.75 \times 60}{3.14 \times 7\frac{1}{2} \times 900} = 0.00212$ second, or if v_k' = the peripheral speed of the commutator in feet per minute, *i.e.* $= \frac{\pi D_k N}{12}$, where D_k is the diameter of commutator in inches, and $b_1 - m$ is in inches, $T = 5 \cdot \frac{b_1 - m}{v_k'}$ seconds. During this brief time the current i must sink to zero and rise again in the reverse direction, *i.e.* with changed sign. If the reversed current be raised to exactly the same value at the end of short-circuit as it had at the beginning, $i = -J$, $i_1 = 0$, and $i_2 = -2J$. The opening of the short-circuit will then find the coil carrying the exact current flowing in the coils of the branch of the armature which it is to join, the whole of the current originally carried by BD and the leading sector having been withdrawn therefrom. Thus in a bipolar machine with 100 sections in two parallels over the resistance of which 5 volts are expended, a net effective voltage of $\frac{1}{10}$th of a volt must be acting on the short-circuited coil as it leaves the brush, in order to correspond with the passage through it of the normal current. The commutation will then be effected without any violent change, and consequently without any sparking. It is evident that the transference of a section of the armature from one branch of the winding into another on the leading side of the brush can only be sparkless if the original current in it is stopped, reversed, and further is reversed to exactly the same value in the opposite direction within the period of short-circuit.

§ 4. **Apparent inductance of short-circuited section.**—Now the current-turns of the short-circuited coil since they surround a portion of the magnetic circuit react on the field system, so that the number or distribution of the lines of the resultant field is different from what it would be if the short-circuited coils were absent. For the consideration of the problem of commutation, then, the coils which are at any time under the brushes must be mentally isolated from the rest of the winding, and their magnetic effect on the system of armature and field-magnet may be accounted for by crediting them with a certain flux of their own. Since the coils are situated at or near to the line of symmetry between the poles, most of the lines of this self-induced flux cross the double air-gap and enter or leave the iron pole-faces, and thence pass onwards through the field-magnet bobbins to complete their circuit. The field-magnet bobbins are practically short-circuited by the armature winding, and as the double air-gap is the chief item in the magnetic reluctance, the system is so far roughly equivalent to a transformer with an air-core and a short-circuited secondary. Under these circumstances, for rapid changes of current comparable with the frequency of commutation in a dynamo, the effect of the secondary exactly counterbalances the action of the short-circuited coil which is the primary, and the latter apparently has no self-inductance. Any change in the total flux linked with the exciting coils and the armature as a whole is, in fact, damped by the great mutual inductance existing between the short-circuited coil regarded as a primary and the field-coils as a secondary. There is, however, a certain smaller number of self-induced lines which do not pass through the field-coils, but which circle round through the air between the pole-tips, or pass immediately across the tops of the slots in which the short-circuited coils lie in the toothed armature. In virtue of these lines a certain "apparent" self-inductance may be attributed to the short-circuited coils, and the value of this is very much less than their true self-inductance.*

The question is, however, still more complex, and demands further analysis. Taking a single section of a drum armature, *i.e.* a coil with two sides, when undergoing short-circuit, there are always other coil-sides adjacent to it which are also short-circuited either at the same brush or at adjacent brushes on either side of it. In the case of a machine in which the number of commutator sectors per pole or $\frac{N_2}{2p}$ is a whole number, whatever is taking place at one set of brushes is also taking place at the adjacent brushes, so that close to the sides of the considered coil A, in which there is a varying current i, there must be the sides of two other coils B and B_x short-circuited at the adjacent brushes and each carrying an identical current, since they are at precisely the same stage in the process of commutation. In addition, therefore, to the E.M.F. from the apparent self-inductance of coil A, or $-L.\frac{di}{dt}$, there is also present in A the E.M.F. from its mutual inductance with coil-sides B and B_x, or $-M.\frac{di}{dt}$. Next, if as usual each brush-set covers more than the width of one section there are other sections A_{I}, A_{II}, etc., lying alongside A which are undergoing short-circuit at the same brush, but which have reached a different stage of commutation, and which are therefore carrying other currents i_{I}, i_{II}, etc. Lastly, there are other coils B_{I}, B_{II}, etc., of which one side is in close neighbourhood to coil A, and which are short-circuited at adjacent brushes on either side; from both of these groups there is mutual inductance M_{I}, M_{II}, etc., giving E.M.F.'s in A which may be concisely summed up as $-\Sigma.\left(M_{I}.\frac{di_{I}}{dt}\right)$. In this case also we need only consider the *apparent* mutual inductance, due to such lines as do not pass through the pole-faces and onwards through the yoke where the variations would be damped out by the exciting coils. Thus the total E.M.F. *of apparent self- and mutual-inductance* in the considered coil is

$$-(L+M)\frac{di}{dt}-\Sigma.\left(M_{I}.\frac{di_{I}}{dt}\right) \quad . \quad . \quad . \quad . \quad (90)$$

In the case of a machine in which $\frac{N_2}{2p}$ is an uneven number, M is absent, but the number of coils to be taken into account under $\Sigma\left(M_{I}.\frac{di_{I}}{dt}\right)$ is increased. What may be the effect of the second term in

* H. N. Allen, "Sparkless Reversal in Dynamos," *Journal Inst. Electr. Eng.*, vol. xxvii. p. 209.

the above expression (90) entirely depends upon the sign of $\frac{di_1}{dt}$ as compared with the sign of $\frac{di}{dt}$; whenever the rate of current-change in any one or more of the coils A_1, B_1, etc. is opposite in sign to the rate of change in the considered coil A, then the coils of which this is true simply have a damping effect, and take up some of the energy which is being freed from coil A, or if the current has reversed render its growth more rapid. But when the rate of current-change, say in coil B_1, has the same sign as $\frac{di}{dt}$, then they mutually support one another, and owing to the proximity of the coil B_1 the energy connected with the coil A and its apparent inductance are increased.

A further complication is that the apparent inductance will, strictly speaking, vary with the position of the short-circuited coil, according as it is in the centre of the interpolar gap or nearer to one pole-tip than to the other, and thus will change somewhat during rotation. Yet as the arc through which the coil moves during the period of short-circuit is but small, the *apparent self and mutual inductance* of a section of the winding may approximately be regarded as constant for a given position of the brushes.*

But however this may be and leaving for the present the further examination of the effect of $M_1 . \frac{di_1}{dt}$, the coil under consideration, at the moment of arrival at the point where short-circuit begins, is possessed of a certain amount of electromagnetic energy stored in its field equal to $\frac{(L+M)\,J^2}{2}$. The question of securing sparkless commutation turns, then, entirely upon our ability to dissipate and to re-store this amount of energy within the brief space of time of a few hundredths of a second.

§ 5. **The equation of short-circuit.** — As the current J sinks to zero, this stored energy is liberated, and is again re-stored when the current is raised to the same value in the reverse direction, *i.e.* to $-J$. The liberation of this stored energy in an electrical form or its re-storage gives rise to the induced E.M.F. $-(L+M)\frac{di}{dt}$ which forms the first term of (90); *e.g.*, if the current begins at once to fall towards zero, and then rises to $-J$ without over-

* Since the percentage of the lines which do not pass through the field-magnet bobbins is greater when the coil is moved away from the symmetrical line, the apparent inductance increases, but in Mr. Allen's experiments the total increase when the coil was brought just up to the pole-tip was not more than about 15 per cent., and this would be less in a toothed armature.

reversal, $\frac{di}{dt}$ is throughout the process negative, and therefore the induced E.M.F. is positive and retards commutation by maintaining the current in the old direction.

At the same time, the coil is moving through an external field due to the fringe of lines within the interpolar gap. This field is the resultant due to the magnet-winding as modified by the presence of the armature ampere-turns that are carrying the full current and are not themselves undergoing short-circuit. The value of the impressed E.M.F. due to the movement through this field will vary as short-circuit proceeds, and may be expressed as a function of the time, $=f(t)$. To complete the expression for the conditions during short-circuit, there must be introduced the ohmic loss of volts over the resistance of the short-circuited coil AB and its commutator connectors, and over the brush contact resistance. Let r be the resistance of the coil AB, and r_c be the resistance of one commutator connector, *e.g.* BD. Let the contact area between the brush and the leading sector be at any time during the period of short-circuit F_u', and simultaneously that between the brush and the trailing sector be F_u''. At the beginning of the short-circuit of any coil AB, F_u'', or the area in contact with the trailing sector (Fig. 336) continuously increases as the brush passes on to sector C, while *vice versa* towards the end of short-circuit the area of contact with D continuously diminishes, or if the width of brush does not exceed that of a sector, the increase and diminution proceed simultaneously and in correlative degree. The contact resistance of either portion is equal to its specific contact-resistance per square inch divided by the area in square inches, and it must be borne in mind that the specific contact-resistance of the two portions need not necessarily be precisely the same, but may depend upon the current density; the latter will vary on the two sides of the dividing mica in relation to time, and if so the specific contact-resistance may also have a temporal variation. Hence if R_k' and R_k'' are the instantaneous specific contact-resistances of the leading and trailing portions respectively, the loss of volts over the contact with the leading sector is $\frac{R_k' \cdot i_1}{F_u'}$, and over the contact with the trailing sector is $\frac{R_k'' \cdot i_2}{F_u''}$. Let every ohmic loss be now reckoned as negative; then by Kirchhoff's laws the algebraic sum of all the E.M.F.'s acting round one short-circuited section must be zero. The internal resistance within the brush or the sectors may be neglected entirely in comparison with the other resistances. The complete equation is therefore

$$-(L+M)\frac{di}{dt} - \Sigma \cdot \left(M_1 \cdot \frac{di_1}{dt}\right) + f(t) - ri - r_c \cdot i_1 - r_c \cdot i_2$$
$$- \frac{R_k' \cdot i_1}{F_u'} - \frac{R_k'' \cdot i_2}{F_u''} = 0$$

or,

$$-(\mathrm{L}+\mathrm{M})\frac{di}{dt}-\Sigma.\left(\mathrm{M}_{\mathrm{I}}^{\cdot\cdot}.\frac{di_{\mathrm{I}}}{dt}\right)+f(t)-ri-r_c(i_1+i_2)$$
$$-\left(\frac{\mathrm{R}_k'.i_1}{\mathrm{F}_u'}+\frac{\mathrm{R}_k''.i_2}{\mathrm{F}_u''}\right)=0\ .\ .\ .\ (91)$$

The positive direction round the short-circuited section is assumed to be that of the original current at the moment when short-circuit begins. The direction of the fall of potential by the three last terms entirely depends upon the algebraic signs which i_1, i_2, and i are found to have, and as these are determined in relation to the short circuit and not to the external circuit, so also are the potential drops.

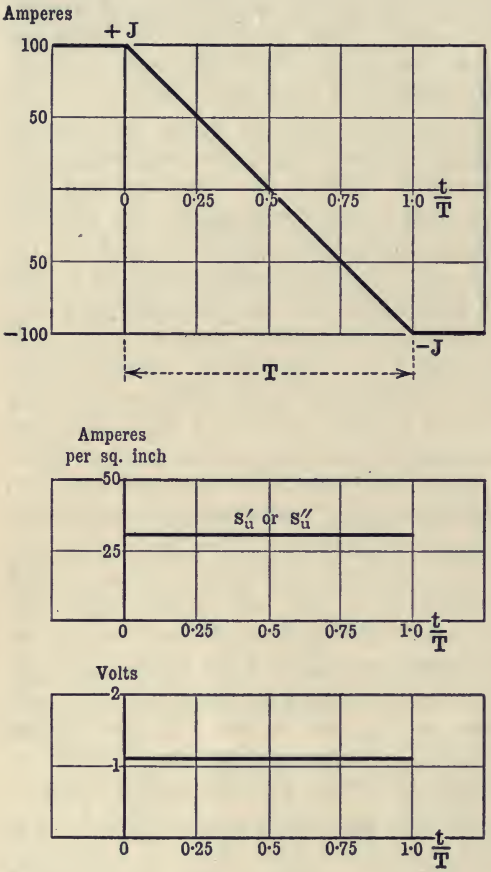


FIG. 337.

§ 6. **Effect of brush contact - resistance.**—The two expressions bracketed together in the last term of the above equation contain the current-densities s_u' and s_u'' in the leading and trailing sector respectively, so that they may also be put in the form $\mathrm{R}_k'.s_u'+\mathrm{R}_k''.s_u''$, and it will be found that they are a most important factor in the problem of commutation. The ideal case of commutation may be regarded as that in which the varying current i when plotted in relation to time yields an inclined straight line passing from the full normal value $+\mathrm{J}$ to the reversed value $-\mathrm{J}$ with a constant rate of change $-\frac{2\mathrm{J}}{\mathrm{T}}$ (Fig. 337), so that its instantaneous value at any time t is

$$i=2\mathrm{J}\left(\frac{1}{2}-\frac{t}{\mathrm{T}}\right)\quad .\quad .\quad .\quad .\quad .\quad (92)$$

When this is the case, it also follows that s_u' is always $=\frac{2J}{F_u}$, and $s_u''=-\frac{2J}{F_u}$, where F_u is the total area of contact of one set of brushes. The signs of the current-densities are therefore different in relation to the short circuit, but their numerical values are equal, and in each case this is equal to the normal current density s_u if the current $2J$ passed uniformly through the area F_u. The characteristic feature of such commutation is then that the current-density over the brush face is throughout constant and uniform at its normal value. The specific contact-resistances R_k' and R_k'' must therefore be alike and equal to the normal specific resistance R_k, and the difference of potential between the brush and the sectors when plotted in relation to time yields a straight line similar to that of the current-density as shown at the foot of Fig. 337. In consequence the two expressions in the bracket of equation (91) cancel out, showing that the normal brush contact-resistance $R_1=\frac{R_k}{F_u}$, although affecting the total voltage of the machine, has under these conditions no effect whatever upon the process of commutation.

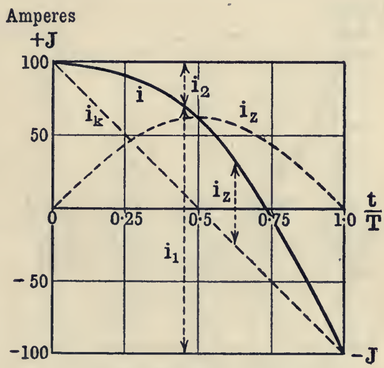


FIG. 338.—Retarded commutation.

When, however, we depart from the ideal case of a change of current which is a linear function of the time, an entirely different set of facts arises. The cases of divergence from a uniform rate of commutation may be grouped under four principal kinds.

(1) Retarded commutation, yielding a curve which when plotted as in Fig. 338 from a starting-point of $+J$ is on the whole convex.

Suppose that at some time t seconds from the commencement of short-circuit the current in the coil has not fallen by its correct amount proportional to t, then the current-density in the leading sector $s_u'=\frac{i_1}{F_u'}$ is greater than that in the trailing sector $s''=\frac{i_2}{F_u''}$. The former is positive, and the latter is negative; for simplicity's sake it is best to regard the densities in themselves as having no sign, since we are not here dealing with them as continuously varying quantities, and to add their signs when required. Hence the expression in the bracket becomes $R_k'.s_u'-R_k''.s_u''$. Even if R_k' does not remain equal to R_k'', yet the two will not vary so much as in inverse proportion to the current-density, so that there results a positive difference, the fall of potential between sector 1 and the brush toe being greater than that between sector 2 and the rest of the brush. The difference of potential which thence arises $\triangle p = -(R_k' . s_u' - R_k'' . s_u'')$ is thus a *negative* E.M.F. in relation to the short circuit or acts round the circuit in the negative direction against the old current, tending to reduce its value; *e.g.*, in Fig. 336 the E.M.F. in question which is solely due to the unequal current densities in the two portions of the brush would be directed from D to C through the coil BA.

Thus the unequal brush contact-resistance acts as an outlet through which the stored energy may expend itself in heating not only the coil, but also the commutator surface; while after reversal of i it is the means by which the energy that has to be re-stored is derived from the

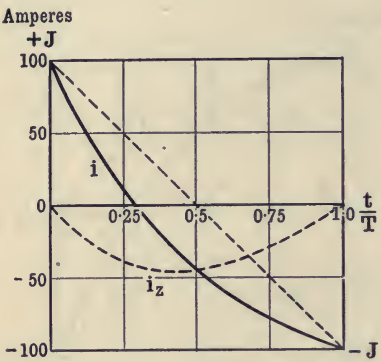


FIG. 339.—Accelerated commutation.

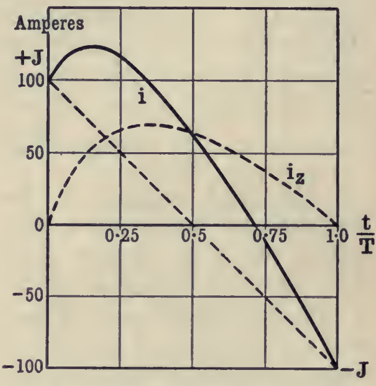


FIG. 340.—Increase of current in old direction.

electrical output of the rest of the winding, for the voltage of the external circuit is during this stage temporarily lowered.

(2) Accelerated commutation, giving a concave curve (Fig. 339). In this case the current in the short-circuited coil is too quickly reduced, and the density in the leading sector is less than that in the trailing sector. In relation to the short circuit the former is positive and the latter negative, so that the expression within the bracket is again $R_k' . s_u' - R_k'' . s_u''$, but as opposed to the former case yields a negative difference. The resulting difference of potential $\triangle p = -(R_k' . s_u' - R_k'' . s_u'')$ is therefore a *positive* E.M.F. checking the fall of the old current and opposing the rise of the reverse current.

(3) If the current is at first actually increased in its old direction above $+J$ (Fig. 340), the current-density in the trailing sector so long as this is the case becomes positive as well as that in the leading sector; in other words, the excess current actually flows through the brush

from one side to the other and round the short-circuit. The two expressions in the bracket have now therefore to be added, and it is $\Delta p = -(\mathrm{R}_k{}' \cdot s_u{}' + \mathrm{R}_k{}'' \cdot s_u{}'')$, which acts negatively to limit the short-circuit current and to bring it back again to its correct amount proportional to t.

(4) If the current towards the end of the period of commutation is *over-reversed* to a value above $-\mathrm{J}$ (Fig. 341), $s_u{}'$ itself becomes negative as well as $s_u{}''$, and it is again the sum of the two which is effective, their corrective difference of potential being $\Delta p = -(-\mathrm{R}_k{}' \cdot s_u{}' - \mathrm{R}_k{}'' \cdot s_u{}'')$, and therefore positive or checking the over-reversal.

Combinations of the above leading cases are also of frequent occurrence, the curve of i crossing the inclined straight line of uniform commutation. In generators the case is often met with in which the commutation is much retarded at first, and is then followed by over-reversal, due to the coil moving through too strong a reversing field.

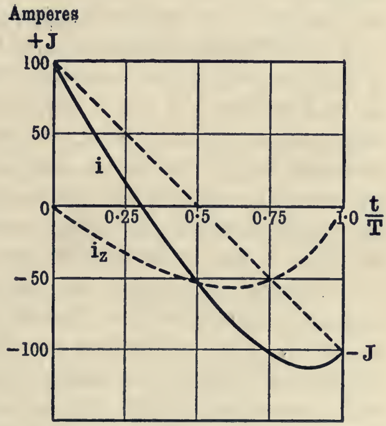


FIG. 341.—Over-reversal.

Thus if there be any divergence from proportionality between the change of current and the time, in every case whether of excess or deficiency, over- or under-reversal, the effect of the *brush contact-resistance* is to keep the current in each sector more nearly proportional to the area of contact, and so to assist in promoting the straight-line change which corresponds to a uniform current density.

§ 7. **The rate of change of the short-circuit current.**—Even with a comparatively small divergence of the short-circuit current from a straight line the current-densities $s_u{}'$ and $s_u{}''$ may vary greatly from the normal towards the beginning and end of commutation, when the area of contact with the trailing or leading sectors respectively becomes very small. At the initial and final moments the current-density is, in fact, determined by the rate of change of the current $\left(\frac{di}{dt}\right)_{t=0}$ or $\left(\frac{di}{dt}\right)_{t=\mathrm{T}}$. Thus at the last moment in all the cases described the current may be finally commuted to the correct value, and end at $-\mathrm{J}$ with $i=0$; yet it must be particularly noted that even so the current density may be very high, and theoretically may reach infinity. But before the mathematical possibility of $\left(\frac{di}{dt}\right)_{t=\mathrm{T}} = \infty$ is reached, the physical result in nature would have been sparking. The rapid change of current towards the end of short-circuit will set up a large E.M.F. of self-induction, and the current-density in the leading sector will rise to a very high value; the two together will cause the copper brush tip to become momentarily overheated and fused with violent sparking, while with the carbon brush the self-induced E.M.F. will cause serious sparking, even if the high current-density does not cause the tip to become red-hot and its edge to be disintegrated. Similarly, at the initial moment if the rate of change of the current is too rapid the current-density may be very high, but it is now the trailing sector in which it has to be feared.

At no time throughout the whole process of commutation can the current-density in either the toe or heel of the brush be allowed to reach such a high value as to melt off particles of the copper brush or of the sectors, nor must its rapid changes set up a self-induced E.M.F. causing sparking. It is from this necessity that the value of $f(t)$ and the position of the brushes become of material importance.

§ 8. **Effect of external field upon current-density of brush.** —If the reader refers to any diagram, such as Figs. 104 or 109, it will be evident that the exact position relatively to the poles which the loops or coils of the winding occupy, at the time when they are short-circuited, depends upon the position of the tips of the brushes as they rest upon the commutator, and that this position of the short-circuited coil can be altered if the brushes are shifted backwards or forwards round the circle of the commutator sectors.

The effect of altering the position of the brushes on the external impressed E.M.F. has now therefore to be considered. First suppose that in Fig. 336 the brushes are shifted slightly backwards away from the line of symmetry or against the direction of rotation. The loops at the commencement of their short-circuit will then be moving through the fringe of the trailing pole-tip; hence an E.M.F. is set up in them directed similarly to that of the coils from which they have just parted company, *i.e.* in the original direction of the current prior to passage under the brushes. The initial value of the impressed E.M.F. $f(t)$, or E_o, is thus positive; although, as rotation continues, it falls and eventually, if short-circuit lasts long enough, will be reversed, yet at the outset at least the commutation will be retarded as in Fig. 338 (case 1). If the angle of trail of the brushes and the initial E_o be large, it may even raise the amperes which the coil is carrying above the normal current being carried by the other armature coils (case 3). In either case it will largely increase the difficulty of commutation by causing an excessive current-density in the leading portion of the brush, and if short-circuit ends with the coil still behind the neutral line of zero field, the whole work of reversal will be thrown on to the electrical action of the brush contact-resistance. Sparking will then ensue between the trailing edge of the sector which has just emerged from under the brush and the tip of the brush itself. Thus the consequence if short-circuit ends while the coil is still moving on the trailing side of the line of symmetry is in general destructive sparking, so that any angle of trail is so far as generators are concerned quite inadmissible.

Now the lines of the field entering into or leaving the armature core shade off more or less gradually as we pass out from beneath a pole-piece into the gap which separates it from a neighbouring pole-piece. The point at which the lines just dip into the core and immediately leave it determines the resultant neutral line, on passing which the direction of the impressed E.M.F. changes. Thus the fall and rise of the impressed E.M.F. during short-circuit may be expressed as $f(t) = E_o - Ht^x$, where E_o is the initial value and is reckoned as positive or negative according as short-circuit begins behind or ahead of the neutral line. The power x to which t should be raised varies in most machines between the first and second powers, and is not strictly constant over any great part of the interpolar region. As, however, the time of short-circuit only coincides with a small angle of movement, the E.M.F. due to the external field may approximately be assumed to be a straight-line function of the time for any one position of the brushes, or $f(t) = E_o - Ht$, the values of E_o and H being such as to suit the exact portion of the curve of impressed E.M.F. which corresponds to the position of the coils during short-circuit. If now the brushes are brought forward into such a position that they have neither lead nor trail, and the diameter of commutation coincides with the line of symmetry, they may be taken as coinciding with the neutral line at no load. But when current is taken out of the armature, then, as explained in Chapter XVII. § 4, the armature ampere-turns react upon the external field, and the cross induction due to the total cross ampere-turns of the armature has the same sign as the induction under the trailing pole-tip. The resultant neutral line is therefore displaced forwards in the direction of rotation, and, in order to overtake this, the brushes must themselves be shifted forwards.

§ 9. **Angle of lead necessary to overtake the neutral line.** —Only, therefore, when the armature current is very small can its reaction on the field be neglected, and a position of the brushes on the symmetrical line be taken as coinciding with the neutral line. If the brushes are given no lead, then, with any appreciable amount of armature current, $f(t)$ may remain positive throughout the whole period of short-circuit owing to the displacement of the field forwards, and commutation is still dependent upon the action of the brush resistance. But if large armature currents are to be reversed, the corrective action of the varying brush surface must in general be supplemented by the additional precaution of giving the brushes a *forward lead* in the direction of rotation, and so shifting the diameter of commutation that it approaches, overtakes, or passes the neutral line. The short-circuit is then not opened until $f(t)$ has become negative and there is a reversing E.M.F. acting on the coil and assisting in the production of a current in it, the same in direction as that which the coil will be called upon to carry as soon as it emerges from under the brush. Our case is now illustrated by Figs. 104 ii or 109, where the coils short-circuited by either brush are slightly in advance of the vertical diameter, and are assumed to be just moving in the fringe of lines from the leading pole-edge when short-circuit ends. Although the E.M.F. impressed by the external field is now in the required new direction, it may not suffice by itself to raise the current to equality with the normal current of the other coils; yet much will have been done to prevent the rise of the current-density, in any portion of the brush and at any time during short-circuit, from reaching an excessive amount.

If the external field becomes negative or reversing while the current in the short-circuited coil is still in its original direction, the E.M.F. $f(t)$ is in the opposite direction to the current, and the coil is itself driving the armature forward as in a motor. We thus have at once a ready means by which the bulk of the initial energy which has to be dissipated may be conveniently absorbed in the form of mechanical work instead of as heat and with any degree of rapidity. When the current has been reversed the prime mover expends mechanical energy with equal rapidity depending upon the value of the reversing field, and this appears not only as heat in the coil but also as stored electro-magnetic energy.

If the brushes are so far advanced that the reversing field becomes too strong, cases (2) and (4) of accelerated commutation and over-reversal arise. It is, however, of great importance to minimise the angle of lead as far as possible; hence in generators at full-load such cases are at once remedied by adopting a lesser angle of lead. They are not therefore of such frequent occurrence, except at no-load or intermediate loads, when the brushes are retained in the correct position for full-load. But in nearly every case, if the effect of sparking on the brushes and commutator be carefully examined, it will be found in generators that it is the trailing edges of the sectors that are first pitted and worn by the sparks, and the leading edge of the brush that first deteriorates, showing that it is the final current-density and the final rate of change which have the greatest importance in generators.

§ 10. **Experimental determination of short-circuit current.** —If an armature coil is severed at some spot and the two free ends are connected to a pair of slip-rings, and if upon these rings rest brushes which are short-circuited by a standard low resistance of known amount, the current in the coil can be traced by the oscillograph through a complete revolution, *i.e.* not only when the coil is under a pole but also in the brief periods when it is short-circuited by the brushes. The armature circuit still remains closed, and the insertion of the low resistance at the one point hardly affects the conditions. Potential leads are taken from the ends of the low resistance to the oscillograph, and the current in the coil can thus be measured. By special arrangements the horizontal scale can be increased, and the curve of current-change during the period of short-circuit be extended, so as to enable the whole process to be carefully watched and recorded. Curves taken by this method * show that the change of the short-circuit current is often extremely irregular owing to obscure secondary causes such as the exact bedding of the brush surface, yet they fully bear out all the conclusions that had been previously drawn on more theoretical grounds. When the brush position in a dynamo is advanced into too strong a reversing field, or is moved backwards into a strong field towards the trailing pole, the heavy current in the new or in the old direction is shown in the curves at the end or at the beginning of short-circuit by sharply pointed peaks which fluctuate violently when excessive sparking takes place. Even when no sparking takes place, if the brushes are too far forwards or backwards, considerable pulsations are set up in the magnetic field, the excess current in the short-circuited coils causing the value of the direct magnetising turns of the armature and their effect on the field to pulsate with the frequency of commutation. The main field through the entire magnetic circuit of yoke and poles is thus set into oscillation, which appears as a ripple in the wave of E.M.F. or current in a coil, while it is passing under the poles, especially towards the pole-tips. This shows that under such circumstances it is only approximately true to regard the apparent inductance of the coil as due solely to the field within the interpolar region. An excessive short-circuit current can, in fact, even affect the voltage given by a machine for the same number of ampere-turns on the field, and after allowance has been made for the actual value of the direct magnetising turns (other than those of the short-circuited coils) due to the angle of lead or trail.†

§ 11. **The action of the angle of lead of the brushes.**—The effect of giving the brushes an angle of lead requires a little further examination. It is evident that if $f(t)$ is to be exactly proportioned to the armature current that has to be commuted, the angle of lead must be increased as the armature current is increased (unless there are

* *See* especially *Journ. Inst. Electr. Eng.*, vol. xxxiii. pp. 548, 557 (Dr. W. M. Thornton); p. 1023 (Dr. D. K. Morris and J. K. Catterson-Smith); vol. xxxv. p. 430 (J. K. Catterson-Smith); vol. xxxviii. p. 176 (Prof. F. G. Baily and W. S. H. Cleghorne), where oscillograms are given of the current flowing through a sector into the brush, *i.e.* i_1 or i_2.

† Cp. "Ueber magnetische Wirkungen der Kurzschlussströme in Gleichstrom-ankern," by Dr. R. Pohl, vol. vi. *Sammlung elektrotechnischer Vorträge* (Stuttgart, Ferdinand Enke, 1905). The similar effect with commutating poles is especially noticeable in the machine investigated by Prof. F. G. Baily and Mr. Cleghorne, *Journ. Inst. Electr. Eng.*, vol. xxxviii. p. 171 ff.

special commutating poles present to provide this proportionate reversing field, and the consideration of these is for the present postponed). But with the increasing armature current the cross ampere-turns are increased, and the resultant neutral line becomes more and more displaced from the line of symmetry; the fringe of lines issuing out of or entering into the leading pole-tip is driven inwards under the leading pole-tip, and a further forward lead is thereby necessitated. At the same time, with each step forward of the brushes the number of back ampere-turns on the armature is itself increased, which tends to weaken the main field. In series- or compound-wound machines this weakening of the main field is partially or entirely neutralised by a corresponding increase of the magnetising turns of the field-winding. By the advance of the brushes armature ampere-turns are progressively shifted from the belt of cross-turns to the belt of back-turns; the cross M.M.F. at the diameter of commutation is thereby reduced, and the short-circuited section can thus be retained in a reversing field. If the total flux is maintained at the same amount, or is even increased in spite of the increase of the back ampere-turns, the reversing density that could be reached by an advance of the diameter of commutation right under the pole would approach the limiting value of the normal B_g. The same action takes place in the simple shunt-wound dynamo, in virtue of the gradual shifting of the armature wires from cross to back ampere-turns, although with less rapidity and to a less degree. For a small advance the decrease in the effect on the main field of the more distant pole, and the shortening of the air-path together outweigh the increase in the back ampere-turns. Yet in a shunt machine with constant excitation the reversing field very quickly reaches its maximum after the pole-edge is passed, and then remains constant at some value * less than B_g.

§ 12. **Disadvantages of a large angle of lead.**—With a large angle of lead the demagnetising turns of the armature increase the necessary weight of copper on the field, and in the case of compound-wound machines, as explained in Chapter XVI. § 19, render the regulation for constant potential less perfect than it would otherwise be. In an extreme case, if the diameter of commutation has to be advanced much beyond the neutral line in order to reach the requisite strength of reversing field, the machine further becomes inefficient as a generator of E.M.F., since some of the active wires are then inducing a back E.M.F. Apart from the above disadvantages, if the conditions for non-sparking require the diameter of commutation to be brought up to the leading pole-corners, the steepness of the gradient of the flux-density near to the pole-corner implies that small movements of the

* *E.g.*, with a smooth core it cannot exceed the difference between the M.M.F.'s of the interpolar excitation X_P and of the armature magnetising turns $\frac{J\tau}{2p}$, divided by $2l_g$.

brushes will produce great variations of the reversing E.M.F.—much more so than when the angle of lead is small and the coils are short-circuited near the middle of the interpolar gap. Hence the brushes must then be very accurately placed to suit the exact load on the armature; or as the corollary of this, they are very sensitive to small changes of load, since a slight alteration in the armature current will materially alter the distribution of the field near the pole-corner.

But the real objection is that even when the brushes can be accurately placed so as to secure entirely sparkless collection at all loads, perhaps with a large angle of lead at full-load, yet if we are dependent upon the values of the reversing field and E.M.F. being exactly suited to the load, the amount by which the short-circuited coils must be advanced will require to be varied every time that the armature current is varied. If the output fluctuates between wide limits—which in practice is usually the case—continuous attention is necessary, and even then it may not be possible to shift the brushes quickly enough to meet rapid fluctuations. The greater the angle of lead at full-load, the greater is the possible inaccuracy of the adjustment, and the more forcibly does the objection apply. It is therefore of the greatest advantage, in order to minimise the necessary attendance, to keep the angle of lead within small limits, and, if possible, to secure a fixed position of the brushes for all loads, so that no appreciable sparking results however widely and rapidly the load may vary. To attain this in the absence of special commutating poles, the utmost possible use must be made of the action of the brush contact-resistance in effecting reversal.

A digression must therefore now be made in order to investigate the specific contact-resistance of brushes as a necessary preliminary to the question of how far the corrective effect can be realised in practice. In this connection it will be found that a marked distinction must be drawn between copper and carbon brushes.

§ 13. **The contact-resistance of copper brushes.**—The specific contact-resistance of brushes per unit area of bearing surface is affected in general by the current-density, the pressure, the peripheral speed of the commutator and the state of its surface, and more especially in the case of carbon brushes by the direction of the current and by the temperature of the working surfaces. The effect of these various conditions has been investigated by a number of experimenters, and especially by Professor Arnold,* from whom the following results are mainly derived.

While the true specific resistance of the contact between brush and commutator when the latter is stationary may be taken to be a constant quantity, giving a fall of potential which rises in a straight line with

* *Die Gleichstrommaschine*, vol. i. chap. xviii. (2nd edit.), and pp. 476 ff. and 362 ff. (1st edit.); *Electr. Eng.*, vol. xxxviii. p. 330, and *E.T.Z.*, March 21, 1907.

increasing current-densities, in all cases the actual curve of the difference of potential when the commutator or slip-ring rotates bends over as the current-density is increased, more or less suddenly or gradually according to the nature of the material. This shows that the apparent contact-resistance on a rotating surface progressively decreases as the current-density rises, and it is the value of the specific running contact-resistance, or R_k defined simply as the quotient of the loss of volts divided by the current-density when the dynamo is running, with which alone we are concerned, the true specific contact-resistance having but little value in practice. Further, for a given pressure and current-density the specific contact-resistance is always greater when the commutator or slip-ring rotates than when they are at rest.

Taking first the case of copper brushes, it is found that with normal brush pressures and conditions of surface the contact-resistance with a rotating commutator decreases but slowly after a current-density of about 40 amperes per square inch is exceeded, and gradually becomes almost constant. It is practically independent of the peripheral speed when once this has passed a low value, but this result is of course dependent upon the commutator surface being smooth, and its running free from vibration so that there is no corresponding vibration set up in the brush-holders. Increase of the pressure for any given speed causes better contact and decreases the resistance.

Coming to numerical figures, with copper brushes a density of 40 amperes per square inch is almost always exceeded, and 200 amperes per square inch may be regarded as the maximum limit. The peripheral speed of the commutator is sometimes as high as 3000 feet per minute, but preferably does not exceed 2500 ft. per minute, and in all cases the lower its speed the better. In a good dynamo running under ordinary conditions, the brush pressure should range from $1\frac{1}{4}$ to $1\frac{3}{4}$ lb. per square inch of contact area; this may be tested by noting the pull required to lift the brush from the commutator surface with a small spring balance. Even under normal conditions the specific contact-resistance of copper brushes shows great variations; on a slip-ring it may fall as low as 0·0002 or on a commutator to 0·0007 ohm per square inch, while, if the periods of vibration of the commutator and brush-holder happen to coincide, it may be as much as 0·0025 ohm per square inch. As an average value for R_k may be taken 0·0016, and in no case is it likely to exceed 0·003 ohm per square inch.

§ 14. **The contact-resistance of carbon brushes under permanent conditions.**—The specific contact-resistance of carbon brushes falls much more rapidly with increasing current-density than that of copper brushes, and, after a medium current-density is reached, almost in inverse proportion thereto. The curve of fall of potential in relation to current-density therefore bends over fairly sharply and becomes nearly flat. On a smooth slip-ring the curve for $\triangle P$ after reaching a maximum may even slowly descend with increasing current-densities, showing that R_k is then decreasing faster than the current-density is rising. This phenomenon is closely involved with the property of carbon by which its resistance falls as it becomes hotter. Owing to the negative temperature coefficient of the carbon, as the current-density is increased, the expenditure of energy in heating the contact is checked; and this effect can only be eliminated by artificially maintaining the commutator at a constant temperature. The true fall of resistance with increasing current-density is to be explained as mostly due to the small carbon particles which are worn off the brushes, especially under a high current-density, when a blackening of the commutator surface results. A more intimate contact between brush and commutator is obtained by this wearing away of the carbon, which proceeds rapidly when the brush is heated and, becoming softer, disintegrates more readily.

Increase of the pressure lowers the contact-resistance, but the true effect is again partially masked by the fact that the increased friction loss raises the temperature and assists in lowering R_k. But as soon as a pressure of about 2 lbs. per square inch is reached there is little further improvement, and the pressure that can be advantageously employed is strictly limited by the mechanical friction and consequent heating that results.

The alteration of the cooling power of the slip-ring or commutator is also a disturbing factor when the effect of different peripheral speeds is to be examined.

A curve connecting specific contact-resistance with current-density may therefore presuppose either that in every case the passage of a given current was maintained long enough for its corresponding constant condition of temperature to have been reached with some normal peripheral speed, or that some given temperature such as may be found in practice is artificially maintained constant throughout.

The temperature of the contact is thus a factor of the greatest importance, and Professor Arnold found marked differences in its effect between different varieties of carbon and between the positive and negative brushes. Between temperatures of 20° C. and 35° C., or say 70° F. and 95° F., there is no great change, but beyond this point there is as a general rule a progressive and more or less rapid fall, which is especially marked when the current flows from carbon to metal, *i.e.* at the negative brushes. *E.g.* at temperatures of 55° C. (131° F.) and 75° C. (167° F.) the contact-resistance or loss of potential at the positive brushes with a current-density of 35 amperes per square inch may be respectively only about 88 and 50 per cent. of the value at 35° C., while at the negative brushes it may be only about 70 and 25 per cent. Efficient ventilation of the commutator to keep it cool is therefore of great assistance in suppressing sparking, and this is amply borne out by the well-known fact that machines which when cool run quite sparklessly, yet may begin to give trouble when they become hot after a prolonged run.

At low temperatures the contact-resistance at the negative brushes is as a rule greater than at the positive brushes, but at high temperatures this is often reversed, so that the curves intersect. This agrees with the general observation that in a machine at full-load and thoroughly heated up the negative brushes usually spark first, and it also accounts for the variability of this effect; since if there is over-commutation either through the brushes being shifted too far forwards, or at no-load through their being retained in the full-load position, the direction of the current in the brush-tip may be reversed, and either the positive or negative brushes might spark first, according to their state of temperature. The divergence between the + and − brushes may be as much as 50 per cent., but varies much with different qualities and different temperatures. Professor Arnold has suggested that if not due to some electrolytic action, it may arise from a greater or less voltage of ionisation with different brush materials.

The condition of the metal rubbing surface has considerable effect. If it be newly polished, the contact-resistance is low; as the surface becomes oxidised, and acquires a brown skin, the resistance rises. But on the other hand, when blackened by the presence of small conducting particles of copper, the resistance falls. Though blackening is to be avoided, a commutator should be allowed to retain the dull brown colour which continuous running will produce in the absence of sparking. An oily surface may more than double the resistance, especially if the current-density be low, but must be strictly avoided owing to the danger of carbonisation by sparking. Paraffin wax, on the other hand, although in itself an insulator, when used sparingly and thoroughly spread on a warm commutator, has little or no effect on the loss of volts under average conditions; it very much reduces the friction and noise of the carbon brushes, and is therefore frequently used as the basis of commutator compounds.*

The effect of speed has been left to the last on account of its great importance, and in this connection the two cases of the smooth slip-ring and the commutator must be clearly distinguished. The former shows lower contact-resistances which are practically independent of the speed (although with some tendency to increase at higher speeds), and correspondingly higher current-densities are permissible than in the commutator. The reason for the difference is to be found in the fact that, even with a commutator which may be regarded as practically smooth, carbon brushes are periodically subjected to momentary vibrations as they pass the mica strips dividing the sectors. The natural elasticity of the copper gauze brush suffices to take up the minute mechanical shocks, but the unyielding carbon brush is kept in a state

* Prof. F. G. Baily and Mr. Cleghorne, *Journ. Inst. Electr. Eng.*, vol. xxxviii. p. 158.

of continuous vibration. In consequence the curve connecting the loss of potential over the contact with the current-density does not become so flat in the case of the commutator as in the case of the slip-ring. In the latter the potential difference between brush and ring, or $R_k \cdot s_u$ reaches a maximum at which within wide limits of current-density it remains nearly constant, and according to the nature of the material this constant loss is about 0·75 volt with hard carbons or 0·45 volt with very soft carbons of high conductivity. But on a commutator, even if similar results may be repeated under very favourable conditions, the curves usually continue to show a gradual increase with increasing current-density, and for the same varieties of carbon the figures would rise, say, to 1 volt and 0·55 respectively.

But the real difference is rather that with the commutator 2, 3, or even 4 volts between brush and commutator are easily reached with sparking hardly, if at all, perceptible, the amount increasing as the speed is increased and the pressure weakened. If the commutator surface is rough, or untrue, and the brush-holders are insufficiently damped, this effect is magnified, and if at some particular speed the natural period of vibration of the brush-holders coincides with the period of the shocks it reaches a maximum, and the loss of volts may be 6 or more (cp. § 28). The same effect may be imitated on the slip-ring by so weighting the brush-holder as to produce resonance at the particular speed employed. In practice it is generally found that the lighter the moving parts of the brush-holders are, the better for all ordinary conditions of speed. Even with the same peripheral speed, the higher the actual number of revolutions per minute, the greater the likelihood of vibration, so that small high-speed machines are more liable to give trouble than large machines with commutators of large diameter running at a low number of revolutions per minute.

Thus a continuous-current dynamo with its commutator may with but little exaggeration be said to run always with its brushes in a stage not far removed from incipient arcing; owing to the slight percussion of the carbon brush as the sectors pass beneath its face, even when the commutator is practically quite smooth and there is no sparking, higher differences of potential are possible than are found experimentally with slip-rings.

With carbon brushes a pressure of $1\frac{1}{4}$ to $1\frac{3}{4}$ lb. per square inch should suffice, although at high peripheral speeds over 2000 feet per minute where there is vibration, it may become necessary to increase it to 2 lbs. per square inch. For peripheral speeds from 1000 to 2000 feet per minute, and average values of the brush pressure from $1\frac{1}{4}$ to 2 lbs. per square inch, Fig. 342 shows for hard and soft carbon brushes the values of the specific contact-resistance which correspond to good conditions of working on a smooth commutator without sparking. The full-line curves presuppose that the running is maintained long enough for the constant temperature to be reached that is proper to the particular current-density, while the dotted curves indicate the effect of a commutator which is maintained at one and the same constant temperature corresponding to the current-density at which the curves cross.* In the curves of Fig. 342 the values at the + and − brushes have been averaged, and it will be seen that for normal current-densities, such as from 30 to 40 amperes per square inch, the specific contact-resistance under ordinary conditions of pressure and temperature may be taken as varying between 0·03 and 0·015 ohm, or on an average for 30 amperes per square inch and hard carbon brushes, $R_k = 0{\cdot}03$ ohm.

In Fig. 343 are given curves of the loss of volts over two sets of brushes (*i.e.* positive and negative) for different kinds of carbon, and with an allowance for about ½″ of length down the two carbons, the

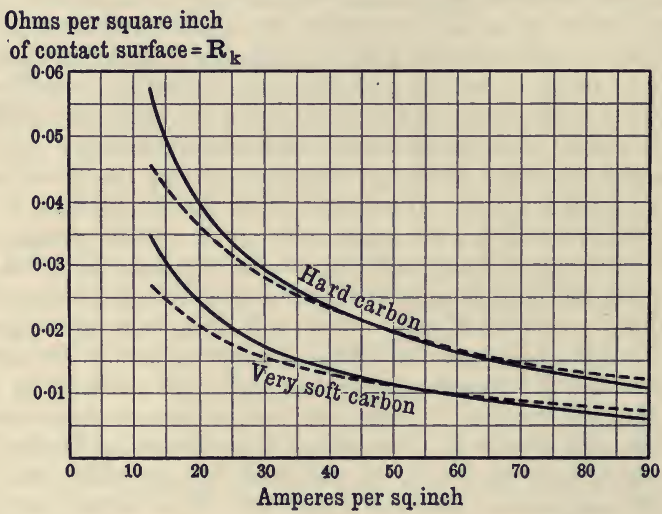


FIG. 342.—Contact-resistance of carbon brushes.

commutator being assumed to have reached its final natural state of temperature after prolonged running at each current-density.† They

* The shape of the curve of R_k at different current-densities with the commutator artificially maintained at a constant temperature throughout is represented approximately by an equation of the form

$$R_k = \frac{\text{a constant}}{s_u} + \text{a constant},$$

and values for the constants are given by Prof. Arnold, *E.T.Z.*, vol. xxviii. p. 266. But this resemblance to a rectangular hyperbola must not be pressed too far by mathematical deductions when the current-density is very small, since the curves of the loss of potential do not then cut the vertical axis but pass through the origin.

† The intermediate curves for soft electrographitic brushes of Le Carbone X quality are derived from Prof. F. G. Baily and Mr. W. S. H. Cleghorne's experi-

may therefore be used to determine the total drop of volts and loss of watts over the brushes and commutator so far as the current-density is uniform over the contact-area.

Although the selection of the best grade of carbon to suit given conditions of voltage, current-density, and speed is to some extent a matter of experience proceeding by trial and error, yet it may be said that for voltages of 250 and upwards the coarser and harder varieties with densities of 30 to 40 amperes per square inch are in general the most suitable. With low voltages, favourable conditions, and in cases where it is imperative to shorten the commutator width as much as possible, high-conductivity *graphitic* brushes with current-densities of 60 amperes per square inch are frequently used. Of this kind are the electrographitic Le Carbone X and Z grades, which are baked in an electric

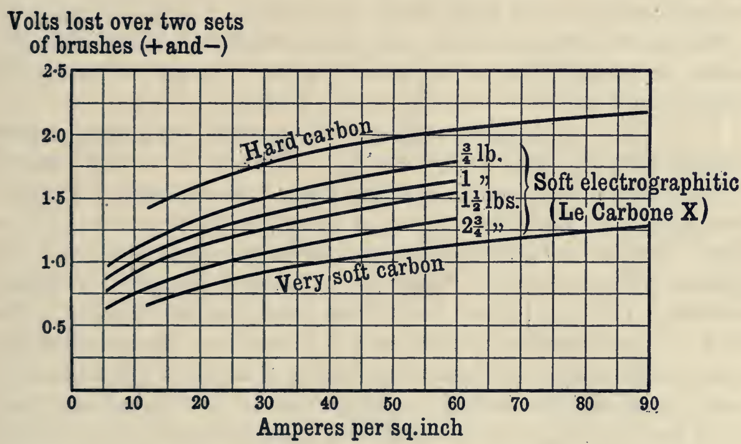


FIG. 343.—Loss of volts over carbon brushes.

furnace, and Morganite Link II. Professor Arnold found that the latter, when of negative polarity, gave a high contact-resistance which was well maintained over a range of temperature up to 40 or 50° C.; he therefore suggested its use for the negative brushes in combination with positive brushes of Le Carbone Z or Siemens S quality, which show high contact-resistances when of positive polarity. Even softer graphitic brushes are also made which have a low friction coefficient and are used on turbo-generators with commutator speeds exceeding 3500 feet per minute. Such brushes disintegrate more readily, blacken the com-

ments (*Journal Inst. Electr Eng.*, loc. cit.), who deduced as an equation expressing closely the shape of the curves and the effect of different pressures

$$\text{volts over two sets of brushes} = \frac{(\text{amperes per sq. inch})^{0 \cdot 28}}{1 + 0 \cdot 88\sqrt{p}}$$

where p is the pressure in lbs. per square inch.

mutator, and in proportion to their increased conductivity oppose less contact-resistance to sparking; further, they should not be used when several sectors are to be covered by the brush simultaneously. They may therefore be regarded as intermediate between copper and the harder varieties of carbon which suppress sparking more thoroughly.

Various types of brushes have also been brought out, in which the attempt has been definitely made to combine the high conductivity of the pure copper brush with the high contact-resistance of carbon, and have met with more or less success. These range from the laminated Endruweit brush to be described in § 42 to mixtures of carbon and bronze as in the Bronskol brush, or of carbon and copper in various proportions as in the KK grades of Le Carbone. The latter are only suitable for the most favourable conditions of very low voltage, or on slip-rings, where commutation does not enter into the problem, and current-densities of 150 amperes per square inch or over are required.

Endruweit copper-carbon and metal brushes, when the + and − values are averaged, show respectively about 0·3 and 0·2 volt at 80 amperes per square inch, and Bronskol 0·15 volt.

§ 15. **The contact-resistance of carbon brushes under rapidly varying currents.**—So far the effect of various current-densities permanently maintained has alone been considered at length. But during the running of a dynamo or motor the carbon brushes are subjected to a very rapid sequence of varying current-densities in the different portions of their surface, the sequence being continually repeated as the sectors pass under and away from the brushes. The effect of rapidly varying current-densities upon the contact-resistance requires, therefore, to be considered, and this has been investigated by passing a periodic alternating or a pulsating current through the brush into a rotating ring; the simultaneous momentary values of current and voltage across the contact-surface are then obtained, so as to determine corresponding values of the instantaneous current-density, and of the specific contact-resistance. The curves so obtained do not repeat the full-line curves of Fig. 342 for long-continued current-densities, but as might be expected resemble the dotted curves. For a given effective current-density the resistance when the current falls to lower values is lower than for similar constant current-densities, owing to the carbon being really at a higher temperature than would correspond to them, while for instantaneous densities above the effective value the resistance is higher than for similar constant current-densities owing to the carbon being cooler. Further, the experiments of Dr. Kahn and Professor Arnold have shown that the curves in relation to instantaneous current-density intersect the curves for permanent current-densities at the point of the effective current-density in the alternating case, *i.e.* where the square root of the mean square of the instantaneous current-densities is equal to the continuous current-density. The effective current-density or R.M.S. value marks, in fact, the point of equal heating effect over the contact area in the two cases. The curves are also independent of the periodicity, so long as this is high.

§ 16. **The contact-resistance of carbon brushes under the rapid changes of commutation.**—Lastly, it is specially to be noted that when the changes of current are so rapid as to be comparable with those in a dynamo, the curve of volts in relation to the instantaneous current-density is not far from a straight line. This shows that the specific contact-resistance remains practically constant, and it follows that this constant value is the value fixed by the effective current-density. Although this is generally speaking true, there is, however, a tendency towards the same effect as appears in such a marked degree with permanent current-densities, namely, for the resistance and loss of volts to be higher with instantaneous densities below the effective value and lower with densities exceeding this value. The experimental ΔP curves are therefore slightly bowed. Yet as they pass through the origin with zero current, and the divergence from a straight line is but small, it suffices to consider the R_k fixed by the effective current-density as practically constant during the rapid changes of commutation. In other words, the heating of the brush and its specific contact-resistance may be taken as uniform over its face from toe to heel, and also as constant for a given effective current-density. It remains to define more closely the meaning of the effective current-density during commutation.

So long as the brush width $b_1 - m \leqq \beta$, the width of one sector and one mica strip, *i.e.* the pitch of the commutator sectors, there are at any instant only two current-densities to be considered, but at each spot these vary temporarily. But if as usual with carbon brushes a number of sectors is covered simultaneously, the current-density at the brush contact surface then varies spacially at a greater number of points, yet conversely the temporal variation at each point is reduced. The position of the sectors relatively to any point of the brush surface varies periodically, and the time of a period corresponds to the passage of one sector and mica past a given point, or is $5\dfrac{\beta}{v_k'}$ seconds, where β is the pitch of the sectors in inches, and v_k' is the peripheral speed of the commutator in feet per minute. As the number of sectors covered is increased, and β becomes less and less relatively to $b_1 - m$, so does the temporal variation of current-density diminish. In the extreme case of a very large number of sectors covered, the current-density at any point would remain constant, and the effective current-density $s_{u\,eff.}$ is the square root of the mean square of the varying current-density along the surface of the brush contact. The practical case of the dynamo with carbon brushes in which $b_1 - m$ is usually $\geqq 2\beta$ falls between the two extremes, but is best approached from the latter hypothesis rather than from the former. It may therefore be approximately assumed that there is no temporal but only a spacial variation of the current-density, and if f_u is the form factor of either the stationary current-densities under the brushes or of the ΔP curve, the effective current-density is $s_{u\,eff.} = f_u \cdot s_u$, where s_u = the average current-density, and the loss of volts over one row of brushes is $\Delta P = s_{u\,eff.} R_k$ and of watts is $(s_{u\,eff.})^2 R_k \cdot F_u$, while the particular value of R_k is itself determined for any load by $s_{u\,eff.}$

§ 17. **Comparison of copper and carbon brushes.**—Returning to the short-circuit equation (91), it remains to consider how far the brushes are capable of exerting a corrective effect in practice.

Since we have now shown that under any particular load R_k' and R_k'' may be identified, the last expression in the bracket of equation (91), giving the corrective E.M.F. from the unequal current-densities, becomes $-R_k(s_u' \pm s_u'')$. If $b_1 - m \leqq \beta$, so that only one coil is short-circuited at a time at each brush, this again becomes simply

$$-\frac{R_k \cdot T}{F_u}\left(\frac{i_1}{T-t} + \frac{i_2}{t}\right) \quad . \quad . \quad . \quad . \quad (93)$$

and $\frac{R_k}{F_u} = \frac{R_k}{b_1 \cdot b} = R_1$, or the contact-resistance of one row of brushes, b_1 being the width of contact in the direction of rotation, b the joint length of a set of brushes measured parallel to the axis of rotation, and R_k itself depending in this case upon the form factor of the temporal variations of current-density.

If $b_1 - m > 2\beta$, so that several sections are simultaneously short-circuited at each brush, then towards the end of short-circuit let t be reckoned from the time when the leading sector is exactly under the tip of the brush with its leading edge about to emerge, *i.e.* let only the remaining time $T' = 5\frac{\beta}{v_k}$, be considered, during which the leading coil continues to be short-circuited. Then the contact-resistance term in relation to the leading coil becomes approximately (the influence of the trailing coils being neglected)

$$-\frac{R_k}{\beta \cdot b}\left(i_1 \cdot \frac{T'}{T'-t} + i_2\right) = -\frac{R_k \cdot T'}{\beta \cdot b}\left(\frac{i_1}{T'-t} + \frac{i_2}{T'}\right)$$

and since $\frac{T'}{\beta \cdot b} = \frac{T}{F_u}$,

$$= -\frac{R_k \cdot T}{F_u}\left(\frac{i_1}{T'-t} + \frac{i_2}{T'}\right) \quad . \quad . \quad . \quad (94)$$

With a given value for the total time T it would therefore in either case be advantageous to make $R_1 = \frac{R_k}{F_u}$ high. But as the material employed has a certain specific contact-resistance, this in practice resolves itself into decreasing F_m, *i.e.* a reduction in b which is equivalent to employing a high normal current-density at full-load. With copper brushes it is not practicable to employ a higher normal current-density than about 175 to 200 amperes per square inch, and R_k as before stated may be reckoned on the average as $= 0{\cdot}0016$ ohm per square inch. The maximum normal loss of pressure over one set of brushes is therefore about $0{\cdot}0016 \times 190 = 0{\cdot}3$ volt, or 0·6 volt over the two sets of opposite sign, and in practice the loss is more often nearer to 0·5 volt owing to the current-density being lower. The normal contact-resistance of copper brushes being so low relatively to the resistance of the short-circuited coil, no appreciable effect on the whole can be obtained out of the brushes until close upon the end of the period of short-circuit, when $\frac{1}{T' - t}$ assumes a preponderating influence. But when this is relied upon to correct even such small inaccuracies of adjustment as must occur in practice, it implies such a high momentary current-density at the trailing edge of the leaving sector that there is a danger of the brush-tip being fused and of sparking volatilising the metal of the brush. Any attempt to increase the normal current-density and the loss of volts above the maxima indicated above would only intensify the likelihood of an abnormal current-density fusing some local portion of the brush. A limit is therefore quickly set to the possibility of calling in the corrective action of the contact-resistance to suppress sparking in the case of copper and brass gauze brushes, and a fixed position for all loads within the capacity of the machine cannot be attained. As the armature current varies, the brushes must be shifted in order that the reversing field may be of about the right strength to keep the local current-density fairly uniform, and the corrective action of the contact resistance can only be relied on to prevent sparking when the brushes are but little removed from the natural position for exact reversal. The proper setting is found in practice by shifting the rocking bar slightly backwards and forwards until a position is observable on either side of which the sparking becomes greater. Hence in all cases where the fluctuations of the armature current are large and rapid, as, *e.g.*, in traction generators, recourse must be had to *carbon*. If some sparking does take place, the carbon brush, being non-metallic, has the advantage that it does not become fused and adhere to the surface of the commutator, so that the evil effects are minimised.

With carbon brushes, so far as the contact-resistance under instantaneous changes of current-density diverges from a constant value, decreasing with higher current-densities, the variation is in the wrong direction, at least when the commutation is simply retarded or accelerated; for the change of resistance then magnifies rather than diminishes the unequal current-density over the face of the brush. If the short-circuit current actually rises above the value $+J$, or is over-reversed above $-J$, which can only arise through the action of the external field, this is no longer true. But in either case this secondary effect is insignificant compared with the outstanding fact that the constant value R_k as fixed by the effective current-density itself varies in the wrong way, *i.e.* diminishing with high current-densities. It is therefore even more than with copper brushes impossible to work with a very high normal current-density. The heating of the brush and commutator from any local increase in the density would become too great, and further disintegration of the material would then be set up so that the contact-surface becomes eaten away. The part of the brush at which the very high current-density is localised, *i.e.* usually the brush-tip with under- or over-reversal towards the end of commutation or the brush-heel if the current is at first increased at the beginning of commutation, becomes red-hot and its resistance falls. The equalising action of the contact area is then defeated, and the carbon brush may be regarded as having failed to meet the end for which it was introduced. The necessity for limiting the normal current-density to such an amount as to leave a considerable margin to meet actual inequality of the density has led in practice to the adoption with hard carbon brushes of a normal value not exceeding some 35 to 40 amperes per square inch. Assuming the specific contact-resistance at this density to be 0·03 ohm, the normal loss of pressure over the two sets of brushes is from 2 to 2·2 volts. Comparison of this loss with that for copper brushes gives a rough measure of the degree in which the superiority of the carbon brush can be utilised in practice, *i.e.* it is nearly four times as effective as copper; it also shows that it is necessarily purchased at the expense of the efficiency of the machine. Such sacrifice is, however, but small as compared with the advantage that the carbon brush offers; although it may not enable us to retain the brushes on the line of symmetry, yet with them an intermediate angle of lead can be found between the best possible positions for zero and full-load, such that neither the too rapid reversal in the first case nor the insufficient reversing field in the second case will cause an excessive current-density and overheating or serious sparking. The brushes can then be retained in this position through all changes of load perhaps up to an overload of 30 per cent.

With low-voltage machines for large currents a softer carbon may advantageously be employed, and the normal current-density raised to 50 amperes per square inch with a consequent decrease in the necessary size of commutator (cp. § 14). The same also applies to the case of dynamos fitted with commutating poles, which supply a reversing field nearly proportioned to the load.

§ 18. **The division of the short-circuit current into two components.**—The above general results can be advanced by a further mathematical analysis of the problem,* and for this it will be necessary in the first place to return to the expression (90) for the total E.M.F. from the apparent self and mutual inductance of the short-circuited coil, namely—

$$-(L+M)\frac{di}{dt}-\Sigma\,.\left(M_I\,.\,\frac{di_I}{dt}\right)$$

From the general construction of the last term it is evident that if certain suppositions are made as to the nature of the currents, i_I, i_{II}, etc., in the other short-circuited coils which are not at precisely the same stage of commutation, two important cases may be distinguished. Firstly, (*a*) if these currents be assumed to have the same rate of change as i in the considered coil A and in the same direction, although their actual values will not at any one moment be the same as i, yet

$$\frac{di_I}{dt}=\frac{di_{II}}{dt}\text{ etc.}=\frac{di}{dt}$$

so that the E.M.F. from their mutual inductance is simply additive to that of the considered coil, or

$$\begin{aligned}-(L+M)\frac{di}{dt}-\Sigma\,.\left(M_I\,.\,\frac{di_I}{dt}\right)&=-(L+M)\frac{di}{dt}-\Sigma\,.\,(M_I)\,.\,\frac{di}{dt}\\&=-(L+\Sigma\,.\,M)\frac{di}{dt}\end{aligned}$$

whence the apparent self and mutual inductance of the coil is $L+\Sigma\,.\,M$. Alternatively (*b*) if the currents in the coils A_I, A_{II}, etc. are simply those which are due to the E.M.F. of mutual inductance with coil A, as in a transformer, and no other external source of E.M.F. is present, let the value of this mutually induced current in, *e.g.*, A_I be i_I'; then by Kirchhoff's laws, and assuming the coils to be similar, so that $L_I=L$,

$$-M_I\,.\,\frac{di}{dt}-L\,.\,\frac{di_I'}{dt}-r_I\,.\,i_I'=o$$

Neglecting $r_I\,.\,i_I'$ as comparatively small

$$\frac{di'_I}{dt}=-\frac{M_I}{L}\,.\,\frac{di}{dt}$$

* The treatment of the question of commutation here follows in many respects the results of its exhaustive investigation by Professor E. Arnold in *Die Gleichstrommaschine* (2nd edit.); compare also *Zeitschrift für Electrotechnik* (Wien), 26th November 1905, and "The Commutation of Direct and Alternating Currents," by Professor Arnold and J. L. La Cour in *Trans. Intern. Electr. Congress St. Louis* (*1904*), vol. i. p. 801. The present acknowledgment must therefore be taken to cover a wide indebtedness which extends over the whole subject.

The secondary current i_{I}' induces in the primary coil an E.M.F.

$$-\mathrm{M}_{\text{I}} \cdot \frac{di'_{\text{I}}}{dt} = \frac{\mathrm{M}_{\text{I}}^2}{\mathrm{L}} \cdot \frac{di}{dt}$$

The total self and mutually induced E.M.F. in the considered coil A is then

$$-(\mathrm{L}+\mathrm{M})\frac{di}{dt} + \Sigma \cdot \left(\frac{\mathrm{M}_{\text{I}}^2}{\mathrm{L}}\right)\frac{di}{dt} = -\left\{\mathrm{L}+\mathrm{M}-\Sigma \cdot \left(\frac{\mathrm{M}_{\text{I}}^2}{\mathrm{L}}\right)\right\}\frac{di}{dt}$$

and the total apparent self and mutual inductance is

$$\mathrm{L}+\mathrm{M}-\Sigma \cdot \left(\frac{\mathrm{M}_{\text{I}}^2}{\mathrm{L}}\right) = \mathrm{L}_{sz}$$

In this case, therefore, the action of the secondary coils is purely damping, since their currents are always so changing as to minimise the variation of the current in A. The inductance is therefore reduced instead of increased as in the first case, and the lesser value L_{sz} is "apparent" in a double sense, since it takes into account not only the damping action of the exciting coils, but also that from the other sections which are simultaneously short-circuited.

Neither of these suppositions holds exactly in the real process of commutation during short-circuit, but since the physical effect of any current may be obtained by dividing it into components which correctly express its total value, and tracing the result of each component separately, a division of the actual short-circuit current into two components may enable us to make use of the two suppositions, one component corresponding to $\mathrm{L}+\Sigma \cdot \mathrm{M}$ and the other to L_{sz}. At any moment, then, let the short-circuit current of the coil A be divided into a part i_k which corresponds to a straight-line change from $+\mathrm{J}$ to $-\mathrm{J}$, and an "additional" part i_z which takes account of any excess or deficiency of the actual current as compared with the straight-line current; *i.e.* $i = i_k + i_z$, as in Figs. 338–341, where the two components of the actual short-circuit current are given by the dotted lines. Similarly, the current in, *e.g.*, coil A_{I} is divisible into two analogous parts, or $i_{\text{I}} = i_{k\text{I}} + i_{z\text{I}}$. *Ex hypothesi*, the curve of $i_{k\text{I}}$ is a straight line with the same slope as that of i_k, and the same holds for all other components $i_{k\text{II}}$, $i_{k\text{III}}$, etc. The mutual inductance of these is then additive to the inductance of the original coil as in the first of the two cases dealt with above, and the total E.M.F. from self and mutual inductance of the considered coil is

$$-(\mathrm{L}+\mathrm{M})\frac{di_k}{dt} - \Sigma \cdot (\mathrm{M}_{\text{I}})\frac{di_k}{dt} = -(\mathrm{L}+\Sigma \cdot \mathrm{M})\frac{di_k}{dt}$$

$$= -(\mathrm{L}+\Sigma \cdot \mathrm{M})\frac{-2\mathrm{J}}{\mathrm{T}} \quad . \quad . \quad . \quad (95)$$

A straight-line current-change not only in coils A and B, but also in all the coils which are simultaneously short-circuited both at the same brushes as coil A and at adjacent brushes is, in fact, the indispensable condition that they may all have throughout the same rate of change and all assist in retarding commutation. The impressed E.M.F. from the field $f(t)$ must also be similarly divided into two components $f(t)_k + f(t)_z$; the former corresponds to that value of the reversing field which suffices throughout to give a straight-line current, and its value will be discussed later. If it is not present at its precisely correct value or is entirely absent, the actual field being in the wrong direction, this simply means that the value of $f(t)_z$ must be so taken as to neutralise the component $f(t)_k$, or the portion of it which is actually non-existent.

§ 19. **The reversing field required for** i_k.—The assumption of such straight-line components for the several coils covered by the brush yields equations to which $L + \Sigma . M$ is strictly applicable. When $b_1 - m > 2\beta$, the process is divisible into three stages. At the beginning of short-circuit there is a period when the trailing sector is gradually passing under the brush; next, both the leading and the trailing sectors are completely covered by the brush; finally, towards the end of short-circuit there is a period when the leading sector is emerging from beneath the brush. Taking the three stages in turn, and constructing the Kirchhoff equation for each, we have

(i) Between $t = 0$ and $t = \frac{\beta}{b_1} . T$

the current in the leading commutator connector must be $2J . \frac{\beta}{b_1}$, and in the trailing connector $-2J\frac{t}{T}$; the areas of the sectors which are covered by the brush are similarly $F_u' = \beta . b$, and $F_u'' = F_u . \frac{t}{T}$. Hence

$$-(L + \Sigma . M)\frac{di_k}{dt} + f(t)_k - ri_k - r_c . 2J\frac{\beta}{b_1} + r_c . 2J\frac{t}{T} - \frac{R_k}{F_u'} . 2J\frac{\beta}{b_1} + \frac{R_k}{F_u''} . 2J\frac{t}{T} = 0$$

$$(L + \Sigma . M)\frac{2J}{T} + f(t)_k - 2J . r\left(\frac{1}{2} - \frac{t}{T}\right) - 2J . r_c\left(\frac{\beta}{b_1} - \frac{t}{T}\right) - 2J . \frac{R_k}{F_u} + 2J . \frac{R_k}{F_u} = 0$$

$$f(t)_k = -2J\left\{\frac{L + \Sigma . M}{T} + r\left(\frac{t}{T} - \frac{1}{2}\right) + r_c\left(\frac{t}{T} - \frac{\beta}{b_1}\right)\right\}$$

(ii) From $t = \frac{\beta}{b_1} . T$ to $t = \left(1 - \frac{\beta}{b_1}\right) . T$

the currents in the leading and trailing connectors are $2J . \frac{\beta}{b_1}$ and $-2J . \frac{\beta}{b}$, while in each case the whole sector is covered, so that $F_u' = F_u'' = \beta . b$.

In the same way as above we find

$$f(t)_k = -2J\left\{\frac{L+\Sigma\,.\,M}{T} + r\left(\frac{t}{T} - \frac{1}{2}\right)\right\}$$

(iii) From $t = T\left(1 - \frac{\beta}{b_1}\right)$ to $t = T$

the currents in the leading and trailing connectors are respectively $2J\,.\frac{T-t}{T} = 2J\left(1 - \frac{t}{T}\right)$ and $-2J\,.\,\frac{\beta}{b_1}$, while $F_u' = F_u\,.\,\frac{T-t}{T}$ and $F_u'' = \beta\,.\,b$. Thence

$$f(t)_k = -2J\left\{\frac{L+\Sigma\,.\,M}{T} + r\left(\frac{t}{T} - \frac{1}{2}\right) + r_c\left[\frac{t}{T} - \left(1 - \frac{\beta}{b_1}\right)\right]\right\}$$

Corresponding, therefore, to the three stages, the reversing field should have three portions of different slope (Fig. 344).

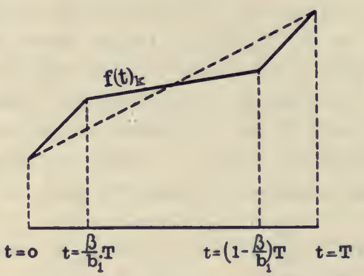


FIG. 344.—Reversing E.M.F. required for uniform commutation.

From the initial and final values of $f(t)_k$, namely—

$$f(t)_{t=0} = -2J\left\{\frac{L+\Sigma\,.\,M}{T} - \frac{r}{2} - r_c\,.\,\frac{\beta}{b_1}\right\} = -2J\left\{\frac{L+\Sigma\,.\,M}{T} - \frac{R}{2}\right\} \quad \ldots (96)$$

$$f(t)_{t=T} = -2J\left\{\frac{L+\Sigma\,.\,M}{T} + \frac{r}{2} + r_c\,.\,\frac{\beta}{b_1}\right\} = -2J\left\{\frac{L+\Sigma\,.\,M}{T} + \frac{R}{2}\right\} \quad \ldots (97)$$

we obtain an average expression with uniform slope, as shown by the dotted line (Fig. 344), for which the equation is

$$f(t)_k = -2J\left\{\frac{L+\Sigma\,.\,M}{T} + R\left(\frac{t}{T} - \frac{1}{2}\right)\right\} \quad . \quad . \quad . \quad (98)$$

where

$$R = r + 2r_c\,.\,\frac{\beta}{b_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

This holds strictly when $b_1 - m \leqq 2\beta$, and the intermediate stage disappears.

If the brush width is less than or equal to that of a sector, or $b_1 - m < \beta$, the currents in the leading and trailing commutator connectors become respectively $2J\left(\frac{T-t}{T}\right)$, and $-2J\cdot\frac{t}{T}$; the same equation then results, save that we have M instead of Σ . M, and R is $r+2r_c$.

$\frac{2J(L+\Sigma \cdot M)}{T}$ is evidently the average value of the E.M.F. from self and mutual inductance, or the constant instantaneous value which results when the commutation is actually performed uniformly; it may therefore be called the "inductive voltage" as opposed to the ohmic voltage JR. The density of the main field, from the field excitation and with allowance for all the reacting ampere-turns of the armature save those of the coils actually undergoing short-circuit, ought then to rise in an inclined straight line by an amount proportional to JR. The sign of the correct commutating field at the commencement of short-circuit may require to be either positive or negative, according as $\frac{R}{2}$ is greater or less than $\frac{L+\Sigma \cdot M}{T}$; usually $\frac{R}{2}$ is the lesser quantity, so that the impressed field-density must be throughout negative or reversing. At the beginning it balances the difference between the inductive and ohmic voltages, and at the end it balances their sum.

The above assumes that $L+\Sigma$. M is constant for any one coil, and further, when applied to determine the requisite strength of reversing field, that it is the same for each coil which is simultaneously short-circuited. When b_1-m much exceeds β, as already assumed in § 16, although the coils are themselves moving, the system of currents within them becomes fixed in space, and so also the magnetic fields to which they give rise would become constant and fixed in space. The component E.M.F.'s set up within the coils can then, if required, be traced by considering them as moving through component fields, partly due to external excitation modified by the reaction of the remaining full-current armature-turns, and partly due to their own currents. It is now assumed that, with $b_1-m>2\beta$, the self and mutually induced field which is regarded as fixed spacially is also of uniform density, and so by movement of the coils through it a constant E.M.F. is yielded which must be balanced by a similar uniform component of the external field.

The assumptions of a constant and similar value for the $L+\Sigma$. M of the short-circuited coils is incorrect, and especially in the case of toothed armatures with several coil-sides in each slot, when the short-circuit curves cannot be exactly alike owing to the different positions of the coil-sides relatively to the commutator sectors. But the curve of the commutating field must in any case be smooth, and cannot have abrupt changes of inclination, so that only an average adjustment of it is possible. A more correct shape for it in the case of a number of sectors simultaneously short-circuited can be theoretically deduced,* but its closer determination is not of practical value owing to the many secondary effects from vibration, etc. that enter into the problem. The equation (98) and the value of the correct reversing density B_k for the resultant field thence deduced are therefore sufficiently accurate for practical use.

§ 20. **The system of i_z components.**—Next taking the components i_z, i_{zI}, etc., the determination of an entirely true differential equation is not easily effected. In the first place, since $\frac{di_{zI}}{dt}$, etc. must have a different sign from $\frac{di_z}{dt}$ for some portion of the time of short-circuit if commutation is actually effected, there is at such times a straightening effect from i_{zI}, etc.; *e.g.*, in the case of a dynamo, as in Figs. 338 and 339, if three coils are simultaneously short-circuited at each set of brushes, and coil A is close to the end of its short-circuit period, in the remaining coils A_I and A_{II} if then situated at the points shown in Fig. 345, the rates of change $\frac{di_{zI}}{dt}$ and $\frac{di_{zII}}{dt}$ as shown by the tangents to the curve of i_z are of opposite sign to $\frac{di_z}{dt}$, and help to reduce the apparent inductance of A. On the other hand, for the same reason the apparent inductance of coil A when itself at the earlier stages of commutation is actually increased, since it is then also carrying the damping current set up by another coil which is approaching the end of its short-circuit. The case is therefore not one of simple damping, and the inductance of coil A in relation to i_z is not a constant, even if the change due to its movement by rotation up to the leading pole-edge is neglected. The complex interdependence of the various rates of change must, however, be replaced by some simpler hypothesis, and it must suffice to treat coil A as moving through a field which gives $f(t)_z$ in close proximity to other coils which, throughout, act simply as dampers and have no other than damping currents within them. We can then return to the second (b) of the two suppositions described in § 18, and can retain the apparent inductance in its special form

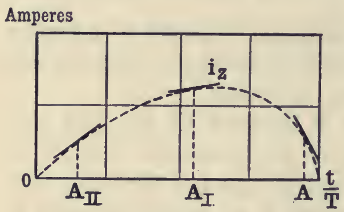


Fig. 345.

$$L_{sz} = L + M - \Sigma \cdot \left(\frac{M_I^2}{L}\right) \quad . \quad . \quad . \quad . \quad . \quad (100)$$

Even in the simple case when $b_1 - m \leqq \beta$, provided that $\frac{N_2}{2p}$ is an

* As shown by Professor Arnold, *Die Gleichstrommaschine*, vol. i. p. 457 (2nd edit.).

uneven number, $\frac{M_I^2}{L}$ and $\frac{M_{II}^2}{L}$ from coils B_I and B_{II}' enter into the problem, although M is absent. The differential equation of i_z in this case would be

$$-L_{sz} \cdot \frac{di_z}{dt} - i_z \left\{R + \frac{R_k \cdot T}{F_u}\left(\frac{1}{t} + \frac{1}{T-t}\right)\right\} + f(t)_z = 0 \quad . \quad . \quad (101)$$

From this equation, if both $f(t)_z$ and L_{sz} were known, the complete curve of i_z, and therefore of i during the period of short-circuit, could be deduced. But this would have again to be corrected to allow for the damping component which the considered coil would itself have carried in relation to other coils at some stages of the period until finally the i_z contained within itself the effects of the i_z in other coils. Such refinement of calculation would be impracticable, but its statement illustrates the real processes. The important point is that any gain from

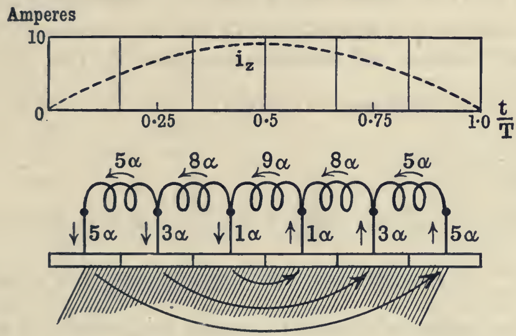


FIG. 346.—Brush covering a number of sectors.

damping can only be obtained at the expense of greater divergence from the straight-line ideal in the initial stages.

In the more practical case of a brush covering a number of sectors or $b_1 - m > \beta$, additional complexity arises from the fact that each coil is electrically connected through the brush with other coils on one or both sides of it. The resistances presented to the short-circuit of any one section considered independently are therefore continually varying not only by reason of the varying contact-resistance of the brush, but also as different combinations of coils are formed in parallel with the connections of the considered coil. An additional current i_z may then be regarded as flowing through the coils and of varying amount from end to end. An imaginary case when a coil near to the centre of the brush carries the maximum i_z is indicated roughly in Fig. 346, and it is seen that this current is progressively fed into the winding from one end of the brush and tapped off at the other end through the commutator connectors. The additional current in connectors, except when at the extreme ends, is not therefore the same as the i_z of the coil. But if we arbitrarily choose the moment when the leading sector is situated under the tip of the brush and consider only the period T′ during which the remainder of the commutation is performed (cp. equation (94), § 17), an equation exactly analogous to (101) can be constructed. In either case other resistances then become negligible in comparison with $\frac{R_k}{F_u} \cdot \frac{T}{T-t}$ or $\frac{R_k}{F_u} \cdot \frac{T'}{T'-t}$. In fact, since the denominator of this term becomes infinitely large at the last moment, it is evident that the total resistance of the short-circuit becomes infinite, and with any finite value of $f(t)_z$ at the last moment i_z becomes zero, although $\frac{di_z}{dt}$ may be very high. In other words, since the contact-resistance between the toe of the brush and the leading sector at the moment of separation is infinite, the above equations, if they held good without other secondary effects, show that the current would always be exactly and automatically commuted at the end of short-circuit by the mere action of the brushes, whatever their position and whatever the value of $f(t)$. It can also be deduced that if the final rate of change $\frac{di_z}{dt}$ is to be finite we must have

$$\frac{R_k \cdot T'}{\beta \cdot b \cdot L_{sz}} \geqq 1 \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

and the higher the better. Since $T' = 5\frac{\beta}{v_k'}$, this may also be put in the form $\frac{5R_k}{v_k' \cdot b \cdot L_{sz}} \geqq 1$, which brings out the advantages of a low peripheral speed of commutator, of a small L_{sz}, and a hard variety of carbon.

Such a deduction from a rigid analysis of the mathematical equation, although useful as a guide, is not however to be implicitly relied upon in practical use, since in the formation of the equation other conditions have not been taken into account. It assumes an infinite resistance between the trailing edge of the sector and the brush as soon as the latter has left the sector, but in practice surface leakage may assist in carrying the final i_z after the time T, and also the effect of capacity between the sector and brush at the moment of separation is ignored.*

§ 21. **Importance of the quantity** $L + \Sigma . M$.—In order to reduce the strength of reversing field necessary to give straight-line commutation and a uniform current-density under the brushes, $L + \Sigma . M$ in equation (98) should be small. The coefficients of mutual induction M_I, M_{II}, etc. between the considered coil and others which are simultaneously short-circuited should therefore be reduced as far as possible. On the other hand, in order that the final rate of change of the additional current should not be unduly high, L_{sz} in equation (101)

* H. Poincaré, *Lumière Électr.*, June 6, 1908; J. Bethenod, *Lumière Électr.*, July 18 and August 22, 1908; and R. Rüdenberg, *E.T.Z.*, April 22, 1909.

or (102) should be as small as possible, and the mutual inductance of other coils in close proximity should be high so as to increase their damping effect. Thus the more L_{sz} is reduced the greater becomes $L + \Sigma . M$, and reductions in both quantities are incompatible. A reduction in $L + \Sigma . M$ should, however, take precedence over any reduction in L_{sz}; the former is the chief factor in the required reversing field, and determines the necessary angle of lead in those cases where we are dependent upon shifting the brushes. The greater the required density of reversing field, the more sensitive becomes the machine to changes of load or to any dissymmetry of the various sections. Further, if there should be no reversing field present and the actual external field is small and negligible, the product of $L + \Sigma . M$ with $\frac{2J}{T}$ becomes identical in numerical value with $f(t)_z$ during the central part of the period of commutation; it is then a sound guide to the amount of work that will be left to the corrective action of the brush contact-resistance to perform.

In order, therefore, to forecast the behaviour of a dynamo as regards sparkless running, it is imperative for the designer to estimate even if only approximately the value of $L + \Sigma . M$. He must so dispose the coils which are simultaneously short-circuited that they have minimum mutual inductance as far as this is not forbidden by other considerations, and in each coil the number of turns must be so small, or, which amounts to the same, the number of commutator sectors for the same number of active wires must be so large, that $L + \Sigma . M$ is reasonably low.

It has already been stated that the self- and mutual-inductance of a short-circuited coil, strictly speaking, varies according to its position on the armature core, and though in a generator it increases during short-circuit as the coil approaches nearer to the pole-tip by rotation, yet that it may approximately be regarded as constant for a given position of the brushes. But now, further, for the purpose of approximate calculation the exact position of the brushes must be ignored, and $L + \Sigma . M$ must practically be identified with the value obtained when the group of coil-sides which are short-circuited at all brushes are as nearly as possible in the centre of the interpolar zone. The $L + \Sigma . M$ of any one considered coil varies according to its position within the group, but in order that any error may be on the safe side it is best to take that position of the coil which leads to the maximum value of the inductance, *i.e.* when it is as nearly as possible in the centre of the group, and to regard this as the constant value. Even then the quantity $L + \Sigma . M$ is by no means easily calculated with any great accuracy, and much could be done by direct experiment in the technical laboratory to bring the methods of calculation into closer relation to the facts. Thus the assumption that has been made that all flux entering the pole-faces also passes through the exciting coils, and there has its variations so completely damped out that it adds nothing to the inductance, is perhaps hardly true for the flux entering the extreme pole-tips, and in this connection a difference must exist between laminated and solid pole-shoes. With the former the value of the apparent inductance is probably higher than with the latter, but to what extent only direct experiment could show.

§ 22. **Method of calculating the self and mutual inductance of armature coils.**—The *inductance* of a coil in absolute units is equal to the number of linkages of its component turns with the lines which thread through them when the current carried is one C.G.S. unit. Let w be the number of wires in one side of a coil, *i.e.* the coil has w turns. The M.M.F. of these wires when carrying 1 C.G.S. unit of current is $4\pi w$, and this acts upon a magnetic circuit of which the permeance has to be determined. But since the wires must necessarily occupy some space, they do not all act upon precisely the same circuit, so that all the flux to which they give rise is not completely linked with all the wires. The inductance of the group of wires is therefore usually given by obtaining an expression for a single equivalent permeance $\mathcal{P}$ such that if it were acted upon by the M.M.F. of all the wires, or by $4\pi w$, the resulting flux $4\pi w\ \mathcal{P}$ linked with all the w wires gives the actual total number of linkages. The general expression for the inductance of the group of wires is then

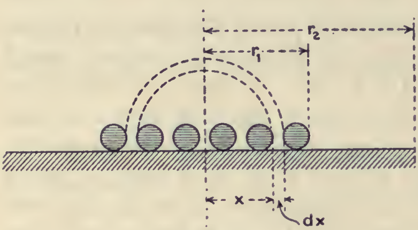


FIG. 347.

$$L = 4\pi \,.\, w^2\, \mathcal{P} \times 10^{-9} \text{ henrys} \quad . \quad . \quad . \quad (103)$$

To take a leading case, in Fig. 347 let there be w wires forming one side of a coil laid on the smooth surface of a flat iron core of axial length l centimetres. Within the radius r_1 = half the width of the coil, consider a small strip of breadth dx distant x centimetres from the centre of the turns; the M.M.F. acting on it for a current of 1 C.G.S. unit per wire is $4\pi w \,.\, \frac{x}{r_1}$. On the assumption that the iron core is infinitely permeable, and that the paths of the lines in the air are semicircles, the permeance of the strip is $\frac{l \,.\, dx}{\pi x}$. Hence the number of lines in the strip is

$$4\pi w \,.\, \frac{x}{r_1} \,.\, \frac{l \,.\, dx}{\pi x} = 4\pi w l \,.\, \frac{dx}{r_1 \pi}$$

and these are linked with $w\frac{x}{r_1}$ wires. Therefore an element of the self-induction is $dL_1 = 4\pi w^2 \frac{l}{r_1^2 \pi} x dx$. The integral $\int_0^{r_1} x dx$ being $\frac{r_1^2}{2}$,

$$L_1 = 4\pi w^2 \cdot \frac{l}{2\pi}$$

Hence for the same number of wires and length of core the inductance within the limits of the coil itself is constant, however close or far apart the wires are, since if the wires are spread out the area and length of path increase equally, so that the same number of lines remains linked with the same number of turns.

Within the region $r_2 - r_1$, where r_2 = half the interpolar gap, the M.M.F. is constant, and the flux is actually linked with all the wires. The permeance of this region is by equation $(71a) = l \cdot \frac{2\cdot 3}{\pi} \cdot \log \frac{r_2}{r_1}$, and

$$L_2 = 4\pi w^2 \cdot l \cdot \frac{2\cdot 3}{\pi} \log \frac{r_2}{r_1}.$$

The total inductance of the group of wires due to the flux between the pole-tips is therefore

$$L = L_1 + L_2 = 4\pi w^2 \cdot l \left(\frac{1}{2\pi} + \frac{2\cdot 3}{\pi} \cdot \log \frac{r_2}{r_1} \right)$$

or the single equivalent permeance is $\mathcal{P} = l \left(\frac{1}{2\pi} + \frac{2\cdot 3}{\pi} \log \frac{r_2}{r_1} \right)$.

Usually $4\pi\mathcal{P}$ is grouped together as Λ in conventional units, so that we have for a single coil-side

$$L = w^2 \Lambda \times 10^{-9} = w^2 l \lambda \times 10^{-9} \text{ henrys} \quad . \quad . \quad . \quad . \qquad (104)$$

and in the above case

$$\lambda = 2 + 9\cdot 2 \log \frac{r_2}{r_1} \quad . \quad . \quad . \quad . \qquad (105)$$

The coefficient λ is therefore essentially an expression for the equivalent permeance per centimetre length of core $\times 4\pi$, and for the case of a coil-side resting on a smooth iron core is composed of a constant term which has to do with the inductance from the flux within the limits of the wires forming the coil-side and a term which continually increases but at a diminishing rate as we widen the boundary of the region considered.

If now there are present j coil-sides, each with w wires, lying alongside of one another, and the inductance of one of the coil-sides is to be calculated, there will be mutual inductance between the considered coil-side and the others which are adjacent to it. Several different methods of procedure can then be followed, dependent upon different ways of grouping the various permeances and fluxes; thus the entire self-inductance of the considered coil may first be calculated, and to this will be added the mutual inductance, an equivalent permeance being found which when acted on by the remaining $(j-1)$ coil-sides will give the actual number of linkages between the considered coil and the flux of the remaining coils. It is, however, usually more convenient to take separately the self-inductance of the considered coil so far as it embraces a local magnetic circuit which is independent of the remaining coils, and then to calculate an equivalent permeance $\mathcal{P}'$ such that, when acted upon by a M.M.F. of $4\pi jw$ from *all* the wires, the flux which results if linked with all the w wires of the considered coil will give the actual number of linkages from both the self and mutually induced flux in that portion of the magnetic circuit which is common to them all. We thus have

$$\begin{aligned} L+\Sigma \, . \, M &= 4\pi w^2(\mathcal{P}+j\mathcal{P}') \\ &= w^2\,(\Lambda+\Lambda') \times 10^{-9} \text{ henrys} \end{aligned}$$

Or if the number of coil-sides affecting the considered coil varies at different parts, the second term is again subdivided, j being given its correct value from one upwards for each of the several subdivisions, so that in general

$$\begin{aligned} L+\Sigma \, . \, M &= 4\pi w^2(\mathcal{P}+j_1\mathcal{P}+j_2\mathcal{P}_2+\ldots) \\ &= w^2(l_1\lambda_1+l_2\lambda_2+l_3\lambda_3+\ldots) \times 10^{-9} \text{ henrys} \quad \ldots \quad (106) \end{aligned}$$

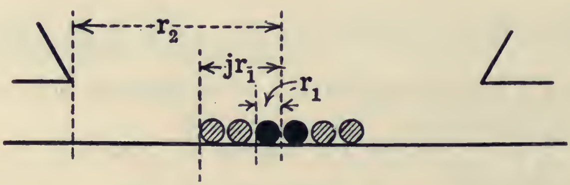


FIG. 348.

A simple case is illustrated in Fig. 348, where there are three coil-sides, each containing two wires lying alongside one another in a single layer on a smooth core. The considered coil is marked black, and there is now mutual inductance between the black coil and the remaining shaded coil-sides. The distribution of the flux when all are carrying 1 C.G.S. unit of current must be exactly the same as in Fig. 347, and the calculation of the self and mutual inductance is made after the same method, but its value will depend upon the position of the considered coil-side within the group, being a maximum if it is at the centre and a minimum if at the outside. Supposing it to be at the centre as shown, and r_1 being again the half width of one coil-side (not of the whole group), the inductance due to the flux within the considered coil-side is as before $2w^2l$. The M.M.F at any distance x from the centre line is $4\pi jw\dfrac{x}{jr_1}=4\pi w\dfrac{x}{r_1}$, and the permeance is $\dfrac{l\,.\,dx}{\pi x}$. Between the limits r_1 and jr_1 the density of the flux is gradually increasing, and the whole of it is linked with the w wires of the centre coil, so that the inductance due thereto is $4\pi w\dfrac{x}{r_1}\times\dfrac{wl}{\pi x}\times\displaystyle\int_{r_1}^{jr_1}dx=w^2l(4j-4)$. Beyond the limits of the group the M.M.F. is constant and is equal to $4\pi wj$, so that the inductance due thereto is $w^2l\times j\times 9{\cdot}2\log\dfrac{r_2}{jr_1}$. The total inductance, self and mutual of the central coil-side is therefore

$$L+\Sigma \,.\, M=w^2 l\left(4j-2+j \,.\, 9{\cdot}2 \log \frac{r_2}{jr_1}\right)\times 10^{-9} \quad . \quad . \quad . \quad . \quad (107)$$

an expression which reduces to the same value as before for L if all the three coils were thrown into one, so that $j=1$. If j is even, the expression for a central coil-side, which however must now lie entirely on one side of the centre line, would be slightly different and would be

$$L+\Sigma \,.\, M=w^2 l\left(4j-4+j \,.\, 9{\cdot}2 \log \frac{r_2}{jr_1}\right)\times 10^{-9}$$

Whether j be odd or even, if the considered coil-side is at the outside of the group

$$L+\Sigma \,.\, M=w^2 l\left(4+j \,.\, 9{\cdot}2 \log \frac{r_2}{jr_1}\right)\times 10^{-9} \quad . \quad . \quad . \quad . \quad . \quad (108)$$

If the width of a coil be only that of one wire, all the lines in the region of radius r_1 are within the copper of the conductor ; the effect of any alteration of these lines is then to modify the distribution of any current within the wire and to render it non-uniform. The constant term becomes thus of less importance, and the flux within r_1 may practically be neglected, with the result that the expressions for a central and outer coil-side become respectively

$$w^2 l\left(4j-4+j \,.\, 9{\cdot}2 \log \frac{r_2}{jr_1}\right)\times 10^{-9},$$

$$\text{and} \quad w^2 l\left(j\times 9{\cdot}2 \log \frac{r_2}{jr_1}\right)\times 10^{-9}$$

§ 23. **The apparent inductance of smooth-surface armature coils.**—(i) In a *ring* armature each coil-side forming an element of the armature winding contains $w=\frac{\tau}{N_2}$ wires. The permeance in relation to a complete coil may be divided into the three portions corresponding to (1) the exterior of the core and (2) its interior, both proportional to the length l of the core in centimetres, and (3) the end-connections on the two flanks, each of l' centimetres. The general expression for the self and mutual inductance of a ring coil is therefore

$$L+\Sigma \,.\, M=\left(\frac{\tau}{N_2}\right)^2[l(\lambda_1+\lambda_2)+2l'\lambda']\times 10^{-9} \text{ henrys} \quad . \quad . \quad . \quad . \quad (109)$$

If the surface of the core be smooth, for the first item λ_1 may be substituted $j\left(2+9{\cdot}2 \log \frac{r_2}{jr_1}\right)$, being approximately an average between the values for a central and an outer position of the coil. Since the short-circuited coils lie side by side, the quantity j is equal to the number of sections simultaneously short-circuited by a brush or $\left(\frac{b_1-m}{\beta}\right)_+$ (the plus sign signifying that the next larger whole number is to be taken if there is any remainder), and r_1=half the width of the coil. The effect from the curvature of the core may be neglected, and for the purpose of calculating the apparent inductance in commutation, the limit r_2 up to which the permeance is integrated may be fixed as in Fig. 348 at the distance between the interpolar line of symmetry and the edge of the pole-tip.

In the interior there is no such limit, and the same expression may approximately be employed if for r_2 be substituted half the pole-pitch measured on the inner periphery, while in the case of the flanks may be substituted for r_2 half the pole-pitch reckoned along the mean of the outer and inner peripheries. If the binding-wires are of steel there may further require to be added a term corresponding to the flux which passes across the group of short-circuited wires by way of the bands.

(ii) In the case of a *drum* armature the number of wires in a coil-side forming an element of the armature winding is, as in the ring, equal to the number of turns in the coil, but is now $w=\left(\frac{\tau}{2N_2}\right)$. It must especially be remarked that in the multipolar drum armature the interpolar zones of the armature core are not necessarily alike; when the number of commutator sectors per pole or $\frac{N_2}{2p}$ is *whole*, if the position of the short-circuited coil-sides in a pair of consecutive interpolar zones is set out, the remaining interpolar zones of the multipolar machine are merely repetitions of the first pair, but this is not the case when $\frac{N_2}{2p}$ is a *fractional* number. Further, unless the pitch of the coil is equal to the pole-pitch, so that it may by analogy from the 2-pole case be called "diametric," the positions of the two sides of any one coil among the groups of short-circuited coil-sides are dissimilar. Strictly speaking, therefore, λ_1 and λ_2 must be determined for each of the two coil-sides separately, and the permeance in relation to a drum coil as for a ring has three terms, one for each of the sides of length l centimetres, and one for the two end-connections, each of length l' centimetres. The general expression for the self and mutual inductance of the drum coil is thus

$$L+\Sigma.M=\left(\frac{\tau}{2N_2}\right)^2[l(\lambda_1+\lambda_2)+2l'\lambda']\times 10^{-9} \text{ henrys} \quad . \quad . \quad . \quad . \quad (110)$$

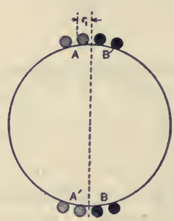


FIG. 349.

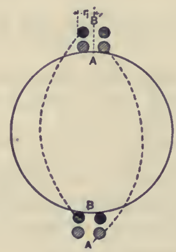


FIG. 350.

With a single layer of winding on a smooth core, if the pitch of the elements is so nearly diametric that all the coil-sides short-circuited in an interpolar zone are adjacent and form a single group of total width $2jr_1$, where r_1 is the half-width of one element (as in Fig. 349, where there are in each zone two elements short-circuited, each of which consists of two wires), then, as in the ring, whatever the position of the considered coil-side, the average expression for the core coefficient of one zone may be used

$$\lambda=j\left(2+9{\cdot}2 \log \frac{r_2}{jr_1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

As the number of coil-sides simultaneously short-circuited in each zone is often alike, and in the present case only differs by one, j_1 and j_2 may be identified with $2\left(\frac{b_1-m}{\beta}\right)$, and the two zones may be treated as alike, so that $l(\lambda_1+\lambda_2)$ becomes equal to $2l\lambda$, and

$$L+\Sigma.M=\left(\frac{\tau}{2N_2}\right).2(l\lambda+l'\lambda')\times 10^{-9} \text{ henrys} \quad . \quad . \quad . \quad . \quad . \quad (112)$$

If the winding be arranged in two layers (Fig. 350), and for simplicity's sake the effect of the greater distance of the upper layer from the core is neglected, the constant term and the ratio $\frac{r_2}{jr_1}$ must be multiplied by 2, and λ may approximately be reckoned to be

$$\text{for a central coil-side} = 8j - 4 + 9{\cdot}2j \log \frac{2r_2}{jr_1},$$

$$\text{or for an outer coil-side} = 8 + 9{\cdot}2j \log \frac{2r_2}{jr_1}$$

Here r_1 is, as before, the half width of a coil-side, so that the total width of the group is jr_1, and on an average

$$\frac{\lambda_1 + \lambda_2}{2} = \lambda = j\left(4 + 9{\cdot}2 \log \frac{2r_2}{jr_1}\right) \quad . \quad . \quad . \quad . \quad . \quad (113)$$

If the pitch of the winding is shortened, so that among the group of short-circuited coil-sides there are interspersed others which are not short-circuited, the value of λ, whether for a single or double layer, must be reduced by multiplying it by a corrective factor diminishing from unity as the coils become more chord-wound.

The end-connections of the drum coils which are short-circuited at one and the same brush always run side by side, and the front and back ends are alike; hence, in relation to the end-connections, j_c is as in the ring always equal to $\left(\frac{b_1 - m}{\beta}\right)_+$ and $\lambda' = c \, . \, j_c$ where c will be given different values according as the armature is barrel-wound or has involute end-connectors lying close up against the core. In the latter case if $j_c = 1$, c ranges from about 8 to 12. As j_c increases c diminishes, since, as will be shown later, the length of air-path of the flux linked with all the wires gradually increases, but the product $c \, . \, j_c = \lambda'$ itself continuously increases.

§ 24. **The apparent inductance of coils of toothed drum armatures.**—In the case of the toothed drum armature which has the greatest practical interest, in addition to the main division of the permeance into portions corresponding to the core-length and end-connections, the former must be subdivided into portions corresponding (*a*) to the slot, and (*b*) to the surface of the core. The first item (*a*) is itself again divisible into a part (*a*′) due to the flux within the slot proper, and a part (*a*″) due to the flux across the opening of the slot which may have overhanging edges; the former depends on the number of layers, etc., but for a given distribution of the winding is proportional to the ratio of the depth of the slot to its width, or to $\frac{h_s}{w_s}$; the latter is proportional to the ratio of the height of the opening to its mean width, or $\frac{h_3}{w_3}$ (Fig. 351).

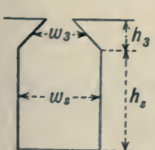


Fig. 351.

The surface-of-the-core permeance is divisible into the portions corresponding to the joint flux linked with all the simultaneously short-circuited sections, and to the local fluxes immediately linked with the section under consideration, or with two or more out of the total number. These several permeances, if expressed in terms of j_b, the total number of coil-sides short-circuited in the zone, may be grouped together as $b \, . \, j_b$.

In the toothed armature, apart from the causes mentioned in § 23 which lead to a dissimilarity in two consecutive interpolar zones in which a considered coil may lie, this also results even when $\frac{N_2}{2p}$ is a whole number if the number of slots is not exactly divisible by the number of poles without remainder. The first operation, then, must be to clearly set out the position of the short-circuited coil-sides in all the interpolar zones so far as these differ, and to choose a certain pair of zones and a particular position for the considered coil therein which it is estimated will give the maximum number of linkages and maximum inductance. The situations of the two sides of one complete coil usually differ so radically that it is best to treat them separately and to add their inductances subsequently. The three items a', a'', and b which are proportional to the length of the core will therefore require to be estimated separately for each of the two zones in which lie the sides of the coil, and to be indicated as a_1', a_2', a_1'', a_2'', b_1 and b_2. For each of the several portions, again, the correct value for j must be taken.

The general expression for the apparent inductance, self and mutual, of a drum coil on a toothed armature is therefore

$$L+\Sigma . M = \left(\frac{\tau}{2N_2}\right)^2 \Big[l\left(a_1' j_{a1}' \frac{h_s}{w_s} + a_1'' . j_{a1}'' . \frac{h_3}{w_3} + b_1 . j_{b1}\right)$$

$$+ l\left(a_2' . j_{a2}' . \frac{h_s}{w_s} + a''_2 . j_{a2}'' . \frac{h_3}{w_3} + b_2 . j_{b2}\right) + 2l' . c . j_c \Big] \times 10^{-9} \ldots (114)$$

$$= \left(\frac{\tau}{2N_2}\right)^2 \Big[l\left(k_1' . \frac{h_s}{w_s} + k_1'' . \frac{h_3}{w_3} + k_1'''\right)$$

$$+ l\left(k_2' . \frac{h_s}{w_s} + k_2'' . \frac{h_3}{w_3} + k_2'''\right) + 2l'\lambda' \Big] \times 10^{-9} \text{ henrys} \ldots (115)$$

which again returns to the original form of (110), namely,

$$L+\Sigma . M = \left(\frac{\tau}{2N_2}\right)^2 \Big[l\left(\lambda_1 + \lambda_2\right) + 2l'\lambda' \Big] \times 10^{-9} \quad . \quad . \quad . \quad (110)$$

(*a*) *The slot inductance* is easily deduced from a few simple cases, on the assumptions that the iron of teeth and core may be regarded as infinitely permeable, so that all the lines pass round through the iron at the root of the tooth, and that all the flux within the slot passes in straight lines across from side to side. The usual case of a barrel-winding in two layers may also be assumed.

Thus with a single coil-side containing w wires at the top of a slot (Fig 352), within the region of its own depth which is practically equal to $\frac{h}{2}$, the self-induced flux across the slot is increasing in density at a uniform rate as we advance upwards towards the opening; at the same time the wires with which this flux is linked are also increasing at a uniform rate. Taking any small strip across the air in the slot, distant x centimetres from the bottom of the coil-side, and within the limit $\frac{h_s}{2}$, the M.M.F. acting across it with a current of 1 C.G.S. unit is $4\pi w \frac{x}{h_s/2}$, and the permeance is $\frac{l \,.\, dx}{w_s}$. Hence the lines in the strip are $\frac{4\pi w l \,.\, 2x dx}{w_s \,.\, h_s}$, and these are linked with $w \,.\, \frac{x}{h_s/2}$ wires. An element of the inductance is therefore

$$dL = \frac{4\pi w^2 \,.\, l \,.\, 4x^2 dx}{w_s \,.\, h_s^2}$$

The integral $\int_0^{\frac{h_s}{2}} x^2 \,.\, dx$ being $\frac{1}{3} \times \frac{h_s^3}{8}$, the total number of line-linkages, or the inductance, is per centimetre length of the slot

$$\frac{2\pi}{3} \,.\, w^2 \,.\, \frac{h_s}{w_s} = 2{\cdot}09\, w^2 \,.\, \frac{h_s}{w_s}$$

Let a short-circuited coil-side be now added in the lower layer

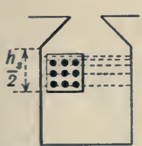


FIG. 352.

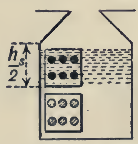


FIG. 353.

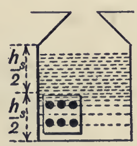


FIG. 354.

(Fig. 353), its wires being shaded in order to distinguish it from the considered coil of which the wires are black; a certain amount of mutual inductance is thereby added. The flux due to the second coil-side has a uniform density $\frac{4\pi w}{w_s}$ across the upper half of the slot, but the black wires with which it is linked gradually increase as $w \,.\, \frac{x}{h_s/2}$. The mutual inductance per centimetre length of slot is then

$$\pi \,.\, w^2 \,.\, \frac{h_s}{w_s} = 3{\cdot}14\, w^2 \,.\, \frac{h_s}{w_s}$$

so that the $L + \Sigma \,.\, M$ of the upper coil-side per centimetre length is $\frac{5\pi}{3} w^2 \,.\, \frac{h_s}{w_s} = 5{\cdot}23\, w^2 \,.\, \frac{h_s}{w_s}$.

If the considered coil-side be at the bottom of the slot (Fig. 354), within the lower half of the slot to the height $\frac{h_s}{2}$, there will be the same flux and the same inductance due to it as in our first case, but to this has now to be added a further self-induced flux of uniform density in the upper half of the slot linked with all the wires; whence the additional inductance is per centimetre length of slot $4\pi w^2 \cdot \frac{h_s}{2w_s}$.

$$= 2\pi w^2 \cdot \frac{h_s}{w_s} = 6{\cdot}28\, w^2 \cdot \frac{h_s}{w_s}$$

so that the L of the single coil-side in the lower layer is

$$\frac{8\pi}{3} \cdot w^2 \cdot \frac{h_s}{w_s} = 8{\cdot}37\, w^2 \cdot \frac{h_s}{w_s}$$

If in either of the two layers there are j coil-sides short-circuited instead of the single one shown above, it is only necessary to multiply the corresponding item by j. The three leading cases when expressed in a form immediately applicable to our main equation are therefore as follows:—

(i) If the flux-density across the slot and the wires with which it is linked are both increasing, the inductance per centimetre length of slot is

$$\left(\frac{\tau}{2N_2}\right)^2 \times 2{\cdot}09\, j_a' \cdot \frac{h_s}{w_s} \times 10^{-9},$$

and $$k' = a' \cdot j_a' = 2{\cdot}09\, j_a' \quad . \quad . \quad . \quad . \qquad (116)$$

(ii) If one is constant, and the other is increasing

$$\left(\frac{\tau}{2N_2}\right)^2 \times 3{\cdot}14\, j_a' \cdot \frac{h_s}{w_s} \times 10^{-9},$$

and $$k' = 3{\cdot}14\, j_a' \quad . \quad . \quad . \quad . \quad . \qquad (117)$$

(iii) If both the flux-density and the wires with which the flux is linked are constant

$$\left(\frac{\tau}{2N_2}\right)^2 \times 6{\cdot}28\, j_a' \cdot \frac{h_s}{w_s} \times 10^{-9},$$

and $$k' = 6{\cdot}28\, j_a' \quad . \quad . \quad . \quad . \quad . \qquad (118)$$

while (iv) the inductance per centimetre length of slot from the flux across the opening of the slot due to j_a'' short-circuited coil-sides within the slot is invariably

$$4\pi w^2 \cdot j_a'' \cdot \frac{h_3}{w_3} = \left(\frac{\tau}{2N_2}\right)^2 \times 12{\cdot}57\, j_a'' \cdot \frac{h_3}{w_3} \times 10^{-9},$$

i.e. $$k'' = a'' \cdot j_a'' = 12{\cdot}57\, j_a'' \quad . \quad . \quad . \qquad (119)$$

The four cases enumerated above rise in the proportion $\frac{2\pi}{3}$, π, 2π, 4π, and into these elements any complicated case may be resolved; thus the value of the coefficient k' for the different distributions of Fig. 355 are

(1)	$k' = 2{\cdot}09 + 6{\cdot}28$	$= 8{\cdot}37$
(2)	$= 2{\cdot}09 + 6{\cdot}28 + 3{\cdot}14$	$= 11{\cdot}51$
(3)	$= 2{\cdot}09 + 3{\cdot}14$	$= 5{\cdot}23$
(4)	$= 2\,(2{\cdot}09 + 6{\cdot}28)$	$= 16{\cdot}74$
(5)	$= 2 \times 2{\cdot}09$	$= 4{\cdot}18$

$$
\begin{aligned}
&(6)\ k' = 2\,(2{\cdot}09+6{\cdot}28)+3{\cdot}14 = 19{\cdot}88\\
&(7)\quad = 2\times 2{\cdot}09+3{\cdot}14 = 7{\cdot}32\\
&(8)\quad = 2{\cdot}09+2\times 3{\cdot}14 = 8{\cdot}37\\
&(9)\quad = 2\,(2{\cdot}09+3{\cdot}14) = 10{\cdot}46\\
&(10)\quad = 2\,(2{\cdot}09+6{\cdot}28+3{\cdot}14) = 23\\
&(11)\quad = 3\,(2{\cdot}09+6{\cdot}28)+3{\cdot}14 = 28{\cdot}25\\
&(12)\quad = 3\times 2{\cdot}09+3{\cdot}14 = 9{\cdot}42
\end{aligned}
$$

With a single bar in each layer the self-inductance within its own layer resolves itself into a question of current distribution over its cross section, and may be neglected.

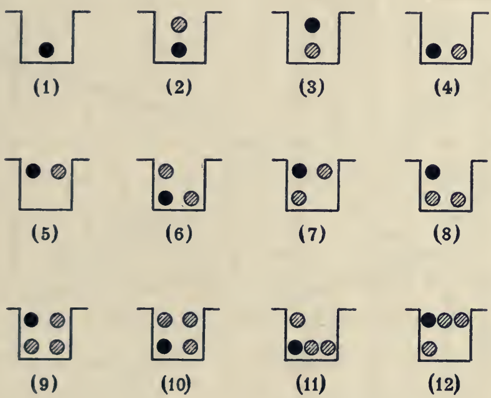


FIG. 355.

The value of k'' for any slot is quickly found by simply counting the number of short-circuited coil-sides in the slot which contains the considered coil (marked black) and multiplying this number by 12·57.

(*b*) The surface of the core.

Secondly, must be taken the inductance from the *surface of the core* up to the pole tips at a distance of r_2 centimetres from the interpolar line of symmetry, the group of short-circuited coil-sides being assumed to be situated midway between the pole-tips for the purpose of obtaining comparative results, although such an assumption is not strictly true when the brushes are shifted away from the line of symmetry.

The leading cases may be grouped into four or five kinds, according as the short-circuited coil-sides are concentrated in one slot or are divided between two, three, four, or five slots. A larger number than five slots containing coil-sides simultaneously short-circuited seldom occurs in practice. The following formulæ and curves are applicable only to a single zone.

(i) When the short-circuited coil-sides are confined to a single slot (Fig. 356), the lines of flux are best assumed to be semicircles spanning the opening of the slot. The permeance in one interpolar zone per centimetre length of core is then, by equation (71*a*), $\frac{2\cdot3}{\pi}\log\frac{r_2}{\frac{1}{2}w_3}$ acted upon by a M.M.F. of $4\pi j_b w$, where j_b is the total number of short-circuited elements in the one zone and w is the number of wires in the coil-side forming an element.

The resulting flux is linked with the w wires of the considered coil, so that the self and mutual inductance is

$$4\pi j_b w^2 \times \frac{2\cdot3}{\pi}\log\frac{2r_2}{w_3} = w^2 \cdot 9\cdot2\, j_b \log\frac{2r_2}{w_3} \times 10^{-9} \text{ henrys}$$

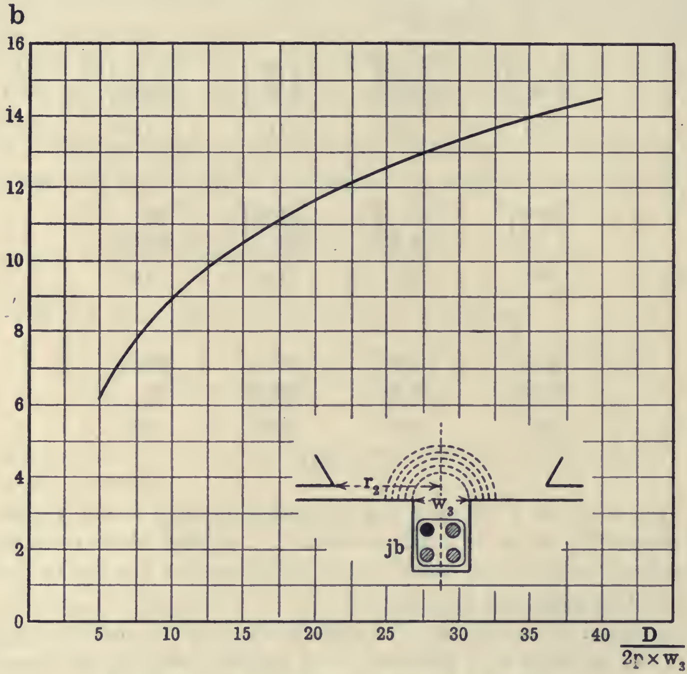


FIG. 356.—Short-circuited coil-sides in a single slot.

Since $w^2 = \left(\frac{\tau}{2N_2}\right)^2$, the coefficient for insertion in our principal equation (115) is for the single zone

$$k''' = b \cdot j_b = j_b \cdot 9\cdot2 \log\frac{2r_2}{w_3}$$

A standard ratio $\frac{\text{pole arc}}{\text{pole pitch}} = \beta = 0\cdot7$ may be assumed as holding throughout, whence $r_2 \doteq \frac{\pi D \times 0\cdot15}{2p}$, and

$$b = 9\cdot2 \log 0\cdot942 \frac{D}{2p \cdot w_3} = 9\cdot2 \log\frac{D}{2p \cdot w_3} - 0\cdot238.$$

The inductance is therefore a function of the quantity $\frac{D}{2p \cdot w_3}$, and the curve of Fig. 356 becomes universally applicable to different numbers of poles and different widths of slot. After having calculated $\frac{D}{2p \cdot w_3}$ in any given case, it is only necessary to read off the value of b from the curve and to multiply it by j_b, or the number of short-circuited coil-sides which are found to be present in the single interpolar zone.

The actual position of the short-circuited coil-sides within the slot is in the present connection immaterial; thus in the diagram the con-

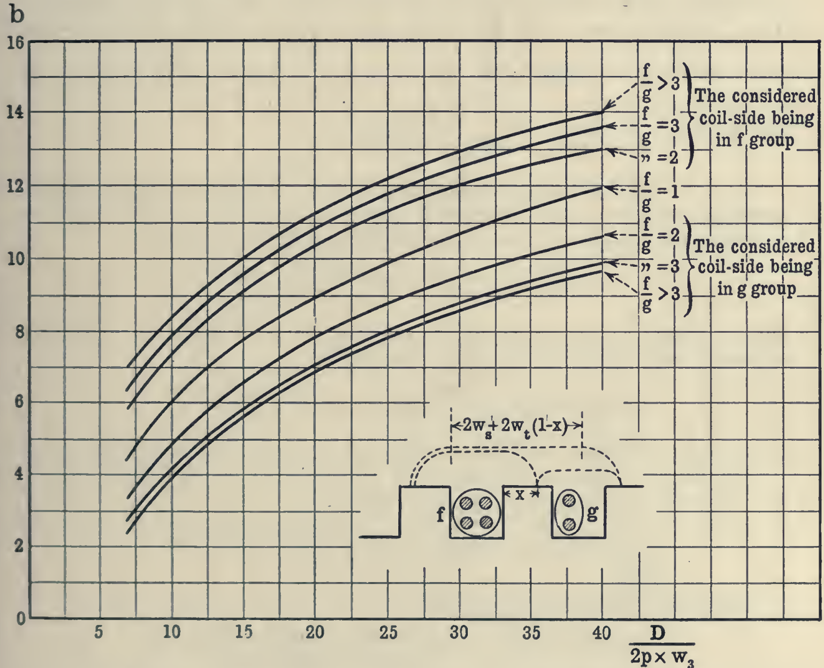


FIG. 357.—Short-circuited coil-sides divided between two slots.

sidered coil-side is marked black, and there are three other short-circuited coil-sides shown shaded, so that $j_b = 4$.

When the short-circuited coil-sides are spread over more than one slot, in addition to the joint flux linked with all the coils, there is also in general a local flux linked immediately with each of the separate slots and occasionally with two or more slots, and this local flux as linked with the slot containing the considered coil-side must be taken into account. In order to deal with these several fluxes it is now best to assume that their paths are quadrants joined by straight lines, so as to fill up more closely the space surrounding the entire interpolar zone;

the assumptions made cannot at best be entirely accurate, but by them correct relative results for comparison are approximately obtained.

(ii) When the short-circuited coil-sides in an interpolar zone are confined to two adjacent slots (Fig. 357), but are not necessarily equally divided between them, let f = the number of coil-sides short-circuited in that slot which has the greater number, and let g = the smaller number of the other slot. At some point along the central tooth the local flux is zero, the general principle of such cases being that the flux becomes zero when the M.M.F. divided by the reluctance of the one path is equal to the M.M.F. divided by the reluctance of the other alternative path in which the lines would have the opposite direction. Thus the fraction x of the intervening tooth which the local flux surrounding f traverses will increase as $\frac{f}{g}$ increases, and is given by the relation $\dfrac{f}{\pi x w_t + w_3} = \dfrac{g}{\pi w_t(1-x) + w_3}$

From the point x the flux surrounding each slot would in strictness increase gradually, but as a first approximation it suffices to calculate by equation (71*a*), the permeance corresponding to each slot at its full value up to the point x,

$$i.e.\ \frac{2\cdot3}{\pi}\log\frac{\pi x w_t + w_3}{w_3},\ \text{or}\ \frac{2\cdot3}{\pi}\log\frac{\pi(1-x)w_t + w_3}{w_3}$$

These are acted upon respectively by a M.M.F. of $4\pi fw$ and $4\pi gw$, so that the coefficient for the inductance due to the local flux is

$$f \times 9\cdot2\log\frac{\pi x w_t + w_3}{w_3},\ \text{or}\ g \times 9\cdot2\log\frac{\pi(1-x)w_t + w_3}{w_3}$$

according to whether the considered coil-side belongs to the f or the g group.

To this has to be added the inductance from the joint flux beyond the limits of the local fluxes; the permeance up to the pole-tips is

$$\frac{2\cdot3}{\pi}\log\frac{\pi\,[r_2 - \{w_3 + w_t(1-x)\}] + 2w_3 + 2w_t(1-x)}{\pi x w_t + 2w_3 + 2w_t(1-x)}$$

so that the coefficient for the inductance is

$$(f+g)\ 9\cdot2\log\frac{\pi\,[r_2 - \{w_3 + w_t(1-x)\}] + 2w_3 + 2w_t(1-x)}{\pi x w_t + 2w_3 + 2w_t(1-x)}$$

Within the usual working limits of $w_t = w_3$ and $w_t = 1\cdot5\ w_3$, the exact relation between w_t and w_3 has but little effect on the whole, so that it is unnecessary to consider it more closely. Thus if $f=g$, $x=0\cdot5$, and the coefficient for the inductance from the joint flux becomes with $w_3 = w_t$

$$(f+g)\ 9\cdot2\log\frac{\pi\left[\dfrac{\pi D \times 0\cdot15}{2p\,.\,w_3} - 1\cdot5\right] + 3}{\dfrac{\pi}{2} + 3}$$

$$= (f+g)\ 9\cdot2\log\left(0\cdot324\frac{D}{2p\,.\,w_3} - 0\cdot374\right)$$

and to this must be added the coefficient for the local flux, say of the f group

$$= f\,.\,9\cdot2\log\left(\frac{\pi}{2}+1\right) = f\,.\,9\cdot2 \times 0\cdot41.$$

Now, if $w_t = 1{\cdot}5 w_3$, the two corresponding items become

$$(f+g)\ 9{\cdot}2 \log\left(0{\cdot}252 \frac{D}{2p\ .\ w_3} - 0{\cdot}341\right)$$

$$f\ .\ 9{\cdot}2 \log\left(\frac{3\pi}{4}+1\right) = f\ .\ 9{\cdot}2 \times 0{\cdot}526$$

Thus with the wider tooth the joint flux decreases, but the local flux increases, and the two changes largely counterbalance one another.

As the disproportion between f and g increases, the coefficient for a coil-side situated in the slot having the larger number f approaches that for a single slot, while conversely the coefficient for a coil-side in

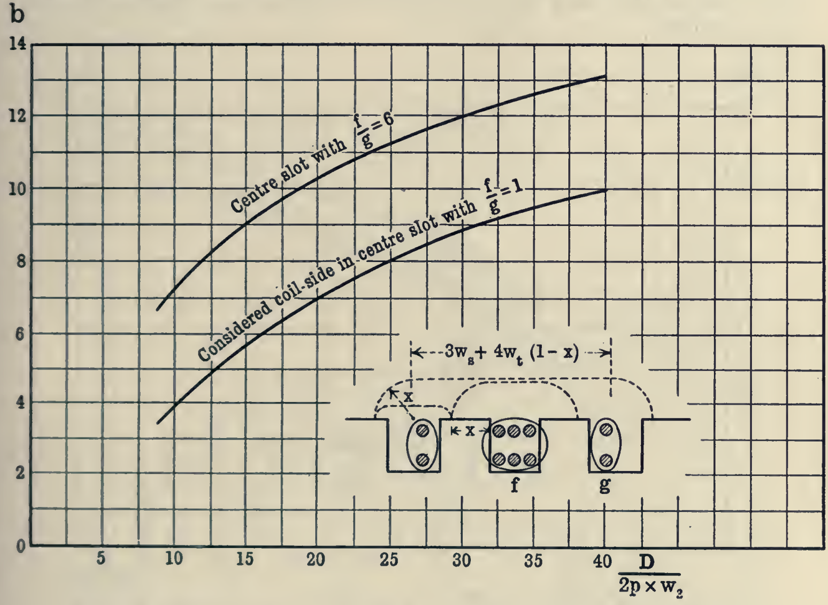


FIG. 358.—Short-circuited coil-sides in three slots symmetrical about centre, and with larger number in central slot.

the smaller g group falls. Thus in both directions for $\frac{f}{g} > 3$ the curves soon reach a limit, the highest approximating to the case of a single slot, and the lower to the curve for the joint flux alone. In every case the value for b is arranged so that it may be multiplied by the full number of short-circuited coil-sides j_b in the zone which is either $2\left(\frac{b_1}{\beta}\right)_+$ or $2\left(\frac{b_1}{\beta}\right)_+ - 1$, and $k''' = b\ .\ j_b$.

Other curves for different values of $\frac{f}{g}$ are easily interpolated.

(iii) If the short-circuited coil-sides are spread over three adjacent slots the centre containing the larger number f (Fig. 358), and further the two outside slots each contain g coil-sides, so that the whole is still symmetrical about the centre line, the point x at which the local fluxes bifurcate is given by the relation

$$\frac{f+g}{\pi x w_t + 2w_3 + w_t} = \frac{g}{\pi w_t(1-x) + w_3}.$$

The upper curve of Fig. 358 is applicable to the centre slot when it contains by far the larger number of short-circuited coil-sides; the lower curve to the centre slot when all are equally filled. In the case of $f=g=h$ there is so little difference between the flux linked with the outer or the inner slot that the lower curve may also be used for the outer slot.

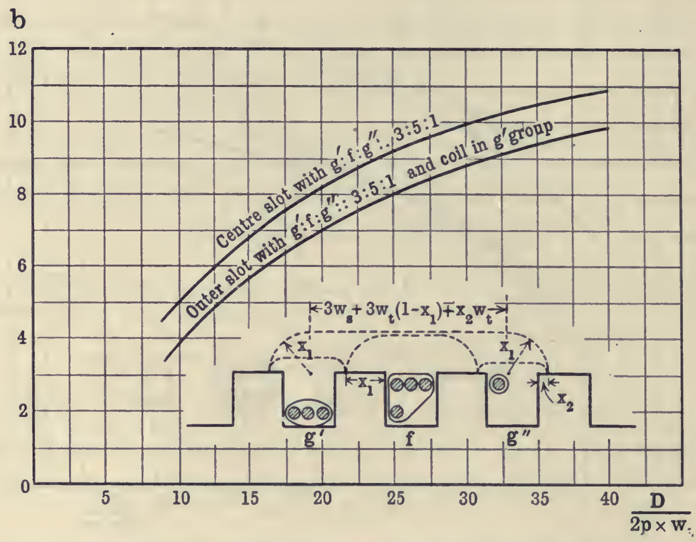


FIG. 359.—Three slots with central slot containing greatest number, but unsymmetrically distributed.

Fig. 359 illustrates the case when the central slot contains the greater number of short-circuited coil-sides, but the distribution is unsymmetrical. It will be seen that the value of b for the central slot falls below the upper curve of Fig. 358, and by comparison with this curve other cases can be dealt with by interpolation.

When an outer slot contains the larger number the distribution may again be symmetrical (Fig. 360) or unsymmetrical (Fig. 361). In such cases, when the value for an outer slot is required, it is almost invariably the outer slot containing the greater number in which the considered coil will lie, and for these the lower curve of Fig. 358 is not far from correct.

(iv) When the short-circuited coil-sides lie in four slots, the two upper curves of Fig. 362 giving the value of b for a coil-side in the f or g groups when $\frac{f}{g}=4$ or $\frac{g}{f}=4$ may be used also for other cases by comparison with the third lower curve for equally filled slots; the upper curve is approximately applicable to an inside slot containing a larger number, and the lower to any outside slot.

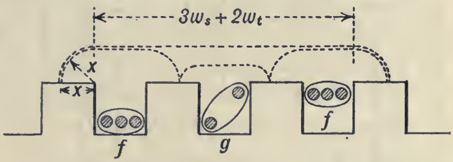


FIG. 360.—Three slots symmetrical about centre, but with larger number in outer slot.

(v) As the number of slots is increased the joint flux predominates more and more, so that the curves for an outer or an inner slot draw together. For the rare case of five slots it therefore suffices to add the approximate curve shown dotted at the foot of Fig. 362.*

In the calculation of (a) and (b), if greater accuracy be desired, from L, the total length of the armature core, should be deducted

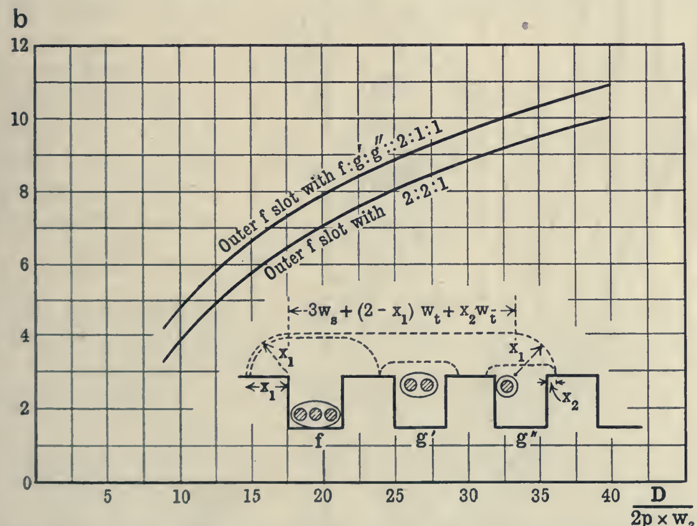


FIG. 361.—Three slots unsymmetrical about centre, and with largest number in outer slot.

the axial length of the air-ducts, to obtain l = the magnetic length of the core; the difference should then be added to l', the length of the end-connections in air. Further, the inductance from steel binding-wires, if present, has above been neglected.

* *Electrical World and Engineer*, vol. xlii. p. 871, "Calculation of the Apparent Inductance of Armature Coils," by C. C. Hawkins, from which the above is abbreviated.

The preceding curves show the gradual decline in the rate at which the permeance and the inductance rise as the diameter of armature increases for the same number of poles and width of opening of slot. Their lower terminations in all cases mark the point at which the width of all the short-circuited coil-sides approaches dangerously close to the total width of the interpolar zone between the pole-tips; as an extreme maximum the number of teeth and slots in which lie the short-circuited coils should always fall short of the width of the whole interpolar zone by at least the width of one tooth-pitch, so that for one, two, three, four, or five slots involved in the span of the sections short-circuited by the brushes, $\frac{D}{2p \cdot w_3}$ should not be less than about 5, 7, 9, 11, and 14 respectively.

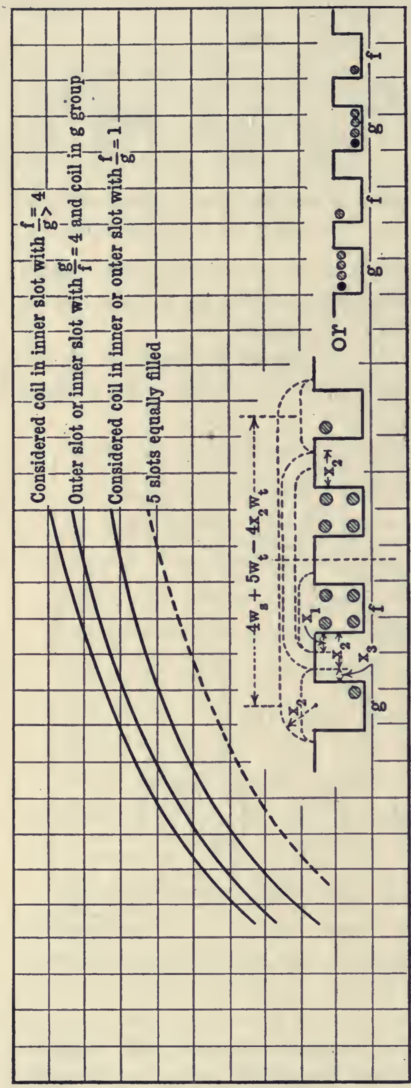


FIG. 362.—Short-circuited coil-sides in 4 or 5 slots.

(*c*) The end-connections.

The inductance of the *end-connections* of a coil of a barrel-wound armature is in strictness not simply proportional to their length, since it depends upon the shape of the coil and the area of the path which is traversed by the lines linked with the ends. In the case of a circular coil entirely in air or half embedded in iron throughout its length, so that every centimetre length is exactly similarly circumstanced, the area corresponding to a centimetre length of the periphery is a wedge-shaped sector; since the density of the lines decreases towards the centre, a square centimetre near the periphery of the ring is of more account than one near the centre; hence, as the diameter and length of a turn are increased, the lines per centimetre length of the periphery and per C.G.S. current-turn rise very slowly, and become fairly constant. When surrounded entirely by air, this point at which the curve of lines per centimetre length becomes nearly flat is reached when a diameter of 50 centimetres is exceeded, and a figure of some 8 to 10 lines per centimetre length is reached. With a rectangular coil free in air the same effect is present; the maximum number of lines per centimetre length is necessarily obtained when the coil is square, but the reduction as the coil is narrowed is not very marked until one pair of parallel sides is less than 20 centimetres apart.* The V-shaped end-connections of a barrel-wound coil, each of length l' centimetres, if grouped together (Fig. 363), approach most closely to the case of a circle in air, and may be replaced thereby if we ignore the influence of the proximity of the iron core; in a bar-wound armature with involute end-connectors which lie more closely to the core, a somewhat higher inductance would be reached.

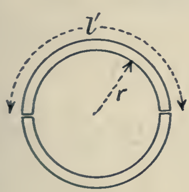


FIG. 363.

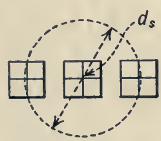


FIG. 364.

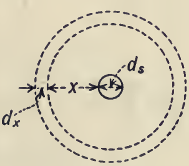


FIG. 365.

It is therefore practically legitimate to treat the end-connectors as linked with a certain flux per centimetre length, and the inductance of the ends of a drum-coil when barrel-wound may be approximated as follows. At each end the end-connections of j_c coils run side by side, where $j_c = \left(\frac{b_1 - m}{\beta}\right)_+$ the number of sections short-circuited simultaneously at one brush, and between the considered coil and the remainder there is mutual inductance. Let d_s = the diameter of a circle whose periphery is equal to that of the group of j_c coil-ends, inclusive of insulation and any air-gaps which there may be between them (Fig. 364, where, *e.g.*, the periphery of the dotted circle is equal to the periphery of the rectangular packet of three coil-ends, each containing four wires). The circle of Fig. 363 equivalent to the two V-shaped sets of a barrel armature has a radius of $r = \frac{l'}{\pi}$. The permeance of a cylinder of air round the coil-ends, of width dx (Fig. 365), and extending along

* H. M. Hobart, "Modern Commutating Dynamo Machinery," *Journ. Inst. Electr. Eng.*, vol. xxxi. pp. 185 ff.

half the circle corresponding to the connections at one end of the armature, is $\frac{l'dx}{2\pi x}$, if we neglect its gradual contraction towards the centre of the circle; this is acted upon by a M.M.F. of $4\pi w j_c$. An element of the inductance is therefore $dL = \frac{4\pi w^2 j_c l' dx}{2\pi x} = 2w^2 j_c l' \frac{dx}{x}$, and the integral of $\frac{dx}{x}$ between the limits of the radius r of the complete equivalent circle and the radius $\frac{d_s}{2}$ of the small circle representing the section of the coils is

$$\int_{\frac{d_s}{2}}^{\frac{l'}{\pi}} \frac{dx}{x} = \log_e \frac{l'}{\pi} - \log_e \frac{d_s}{2} = 2{\cdot}3\left[\log_{10}\frac{l'}{d_s} - \log_{10}\frac{\pi}{2}\right] = 2{\cdot}3 \log_{10}\frac{l'}{d_s} - 0{\cdot}45$$

The self and mutual inductance of one end of length l' is therefore

$$w^2 l'.\ 2j_c\left(2{\cdot}3 \log\frac{l'}{d_s} - 0{\cdot}45\right) \times 10^{-9} \text{ henrys}$$

$$= w^2 l'.\left(4{\cdot}6 \log\frac{l'}{d_s} - 0{\cdot}9\right) \times \left(\frac{b_1 - m}{\beta}\right)_+ \times 10^{-9}.$$

The value λ' to be inserted in our principal equation (115) is therefore

$$\lambda' = \left(4{\cdot}6 \log\frac{l'}{d_s} - 0{\cdot}9\right) \times \left(\frac{b_1 - m}{\beta}\right)_+ . \qquad (120)$$

§ 25. **Influence of pitch of armature coils.**—It is instructive to note the influence which the pitch of the armature winding has upon the slot and core inductance, and in order to illustrate this a number of diagrams of two interpolar zones are collected in Fig. 366 for the particular case of three sectors or 6 coil-sides per slot in two layers and a ratio $\left(\frac{b_1 - m}{\beta}\right)_+ = 4$.

When the number of slots is an exact multiple of $2p$, *i.e.* $\frac{n_s}{2p}$ is an integer, true *diametric winding* is obtained when the back-pitch of the armature coil reckoned in elements is $y_R = \frac{u_n \times n_s}{2p} + 1$, where $u_n =$ the number of coil-sides per slot, and $n_s =$ the number of slots; or when reckoned in terms of slots, $y_R' = \frac{n_s}{2p}$. In this case each of the sides of a complete coil will also have exactly above or below it in the same slot another coil-side which is short-circuited at a neighbouring brush of opposite polarity, so that the inductance for a given value of $\left(\frac{b_1 - m}{\beta}\right)_+$

is a maximum. To shorten y_R, its preceding diametric value must be reduced in steps of two elements, *i.e.* the next lower possible values are $\frac{u_n \times n_s}{2p} - 1$, $\frac{u_n \times n_s}{2p} - 3$, and so on. After passing through a number of such steps $= \frac{u_n}{2} - 1$, the rear pitch in slots is itself shortened and becomes $y_R' = \frac{n_s}{2p} - 1$. The winding may then be called *long chord*, the definition of this term being that with narrow brushes the two coil-sides which are then short-circuited in each zone are separated by an intervening tooth. After the same number of intermediate steps as before, the rear pitch in slots is again shortened to $\frac{n_s}{2p} - 2$; the two coil-sides short-circuited by narrow brushes in each zone are now separated by two teeth and one slot, and the winding may be termed *short chord.*

The intermediate cases when the amount by which the pitch is shortened is not equal to a complete slot do not yield interchangeable coils, so that in practice they are seldom used. When the diagrams of Fig. 366 are examined it is seen that as we proceed down the scale from diametric to long chord, and thence to short chord, the two layers of short-circuited coil-sides are, as it were, gradually sheared apart, and the greatest $L + \Sigma . M$ of any one coil out of a given number short-circuited by the brush width is reduced.

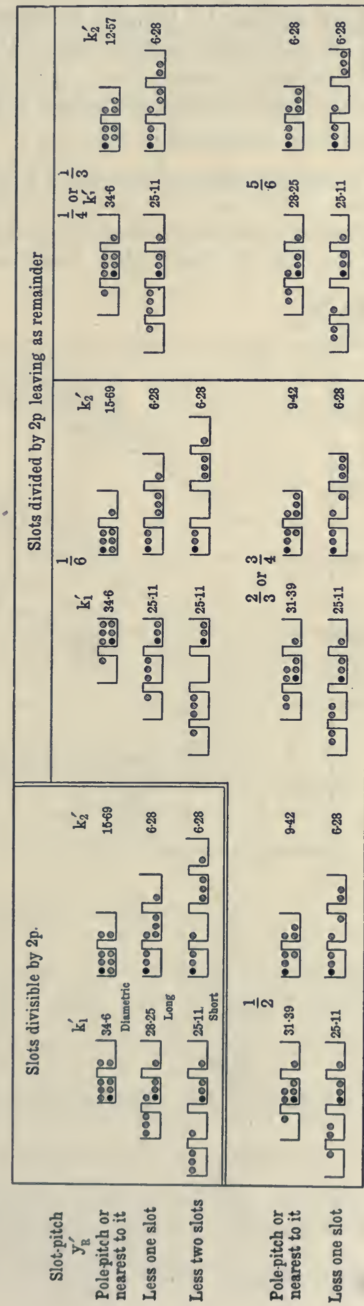


FIG. 366.—Six coil-sides per slot, $\left(\frac{b_1 - m}{\beta}\right)_+ = 4$.

When $\frac{n_s}{2p}$ is fractional, the smaller the remainder the nearer can the rear pitch in slots be brought to equality with the pole-pitch, so as to resemble the true diametric case. From this point a similar gradual shearing apart is produced as y_R' is shortened by one or more complete slots. The reduction in the values of k_1' and k_2' for the slot inductance is shown by the figures in Fig. 366.

Taking any particular values of $\left(\frac{b_1 - m}{\beta}\right)_+$ and of u_n, if the surface-of-the-core inductance is calculated and added to the slot inductance it will be found that their sum is appreciably reduced by

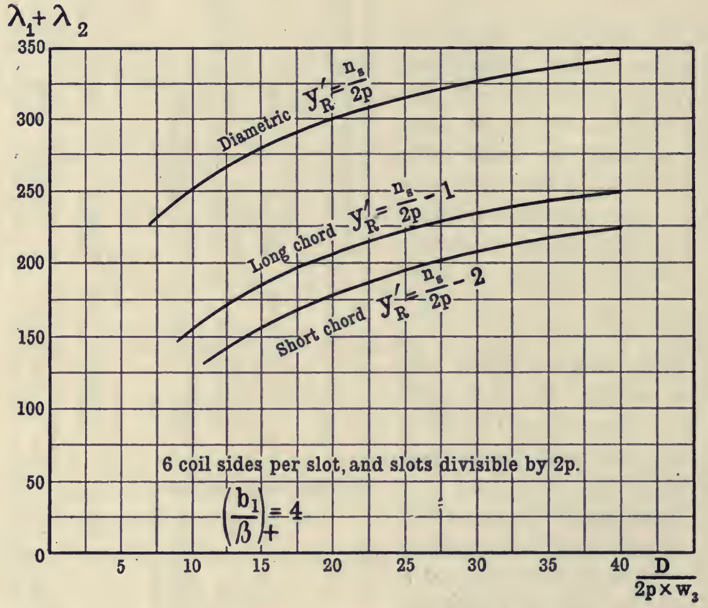


FIG. 367.—Effect of shortening the pitch of coils.

shortening the coil-pitch and so spreading out the two layers of short-circuited coil-sides in each zone. Fig. 367 shows this for 6 coil-sides per slot, and $\frac{b_1 - m}{\beta} = 4$, and on the assumption that the slot is open with a ratio $\frac{h_s}{w_3} = 2{\cdot}5$. The percentage reduction is very considerable for the first shortening of the slot-pitch from $\frac{n_s}{2p}$ to $\frac{n_s}{2p} - 1$, and is much less when the process is carried still further.

With fractional windings the number of coil-sides short-circuited in one zone may be either $\frac{2(b_1 - m)}{\beta}$ or $\frac{2(b_1 - m)}{\beta} - 1$, according to the circumstances of the brush-width and pitch; in the latter case the curves of the surface-of-the-core inductance are somewhat flatter in large armatures where the joint flux predominates than in whole windings with the same width of brush. There is therefore a slight advantage in fractional as compared with whole values of $\frac{n_s}{2p}$, but the difference is not great.

In conclusion, it may be pointed out that occasionally a coil is wound with a number of wires in parallel; but if so, they must be wound in the same slot or slots as if they were a single conductor, since otherwise eddy-currents would be set up within the coil, due to the different E.M.F.'s generated by the wires in the different slots.

§ 26. **The time of commutation.**—While in the simple lap-wound armature $T = \frac{b_1 - m}{v_k}$, in the multiplex lap-wound armature for the same brush width the time is reduced to

$$T = \frac{b_1 + \beta\left(1 - \frac{a}{p}\right) - m}{v_k}$$

where $\frac{a}{p}$ is greater than unity.

In the simple wave-wound armature with as many sets of brushes as there are poles, a coil is first short-circuited as part of a complete zigzag round the armature which returns to a sector adjacent to that from which it starts and under the same brush. The short-circuit current can then pass not only *viâ* the complete zigzag, but also through all the leads connecting the brushes of the same sign; and the duration of this stage is exactly as in the simple lap armature given by the passage of a mica strip past the brush, or $t = \frac{b_1 - m}{v_k}$. The second stage consists in the successive reduction of the number of parallel paths open to the short-circuit current through the brush leads until only one is left through the nearest brush of the same sign, and finally this remaining path is opened by the adjacent brush of the same sign leaving the sector which forms one end of the coil under consideration. Thus the second stage is itself divisible into $p - 1$ successive reductions of the parallel paths open to the current of the coil. The difference between the commutator pitch and the pitch of the brushes of similar sign, *i.e.* $y_k - \frac{N_2}{p}$, is the fraction of a sector corresponding to each of the $p - 1$ changes, and this difference is equal to $\pm \frac{1}{p}$. The space traversed during each of the changes is therefore $\frac{\beta}{p}$, and the movement throughout the second stage during the $p-1$ changes is $\frac{\beta}{p}(p-1)$. The total time of short-circuit at one brush and between brushes of the same sign is thus extended to $T=t+\frac{\beta(p-1)}{pv_k}=\frac{b_1+\beta\left(1-\frac{1}{p}\right)-m}{v_k}$

This extension of the time is in itself advantageous to the commutation,* so that the use of as many sets of brushes as there are poles is to be recommended so long as they are easily visible and accessible, and do not give too much friction in small machines. What percentage increase in the time of commutation is obtained entirely depends upon the width of the brush; thus if the width of the brush expressed in terms of the width of a sector is $y\beta$, $T=t\left(1+\frac{1-\frac{1}{p}}{y}\right)$; *e.g.*, with four poles and $y=1$, $T=1\frac{1}{2}t$, but if the brush width is twice that of a sector, $T=1\frac{1}{4}t$. With wide brushes therefore the increase becomes of less and less importance.

With multiplex wave-wound armatures the case is slightly different. If $b_1-m=\beta$ a complete zigzag round the armature is never short-circuited; there are then, as above, $p-1$ successive stages of reduction in the number of paths through the leads connecting brushes of the same sign, each lasting a time corresponding to $\frac{a}{p}\cdot\beta$, and the total time corresponds to $\beta\cdot\frac{a}{p}(p-1)$. So small a brush width would, however, not be used in practice. As soon as $b_1-m>(a-1)\beta$ an initial stage is added on during which a complete zigzag is short-circuited, and the duration of this depends upon the amount by which b_1-m exceeds $(a-1)\beta$. The time of this first stage is therefore proportional to $b_1-m-(a-1)\beta$, and the total time of all the stages is proportional to $b_1-(a-1)\beta+\beta\cdot\frac{a}{p}(p-1)-m=b_1+\beta\left(1-\frac{a}{p}\right)-m$. Thus the general formula

$$T=\frac{b_1+\beta\left(1-\frac{a}{p}\right)-m}{v_k} \qquad \qquad (121)$$

holds in all cases, whether the armature be simplex or multiplex, wave or lap-wound, so long as there are as many sets of brushes as there are poles, or if b_1-m and β are in inches, and v_k in feet per minute.

$$T=\frac{5}{v_k'}\left\{b_1+\beta\left(1-\frac{a}{p}\right)-m\right\}\text{ seconds} \qquad \qquad (121a)$$

* Cp. J. K. Catterson-Smith, *Journ. Inst. Electr. Eng.*, vol. xxxv. p. 430; and Professor F. G. Baily and Mr. W. S. H. Cleghorne, *Journ. Inst. Electr. Eng.*, vol. xxxviii. p. 168.

§ 27. **The true criterion of sparking.**—It remains to estimate quantitatively the factors upon which sparking at the brushes ultimately rests, for the guidance of the designer as to the degree in which sparkless running is likely to be attained in any given machine.

The true measure of the limitation of the output by sparking is not simply the maximum current-density which occurs at any time during short-circuit at the face of the brush, nor is it the maximum value which the difference of potential between brush and commutator reaches. Neither current nor voltage in themselves are to be feared; it is the integral effect of their product, namely, electrical energy, which alone can give rise to any destructive effect from sparking or overheating. Yet even this statement must be qualified at least in the case of sparking by a further proviso. As Professor Arnold * has pointed out, a certain voltage must be present between the surfaces of brush and commutator before even a high watt-density becomes detrimental in a dynamo.

That the actual expenditure of energy *in the air* by a visible spark *need* not in itself damage the surfaces is shown by the discharge points of an induction coil which are uninjured by the sparks. And on this account Mr. Thorburn Reid † has given as the criterion of the sparking limit the maximum energy-density or rather watt-density at the trailing edge of a sector. But actual facts show that very high watt-densities which would lead to no objectionable consequences with plain slip-ring contacts would quickly cause sparking on the commutators of direct-current machines. The difference lies in the value of the voltage which forms one of the components of the watts, and which for the same value of the watt-density will be higher in the commutator than with the slip-ring. The two cases are, in fact, decisively differentiated by the presence of the slight percussions and vibration to which the brushes are subjected on the commutator as they traverse the mica and copper strips, even when the surface is as smooth as it is possible to attain in practice. The predisposing cause is therefore to be found in a mechanical consideration, which for the same current-density raises the voltage at the point of contact, or the contact-resistance, in the commutator as compared with the slip-ring.

Accompanied, then, by voltages exceeding certain limits which may vary according to the conditions, the watt-density which finally causes the visible spark does materially damage the surface of both commutator and brush. Especially has a *rapid variation* of the current-density to be guarded against; for it is this which, through the action of self-induction, supplies the E.M.F. necessary for damage to the surface to result under the conditions of the direct-current dynamo, and apart from any electrical or magnetic cause for it, such rapid variation may

* *Die Gleichstrommaschine* (2nd edit.), vol. i. p. 406.

† *Proc. Amer. Inst. Electr. Eng.*, vol. xxiv. p. 297.

itself be solely due to mechanical jarring. Too high a watt-density alone may cause carbon brushes to glow, and this must be avoided as causing deterioration of their surface, but comparatively low values of the current-density, if accompanied by voltages increasing roughly in inverse proportion to the current, will still suffice to produce injurious sparking. The slight mechanical chattering of the brushes as modifying their true steady contact-resistance, and the effect of rapid current-variation, are therefore in combination the primary causes of the sparking difficulty.

A calculation of the total energy which may have to be expended in the spark at the moment of opening the short-circuit of a section suggests itself, in the first place, as a guide to the relative merits of different machines. If $i_{z'}$ is the value of the additional current at the instant immediately before the appearance of a spark, and this is reduced to zero at the end of short-circuit, then, neglecting the resistance of the coil, the total energy which is liberated in the spark during the disappearance of the additional current is

$$\int_{i_z = i_{z'}}^{i_z = 0} i_z \,.\, \mathrm{L}_{sz} \,.\, di_z = -\tfrac{1}{2}\, \mathrm{L}_{sz}\, (i_{z'})^2$$

The difficulty in applying any such formula lies not only in the uncertainty of the calculation of i_z, but also in the ambiguity in the time or starting-point $i_{z'}$. By taking a certain percentage error of excess or deficiency in the field, and calculating the product of the additional current and its difference of potential at a fixed time, say, $\frac{1}{10{,}000}$th of a second, before contact is broken, comparative figures might be obtained * for various machines. Professor E. Arnold and J. L. La Cour † have proposed to take the *mean* time-rate at which electromagnetic energy is released per unit length along the line of a set of brush tips, assuming the maximum additional current to persist until the leading sector of the coil is exactly at the brush tip, and reckoning time from this moment. The remainder of the commutation is effected in a time $5\frac{\beta}{v_k'}$ seconds, and the mean rate of release of energy is $\frac{i_{z\,max.}^2 \,.\, \mathrm{L}_{sz} \,.\, v_k'}{10\beta\,.}$. This is distributed over b inches of brush length at right angles to the direction of rotation. From oscillograph experiments, in which the rate of release of energy which was accompanied by sparks could be measured, Professor Arnold suggested a certain limiting value for the watts per cm. or inch of brush length. There

* See W.A.P. in *Electrical Review*, vol. xlvi. p. 229, February 23, 1900.

† "The Commutation of Direct and Alternating Currents," *Trans. Intern. Electr. Congress St. Louis (1904)*, vol. i. p. 801.

remains, however, the objection that $i_{z\ max.}$ in the case of a dynamo that has not been built can only be calculated by means of certain simple assumptions which introduce so many elements of doubt as to seriously invalidate their accuracy, and even such an approximate calculation would demand a more prolonged and minute analysis than can be given during the process of design.

§ 28. **Practical voltage criterion of sparking.**—Abandoning, therefore, the calculation of the energy as the product of voltage and current, we must perforce fall back upon the calculation of a voltage only, and in the first place the final value of the difference of potential between the brush-tip and the leaving sector suggests itself as a criterion of sparking. In a dynamo which has been built and can be tested the curve of brush potential relatively to the commutator does enable us to gain much information as to the probable behaviour of the machine in regard to sparking, and a maximum limit of 3 to 4 volts between commutator and brush is a valuable guide.

But even such a rule may require to be considerably modified according to the absolute value of the current as the other factor which determines the energy expended. By means of oscillographic experiments, Professor F. G. Baily and W. S. H. Cleghorne * have measured the actual sparking E.M.F. between brush and commutator at the last moment, and have found such values as 15 to 20 volts with carbon brushes without very violent sparking. Moreover, there remains a very similar objection to that which meets us in an attempt to calculate the energy of the spark; during the process of design a calculation of the final sparking potential, even if only approximate, requires that we should know the contact-resistance R_k as dependent upon the effective current-density, and also L_{sz}. But R_k is extremely variable at brushes of different sign, especially under high temperatures and when sparking is likely to occur, while the value of L_{sz} is not easy of calculation, owing to the difficulty of predicting the damping effect of other short-circuited coils, especially when not situated in the same slot.

Finally, therefore, we are led to adopt as a criterion of sparking a more easily calculated quantity ΔE, or the additional E.M.F. set up between the outer sectors lying at the extreme edges of the brush, so far as commutation departs from the straight-line law. To the importance of this quantity Professor Arnold has directed especial attention. To it is due the difference of the potentials between the brush and the trailing and leading sectors at its two edges.

§ 29. **The shape of the potential curve.**—The processes which are occurring in the short-circuited coils are brought to light by the shape of the curve which is obtained by plotting the difference of potential between brush and commutator for a number of points along the face of the brush. When the current is commuted at a uniform rate,

* *Journ. Inst. Electr. Eng.*, vol. xxxviii. p. 163.

and i_z is throughout $= 0$, such a potential curve becomes a straight line at a uniform height above the zero line at the positive or below at the negative brush, the brush itself being in each case reckoned as at zero potential; this height corresponds to the normal loss of volts due to the passage of $2J$ over the resistance of contact. But when additional currents are also flowing, the straight line is deformed, so that, generally speaking, one end rises and the other sinks. With every change in the conditions the curve of the commutator potential changes its shape, so that we have the characteristic shapes of Figs. 368–371, corresponding to the four cases of Figs. 338–341. With simply retarded or accelerated commutation in a generator the curve is bowed, rising in the former

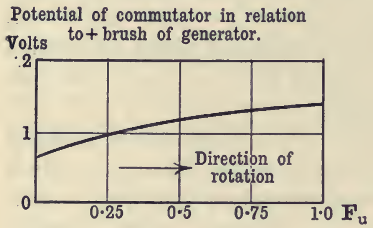


FIG. 368.—Retarded commutation.

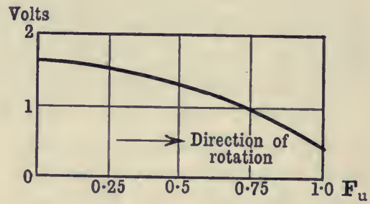


FIG. 369.—Accelerated commutation.

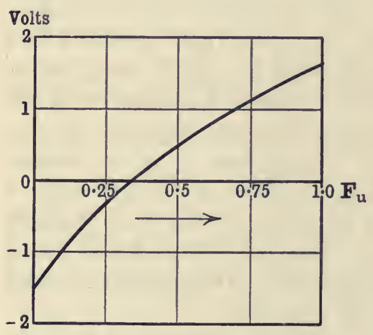


FIG. 370.—Increase above $+J$.

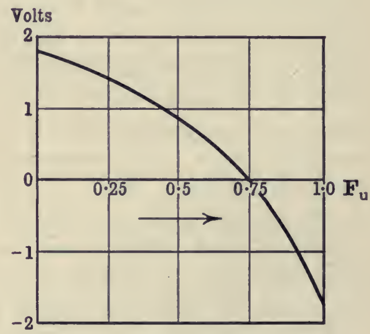


FIG. 371.—Over-reversal above $-J$.

case (Fig. 368) and sinking in the latter (Fig. 369); but if the current is either increased above $+J$ or over-reversed to a value exceeding $-J$, the curve crosses the zero line (Figs. 370 and 371), and the direction of the potential difference changes, since the actual current is in different directions under the same brush. But in every case, unless the commutation is first retarded and then accelerated (Fig. 372), or *vice versâ*, so that the two errors practically balance one another, one end of the curve rises and the other sinks; on the whole, therefore, the curve is inclined, and the greater the error in the field the steeper the slope. There thus arises a difference of potential Δp between the two sectors situated at the extreme edges of the brush, *i.e.* Δp, the algebraic difference of the potential at the two edges in the case of a wide brush covering several sectors, is practically a definite quantity depending upon the load and position of the brushes. When the curve of potential departs from a uniform level, if a horizontal line be found such that it encloses equal areas on either side of the curve, its height above the zero line measures the normal loss of volts due to the passage of the normal current with the resistance of the carbon in its modified condition as determined by the effective current-density under the brush. The special case of commutation first retarded and then accelerated is such as may be

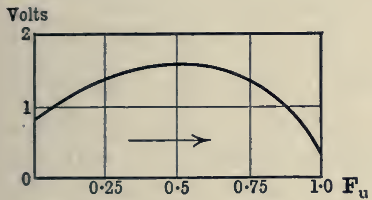


FIG. 372.—Retardation followed by acceleration.

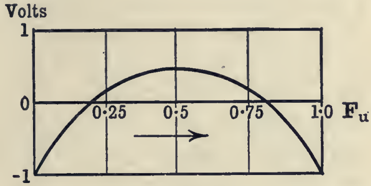


FIG. 373.—Potentials at no-load with wide brushes on line of symmetry.

found in a generator at no-load with the brushes fixed in the geometrical centre, and gives such a curve as Fig. 373, while if the brushes are fixed in the correct position, say, for half-load, and so are too far forward at no-load, we have Fig. 374, in both of which cases it will at once be seen that equal areas are enclosed with the zero line since there is practically no normal current passing.

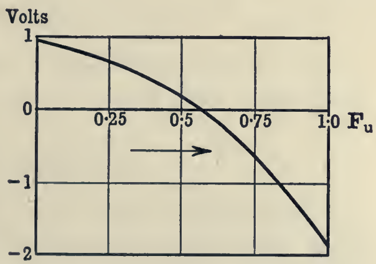


FIG. 374.—Potentials at no-load with brushes in half-load position.

Next, the potential curve is itself flatter than the E.M.F. curve to which it is due, since the latter also contains the ohmic drop over the resistance of the coils and their commutator connectors. The drop of volts across the substance of the carbon brush itself is practically negligible. Corresponding, therefore, to the difference of potential Δp between the sectors at the extreme edges of the brush, there is a still greater difference of E.M.F. ΔE acting through the coils, and the curve of the E.M.F. impressed by the incorrect field is steeper than that of the potential. The system of i_z currents is therefore primarily dependent upon the quantity ΔE, and this may be regarded as constant in point of time when several sectors are covered by the brush. It is certainly an experimental fact that the steeper the gradient of the curve of potential between brush and commutator, *i.e.*

the greater the value of Δp, the more the likelihood of sparking, so that the magnitude of the quantity ΔE will serve indirectly as the required criterion of sparklessness in the design of a dynamo.

§ 30. **The calculation of ΔE.**—In regard to the quantity ΔE, a marked distinction exists between the two cases (*a*) when commutation is due mainly or wholly to an actual reversing field suited to different loads and obtained either by so shifting the brushes as to reach such a field or by means of special "commutating" poles, and (*b*) when the brushes must be retained in some one fixed position without the assistance of commutating poles, and commutation is as it were "forced" by the action of the brush contact resistance.

If a drum coil is moving in any field of density B, its E.M.F. is $\frac{\tau}{N_2} . Blv \times 10^{-8}$ volts, where l is the length of each active inducing side in centimetres, and v is its peripheral speed in centimetres per second. Hence in the former (*a*) case for perfect commutation in a straight line, the E.M.F. impressed on the coil by the resultant external field must be from equation (98)—

$$f(t)_k = \frac{\tau}{N_2} . B_r . l_r v \times 10^{-8} = -2J\left\{\frac{L + \Sigma . M}{T} + R\left(\frac{t}{T} - \frac{1}{2}\right)\right\}$$

where l_r is the axial length of the reversing field acting on each side of the coil and B_r is the density which holds over this length. Thus

$$B_r = -\frac{N_2 . 2J}{\tau . l_r . v \times 10^{-8}}\left\{\frac{L + \Sigma . M}{T} + R\left(\frac{t}{T} - \frac{1}{2}\right)\right\}$$

If $l_r = l$, the axial length of the armature core, and if B_k is the special value which B_r then assumes,

$$B_k = -\frac{N_2 . 2J}{\tau . l . v \times 10^{-8}}\left\{\frac{L + \Sigma . M}{T} + R\left(\frac{t}{T} - \frac{1}{2}\right)\right\}$$

These densities must be the resultant difference between that which would exist without the presence of the armature cross ampere-turns and the density due to their action alone.

The latter equation also holds for the dynamo without commutating poles, but B_k must then result from the initial symmetrical field as weakened by the back ampere-turns carrying the full current of the armature, and as distorted by the cross ampere-turns which also carry the full current. Let B_o = the density of the main symmetrical field at the given brush position which would result from the field excitation corresponding to J, and with allowance made for the back ampere-turns; let B_q = the density at the same brush position which would be due to the cross ampere-turns of the armature. The two, so far as the interpolar gap on the leading side is concerned, are opposed in direction, so that it is their difference which alone is effective, and this on the whole must yield a negative or reversing effect. Hence

$$B_k = -B_o + B_q = -\frac{N_2 . 2J}{\tau . l . v \times 10^{-8}}\left\{\frac{L+\Sigma . M}{T} + R\left(\frac{t}{T} - \frac{1}{2}\right)\right\}$$

or

$$-B_o = -\left[B_q + \frac{N_2 . 2J}{\tau . l . v \times 10^{-8}}\left\{\frac{L+\Sigma . M}{T} + R\left(\frac{t}{T} - \frac{1}{2}\right)\right\}\right] \quad . \; . \; . \quad (122)$$

Neglecting, then, the ohmic resistance term and its variation with time as of comparatively small influence, if either B_r or B_k is on the whole of such amount as to balance the inductive voltage $\frac{2J(L+\Sigma . M)}{T}$, the only remaining cause that may still set up sparking lies in the difficulty of securing such exact grading of the reversing field as to suit the true variations of $L+\Sigma . M$. Both B_o and B_q vary as movement of the coil proceeds by rotation, and so also does $L+\Sigma . M$ not only as the distance from the main pole-tip or commutating pole-face varies, but also from the different location of the short-circuited coils on either side or all on one side of the considered coil with which they are linked by mutual inductance. An average rate of change of the current or $\frac{2J(L+\Sigma . M)}{T}$ with $L+\Sigma . M$ regarded as constant, and an average adjustment of the reversing field is all that can be taken into account. The actual amount of divergence does not admit of practical calculation, but it can be said that the greater the necessary value of B_r or of the angle of lead when the brushes are shifted, the greater the difficulty of balancing the reversing and the inductive voltages, so that under all conditions of load ΔE may remain $= 0$.

But in case (*b*), when the brushes are to be fixed in one position without the assistance of commutating poles, a more definite calculation of ΔE can be made. If the fixed position corresponds exactly to the interpolar line of symmetry, *i.e.* to the no-load position, then at full-load there is present in each short-circuited coil an E.M.F. in the wrong direction from B_q, and also an E.M.F. in the same incorrect direction from the inductance. These must cause the difference of E.M.F. between the brush and sectors at its two edges, and as a first approximation it is sufficiently accurate to assume that at full-load the inclined curve of E.M.F. between brush and commutator rises in a straight line. If B_q be reckoned for a point midway between the edge of the brush and its centre, the latter being also in our present case the geometrical centre between the poles, an average value is obtained for the E.M.F. due to the cross field, which each short-circuited coil may be assumed to give throughout the period of commutation, namely, $\frac{\tau}{N_2} . B_q . lv \times 10^{-8}$ volts.

The total E.M.F. therefrom, when summed up between the edges of the brush, is S_k times that of a single coil. This number S_k is not simply equal to the number of adjacent sections simultaneously short-circuited by the brush width, but also is proportional to the number of pairs of poles, each of which corresponds to a drum coil, so that, *e.g.*, in the simple wave-wound armature p coils lie in series between two adjacent sectors touched by the brush: it may therefore be generally expressed in the notation of Professor Arnold as

$$S_k = \left(\frac{b_1 - m}{\beta}\right)_+ \frac{p}{a} \quad . \quad . \quad . \quad . \quad . \quad (123)$$

where the + sign indicates that when $\frac{b_1 - m}{\beta}$ is a fraction, the next higher whole number must be taken, and $\frac{p}{a}$ may be either whole or a fraction.

The average E.M.F. from the self and mutual inductance when summed up through all the S_k coils will similarly be $S_k . \frac{2J}{T} . (L + \Sigma . M)$.

The ohmic resistance term which should be present for ideal commutation rises as an inclined straight line from $+JR$ to $-JR$, changing sign midway during the period of commutation, *i.e.* under the centre of the brush. So also the small symmetrical field which is present changes its sign midway under the brush and is similar on either side of the centre. Hence, whatever the degree of divergence between these two for any point on the one side of the centre, the same divergence but in the opposite direction exists at the corresponding point on the other side. Thus when summed up between the limits of the brush edges, the effects, whether of the ohmic resistance or of the symmetrical field, cancel out, and we are left with the cross and inductive volts only to cause ΔE. The total difference of E.M.F. between sectors and brush at the two extreme edges which must be counterbalanced by the brush contact-resistance is therefore at full load with the brushes fixed at the no-load position

$$\Delta E_1 = S_k \left\{ \frac{\tau}{N_2} . B_q l v \times 10^{-8} + \frac{2J(L + \Sigma . M)}{T} \right\} \text{volts} \quad . \quad . \quad . \quad (124)$$

For approximate purposes of comparison between different machines it is easier and sufficiently accurate to determine B_q for the actual centre between the poles, *i.e.* $B_q = \frac{1 \cdot 257 J . \tau}{2p . 2 l_{gx}(\text{real})}$, where l_{gx} in relation to the cross flux is calculated as in § 5, Chap. XVII., from Fig. 318.

But if the brushes may be fixed at the position corresponding to half-load or $\frac{J}{2}$, and it is assumed that ΔE then completely disappears with current $\frac{J}{2}$, and armature field-density $\frac{B_q}{2}$, the value of the initial reversing field $-B_o$ from the main excitation must be

$$-\left[\frac{B_q}{2}+\frac{J \cdot N_2}{\tau \cdot l \cdot v \times 10^{-8}}\left\{\frac{L+\Sigma \cdot M}{T}+R\left(\frac{t}{T}-\frac{1}{2}\right)\right\}\right]$$

This negative field now persists at no-load, although practically no reversing field is required. If B_q is calculated for full-load and for a point under the centre *of the brush*, then, analogously to the previous case, the average value of the incorrect E.M.F. is

$$-\left\{\frac{\tau}{N_2} \cdot \frac{B_q}{2} \cdot l v \times 10^{-8}+\frac{J \cdot (L+\Sigma \cdot M)}{T}\right\}$$

since, when summed up between the extreme limits of the brush width, the inaccuracies due to the ohmic resistance cancel out. Hence

$$\Delta E_2 = -S_k\left\{\frac{\tau}{N_2} \cdot \frac{B_q}{2} \cdot l v \times 10^{-8}+\frac{J \cdot (L+\Sigma \cdot M)}{T}\right\}\text{volts} \quad . \quad . \quad . (125)$$

On the other hand, at full-load the reversing field should be doubled, so that ΔE_2 is positive and equal to

$$\Delta E_2 = S_k\left(\frac{\tau}{N_2} \cdot \frac{B_q}{2} \cdot l v \times 10^{-8}+\frac{J(L+\Sigma \cdot M)}{T}\right)\text{volts.}$$

Thus the numerical value of ΔE_2 for constant field-excitation is approximately the same at full and at no-load. In practice with a shunt-wound machine the degree of incorrectness is underestimated, since the negative reversing field is at no-load slightly increased by the absence of the back ampere-turns of half-load, and at full load is weakened by the increase of the back ampere-turns; on the other hand, with compound-wound machines the conditions are considerably more favourable for the reverse reason.

It will be seen that the amount of ΔE is practically halved in the second case as compared with the case of brushes fixed at the geometrical centre, although it must be remembered that, owing to the angle of lead in the second case, B_q is slightly larger than in the first case.

Of the two, ΔE_1 is the more readily calculated, since it does not require a preliminary determination of $\frac{B_q}{2}$ in the correct brush-position for half-load. The term $L+\Sigma \cdot M$ of a drum coil is by equation (110)

$$=\left(\frac{\tau}{2N_2}\right)^2\{l(\lambda_1+\lambda_2)+2l'\lambda'\} \times 10^{-9}$$

Hence if B_q is expressed in terms of the ampere-wires per pole, *i.e.* as $\frac{1{\cdot}257\, J\tau}{2p \cdot 2l_{gx}}$, and v in terms of the armature diameter in centimetres and revolutions per minute, *i.e.* as $\frac{\pi DN}{60}$, equation (124) becomes

$$\Delta E_1 = S_k \cdot J \cdot \left(\frac{\tau}{N_2}\right)\left\{\frac{0{\cdot}329\, l\, N \cdot \tau}{2p} \cdot \frac{D}{l_{gx}} + \frac{\tau}{N_2} \cdot \frac{l(\lambda_1+\lambda_2)+2l'\lambda'}{2T}\right\} \times 10^{-9}$$

volts, where all dimensions are in centimetres. Finally, with as many sets of brushes as there are poles, and giving T its value as in equation (121),

namely $= \dfrac{b_1+\beta\left(1-\frac{a}{p}\right)-m}{v_k}$ seconds, where $v_k = \dfrac{\pi D_k N}{60}$,

$$\Delta E_1 = 0{\cdot}026 S_k \cdot J \cdot N \cdot \frac{\tau}{N_2}\left[12{\cdot}57\,\frac{l\tau}{2p} \cdot \frac{D}{l_{gx}} + \right.$$

$$\left.\frac{\tau}{N_2} \cdot \frac{D_k}{\left\{b_1 - m + \beta\left(1-\frac{a}{p}\right)\right\}} \cdot \left\{l\left(\lambda_1+\lambda_2\right)+2l'\lambda'\right\}\right] \times 10^{-9} \text{ volts} \quad . . (126)$$

Since in each part of the right-hand expression there is a ratio of two lengths, $\frac{D}{l_{gx}}$ and $\dfrac{D_k}{b_1 - m + \beta\left(1-\frac{a}{p}\right)}$ can be left in inches, and only l and $l(\lambda_1+\lambda_2)+2l'\lambda'$ need be in C.G.S. units.

§ 31. **The permissible value of ΔE, or the sparking limit of output.**—With metallic brushes, say, of copper gauze, so feeble is the action of their contact-resistance that, roughly speaking, there must be a reversing field of value suited to each load as given by equation (122); in other words, ΔE must very nearly = 0. This implies either that there must be special commutating poles, or that the brushes must be shifted into a reversing field, and the want of exact balance must be confined within quite small limits, say up to variations of 20 per cent. or 10 per cent. on either side of the correct load for the given brush position. Further, it implies that the armature core must almost necessarily be smooth, so that $L + \Sigma \cdot M$ may not be increased by embedding the wires in iron.

But with carbon brushes which are a practical necessity with slotted armatures (at least if not provided with special reversing means), a much greater inaccuracy of adjustment may be allowed, and a fixed brush position becomes possible. In order to meet the more stringent case when the brushes must be fixed at the line of symmetry, a greater latitude must be allowed in the value of ΔE_1 than of ΔE_2, and as practical limits which will secure sparkless commutation under average conditions may be given for carbon brushes

$$\Delta E_1 \leqq 6 \text{ to } 8 \text{ volts}$$
$$\Delta E_2 \leqq 4 \text{ to } 5 \text{ volts}$$

Even when the brushes may be continuously shifted so that commutation need not be entirely "forced," the calculation of ΔE_1 serves as a practical guide to the good or bad qualities of the dynamo, since it is a measure of the maximum amount of work that the brushes would be called upon to perform should they receive no adjustment to suit different loads. In fact, according to the degree of attention that they will receive may ΔE_1 be given different values, although in every case the ideal should be to reduce it to the lowest possible amount.

We are thus met with a condition for sparklessness which, entirely apart from any question of heating, limits the maximum current that can be passed through an armature, and therefore for a given speed of rotation and voltage limits the output of the machine.

§. 32. **Advantage of the multipolar machine.**—The advantage of a large number of poles from the point of view of sparking is now on general grounds evident, and lies primarily in the reduction of the inductive volts per section which it renders possible. With the lap-wound armature by the passage from a smaller to a larger number of poles, even though the product of the current J flowing in each of the active wires and their total number τ, or the *ampere-wires*, A_w, of the armature remains the same, yet the value of J can always be proportionately reduced; *e.g.*, by the passage from a bipolar to a 4-pole machine, while τ is doubled, yet with $\frac{\tau}{N_2}$ unchanged J is halved. Or again, with a wave-wound armature, since the multipolar lends itself on other grounds to larger diameters of armature and shorter lengths, a larger number of poles, while leaving both J and τ unchanged, may enable N_2 to be increased owing to the larger diameter of commutator which becomes possible.

Further, for the same value of $J\tau$, the ampere-wires per pole or $\frac{J\tau}{2p}$ are reduced in the multipolar machine in proportion to the number of poles, and this especially in the absence of commutating poles has very important consequences, by reason of the reduction of the cross ampere-turns of the armature as acting upon one magnetic circuit. When the ratio of the polar arc to the pole-pitch or β is kept at the same uniform value (and this is usual in order to actively utilise the same proportion of the armature surface), by increasing the number of poles the value of l_{gx} in equation (126) is not reduced so much as the cross ampere-turns are reduced, so that B_q tends to fall slightly. But this is much more marked when the brushes are to be shifted to meet variations of load. Without attempting exact calculation of the distribution of the field throughout the interpolar gap, comparison between different designs or between different numbers of poles may be made by assuming that in every case the brushes are advanced as far as the edge of the leading pole-tip; l_{gx}, in relation to the cross ampere-turns, then becomes equal to the normal air-gap l_g in relation to the main excitation. Not only must the density of the initial field $-B_o$ at the brush position counterbalance B_q, but there must be left in reserve a definite reversing field. Let B_c = the assumed value that precaution requires at full load for the density of the commutating field at the leading pole-corner.

With the smooth-surface armature the M.M.F. of the cross ampere-turns $1{\cdot}257\ \frac{J\tau}{2p}\,.\,\beta$ may be regarded as expended in equal proportions over the two air-gaps. Then, as in Chap. XVII. § 5, the resultant density at the pole-edge, being the difference between the normal B_g and B_q, is

$$B_c = B_g - 1{\cdot}257\ \frac{J\tau}{2p}\,.\,\frac{\beta}{2l_g}$$

whence

$$\frac{J\tau}{2p}\,.\,\beta \leqq X_g - 0{\cdot}8\,B_c\,.\,2l_g \quad . \quad . \quad . \quad . \quad (127)$$

or

$$A_w = J\tau \leqq \frac{X_g - 0{\cdot}8 B_c\,.\,2l_g}{\beta} \times 2p$$

Thus for the same l_g the maximum number of ampere-wires with which a given armature may be loaded, so far as sparking is alone concerned, varies directly as the number of poles.

In the toothed armature the length of path over the saturated teeth at the trailing pole-corner plays a part equivalent to a virtual increase in the length of the air-gap. From Chap. XVII., §§ 10 and 11, the total reluctance of a strip of the cross circuit which is one centimetre wide on the leading side and y centimetres wide on the trailing side and one centimetre in length along the core is

$$ml_g + \frac{ml_g + \text{reluctance of trailing teeth per square centimetre}}{y}$$

By comparison with the air-gap and saturated teeth the reluctance of the unsaturated leading teeth may be neglected. If B_t'' = the resultant density at the centre of the trailing teeth immediately under the pole-edge, so that $f(B_t'')l_t$ is approximately the loss of magnetic potential over their length of path, the reluctance per square centimetre of cross circuit (not square centimetre of iron) is $\frac{f(B_t'')l_t}{B_g''}$. The resultant density close within the leading edge is therefore approximately

$$B_g' = \frac{1{\cdot}257\left(X_g + X_t - \frac{J\tau}{2p}\,.\,\beta\right)}{ml_g + \frac{1}{y}\left(ml_g + \frac{f(B''_t)l_t}{B_g''}\right)}$$

whence

$$\frac{J\tau}{2p}\,.\,\beta = X_g + X_t - 0{\cdot}8\,B_c\,.\left\{ml_g + \frac{1}{y}\left(ml_g + \frac{f(B_t'')\,.\,l_t}{B_g''}\right)\right\}$$

The last term cannot be given until the whole case has been worked out, but the instance analysed in Chap. XVII. § 11 indicates the order of the figures for a normal induction B_g of about 8250 and $l_t = 4 \cdot 5\, ml_g$. It was then found that $B_g'' \doteq 11{,}000$, $f(B_t'') = 1500$, and $y = 1 \cdot 28$.

Thence

$$\frac{J\tau}{2p} \cdot \beta \leqq X_g + X_t - 0 \cdot 8\, B_c \cdot 2 \cdot 25 ml_g \quad . \quad . \quad . \quad . \quad (128)$$

showing that the reluctance of the teeth on the trailing side has virtually increased the length of the single air-gap by $12\frac{1}{2}$ per cent. As in the case of the smooth-surface armature, it is evident that for the same value of l_g, A_w for a given armature may be increased roughly in proportion to the number of poles, and that by suitably increasing these the maximum permissible number of ampere-wires per pole as limited by the requirement of a reversing field need never be exceeded, and in itself presents no difficulty.

§ 33. **Determination of angle of lead.**—To predict the exact angle of lead which the brushes require for any particular armature current, it would be necessary to map out accurately the distribution of the displaced field in the interpolar region,—in itself a complex problem, and to determine the curve of change of current in the short-circuited section, as modified by the brush contact-resistance. The process would be therefore tedious, and at best only a mere approximation. In practice, therefore, when it becomes necessary to know $X_b = J\tau \cdot \frac{2\lambda}{360°}$ in the calculation of the field-winding, the designer must fall back on the evidence of similar machines already built and tested. If the short-circuited coils at full-load were brought up to the pole-tips, and the normal polar angle is taken as $\phi° = 0 \cdot 735 \cdot \frac{360°}{2p}$, the assumed maximum angle of lead would be $\lambda = \frac{1}{2} \times (1 - 0 \cdot 735)\frac{360°}{2p} = 0 \cdot 13 \cdot \frac{360°}{2p}$. Usually the angle of lead at full-load, even in a smooth-surface armature with gauze brushes, is only about 8° to 13° in a 2-pole machine, and with carbon brushes, as already explained, it can be reduced to smaller amounts.

§ 34. **Limiting number of ampere-wires per pole.**—Now although the brushes do not usually require to be so far advanced as to make full use of the reversing field at the extreme pole-tip, yet such equations as (127) and (128) at least secure a reasonable angle of lead falling within the interpolar gap, and when B_c is given such values as 2500 in the smooth-surface drum with copper brushes, and of 1500 in the toothed drum with carbon brushes, they afford a ready means of testing the sparking limits of various machines.

The expressions on the left-hand side of equations (127) and (128) give the maximum permissible *number of ampere-wires under a pole-face*, or the ampere-turns acting athwart a pole as the limiting factor of the armature output from the sparking point of view. The ratio β having usually a constant value of about 0·735, the total ampere-wires within the pole-pitch, *i.e.* per pole, are about 35 per cent. higher. As the same ampere-turns also act on the corresponding cross circuit of another pole, the permissible number of cross ampere-turns per pole may be interpreted to mean half the number which act across any one pole, and is therefore an ambiguous expression. This ambiguity is entirely removed if the number of *ampere-wires* is always spoken of instead of the number of ampere-turns.

The higher the value of the normal B_g or of X_g and X_t, the greater may be the ampere-wires per pole without trenching upon the margin of difference that is required for commuting. It is thus evident that the relation between the maximum permissible number of ampere-wires per pole and the normal air-gap density, although not one of simple proportionality, yet is very close, the former quantity rising with increasing l_g. In the case of Chap. XVII. §§ 11 and 12, we have in equation (128)

$$\frac{J\tau}{2p} \cdot \beta = 12{,}500 - 2380 = 10{,}120 \text{ ampere-wires under a pole}$$

or within the pole-pitch with a ratio $\beta = 0{\cdot}735$

$$\frac{J\tau}{2p} = 13{,}750 \text{ ampere-wires per pole.}$$

Such limiting values of the ampere-wires per pole afford a useful standard in design, but as rules are only applicable to a more or less constant air-gap, say, 0·7 to 1 centimetre, or $\frac{1}{4}''$ to $\frac{3}{8}''$, and also for some normal value of the air-gap density. In large slow-speed machines the average B_g may be as high as 9000, and with the same margin for reversing the permissible armature load becomes as much as 15,000 to 16,000 ampere-wires per pole. Economy in manufacture always dictates as large a number of ampere-wires as possible upon the armature until limited by heating or sparking, since out of the total cost of the machine nearly one-half must be debited to the armature. Economy in exciting copper, etc. also dictates a high value for the ampere-wires per pole, so that $\frac{J\tau}{2p}$ is pushed towards certain definite limits as far as prudence permits without introducing danger from sparking. A great part of the art of design consists in so choosing the number of poles, the length of air-gap, and the winding that the heating and sparking limits are reached at the same output.

§ 35. **Limiting value of ampere-wires per unit of circumference of armature core.**—Since $2p = \frac{\pi D}{\text{pole-pitch}}$, the expression

$\frac{J\tau}{2p}\beta$ may also be written as $\frac{J\tau}{\pi D} \times \beta \times$ pole-pitch, *i.e.* the ampere-wires per unit length of circumference multiplied by the length of the polar arc. Thence from the equations of the preceding section

$$\frac{J\tau}{\pi D} \leqq \frac{X_g + X_t - 0{\cdot}8\ B_c\ .\ 2{\cdot}25\ ml_g}{\beta \times \text{pole-pitch}}$$

In small machines both the normal air-gap density and the length of the air-gap bear smaller ratios to the length of the pole-pitch than in large machines; with large multipolars there is a tendency towards the adoption of a certain maximum B_g, l_g, and length of pole-pitch, so that the exciting copper and proportions of the poles remain the same and their number is simply multiplied. Thus if $a_w = \frac{J\tau}{\pi D}$, the ampere-wires per unit length of circumference, its value, so far as sparking alone is in question, rises from say $a_w = 200$ per centimetre, or 500 per inch of circumference in small machines, and tends to become constant at a maximum value of, say, $a_w = 315$ per centimetre, or 800 per inch of circumference in large machines.

But such figures must be checked by considerations of heating, and in small machines the heating limit is reached first, so that the actual values of a_w may be only half that given above. In larger machines the two limits are reached more nearly simultaneously, so that values of a_w from 400 to 650 in medium sizes and of 800 in large sizes correspond very closely with the limits imposed at once by heating and sparking (cp. Chap. XX. §§ 3 and 4 and Fig. 393).

Although the values of a_w in smooth-surface armatures are somewhat lower than with toothed armatures and carbon brushes, yet there is no very great difference between the two for the same relative sizes. The increase in the depth of the copper envelope of the smooth-surface armature with increasing sizes corresponds to the increasing depth of the slot, and the weaker density employed in the air-gap of the smooth armature is balanced by its much longer air-gap, so that the X_g of the latter is not so far different from the $X_g + X_t$ of the toothed armature.

While the permissible number of ampere-wires per pole is but little else to the designer than a warning, the quantity a_w conveys definite information as to the necessary dimensions of the armature, and is therefore of much greater practical value.

§ 36. **The separate factors influencing sparking.**—Although equation (126) is not immediately applicable to every case with and without commutating poles, yet from an examination of it a clear idea can be gained of the various factors upon which the sparkless running of a dynamo chiefly rests. The same essential relations may be expressed in many other ways, but when analysed they will always be found to resolve themselves into the combined effect of two fundamental factors, the first depending upon the armature ampere-wires per pole or the cross field corresponding to them, and the second upon the self and mutual inductance of a section and the number simultaneously short-circuited. The sum of their effects across the width of a brush must not by comparison with the external field through which the coils are moving yield a quantity ΔE exceeding a certain voltage. It remains to consider how far it is possible for the designer, within the limits imposed by the requirements of commercial economy, to favourably influence the values of the different items.

The disadvantage of a high peripheral speed, whether of armature core or of commutator, is at once evident. Especially is this disadvantageous in the case of the commutator, and herein lies the difficulty of the design of continuous-current dynamos for direct coupling to steam turbines, since the inductive voltage reaches such high values that special devices to secure more favourable conditions become imperative. There is too a limit to the output of kilowatts which can be satisfactorily reached with each voltage, although opinions may differ widely as to the exact point at which sparking sets a limit to the possible size of the machine.*

In regard to the number of revolutions per minute, the designer has in almost all cases to accommodate his design to the requirements of the prime mover, so that N is virtually fixed. The quotient of the watts of output divided by the revolutions per minute is therefore the fundamental datum of the design. As will be explained in Chapter XX., the given value of this very important ratio, even apart from any other considerations, necessitates a certain minimum value for the product of the square of the diameter and of the length of the armature core, *i.e.* of D^2L, in order to comply with usual heating conditions. Although the division of the product into its two factors is not thereby prescribed, the designer is now, generally speaking, enabled by reference to standard sizes and patterns to decide simultaneously the most suitable number of poles and style of winding in accordance with the principles of § 23, Chap. XI., and thence the separate dimensions D and L. The ampere-wires $A_w = J\tau$, as already stated, are dependent upon the diameter, so that this product is more or less fixed, as also the value of τ with the given magnetic frame which is to be employed. In this early stage of the design preference will be given to the largest practicable number of poles; the reason for this preference is that with a simplex lap armature J is thereby reduced, and although τ is proportionally increased, the number of wires per section or $\frac{\tau}{N_2}$, which is a most important factor of the whole expression, can at least theoretically be still maintained at the

* Cp. S. Senstius, "Limitations in Direct-Current Machine Design," and the following discussion, *Proc. Amer. Inst. Electr. Eng.*, vol. xxiv. p. 689.

same value by proportionally increasing the number of commutator sectors. There are, however, practical limits both to the decrease of J and to the increase of N_2. On this account J in practice averages from 150–200, but seldom exceeds 300 amperes even in low-voltage machines, unless commutating poles are fitted, in which case it may be raised as high as 400–500 under favourable conditions. Next, the limiting value of $\frac{\tau}{N_2}$, namely, 2, which is reached when each section of the drum winding consists of a single turn, is always most desirable, but may not be attainable owing to mechanical reasons, for there is a minimum thickness of sector which permits of satisfactory connection to the armature winding by a lug soldered into a saw-cut or riveted to the side of the sector. Hence as J is reduced it may become necessary to pass to $\frac{\tau}{N_2} = 4$, or $\frac{\tau}{N_2} = 6$, and so on. At each of these critical stages the designer must consider the possibility of slightly modifying the dimensions so as still to be able to retain the lower value of $\frac{\tau}{N_2}$; or the effect of adopting a wave instead of a lap winding, or *vice versâ*, on the lines laid down in § 23, Chap. XI.

The introduction of the multiplier S_k in any criterion of sparking has the effect of limiting the possible use of very wide brushes. By increasing the width of brush and the number of coils simultaneously short-circuited with a given commutator, T is also proportionately increased, while $L + \Sigma . M$ does not increase in proportion when the short-circuited coils become distributed over more than one slot in each interpolar region. But although $\frac{2J(L + \Sigma . M)}{T}$ is thus slightly reduced, S_k has been increased, so that not only the total E.M.F. from the armature cross field, but also that from the inductance, is finally greater across the width of a brush.

But now a sharp distinction must be drawn between the cases with and without commutating poles. With commutating poles the first item of the sparking voltage which deals with the armature cross effect calls for an excess of excitation in the winding of the commutating poles sufficient to neutralise the cross ampere-turns which act upon them and to leave a margin for a resultant reversing field. The second item calls for the strength of the resultant reversing field to be so balanced against the inductive effect of the short-circuited sections that no great divergence ΔE arises under any conditions of load. It can then only be said that the higher the value of $\frac{2J(L + \Sigma . M)}{T}$, the greater the likelihood of such a divergence through inaccurate balancing.

But without commutating poles the length of the normal air-gap enters into the first question, and it is now the main field excitation which must keep in check the cross field of the armature. The first item is therefore closely related to the ratio $\dfrac{\text{field ampere-turns}}{\text{armature ampere-turns}}$, or more strictly to the ratio $\dfrac{\text{ampere-turns over air-gaps and teeth}}{\text{armature ampere-wires under a pole}}$. Hence a more or less accurate and similar result as to the sparking limit is reached if only the second item, *i.e.* the inductive voltage, is calculated, and this is accompanied by a further condition which makes it allowable to increase the limiting value for $\dfrac{2J(L+\Sigma . M)}{T}$ as the ratio $\dfrac{X_g+X_t}{\text{ampere-wires under pole-face}}$ becomes higher. Thus $S_k . \dfrac{2J(L+\Sigma . M)}{T} \leqq 7$ might be laid down as a maximum permissible limit for satisfactory running, or for complete sparklessness with a fixed brush position $\leqq 4$, when these limits are accompanied by the secondary condition that the ampere-turns expended over the double air-gap and teeth should not be less than $1\frac{1}{4}$ times the ampere-wires under a pole or $1\frac{1}{2}$ times for a fixed brush position, *i.e.*

$$\frac{X_g+X_t}{A_w \text{ under pole-face}} > 1{\cdot}25 \text{ to } 1{\cdot}5$$

or if the ratio of pole-arc to pole-pitch be 0·735,

$$\frac{X_g+X_t}{A_w \text{ per pole-pitch}} > 0{\cdot}92 \text{ to } 1{\cdot}1$$

There is thus considerable room for practised judgment in choosing the right values for the two quantities to suit the degree of stringency in the terms of the specification to which the dynamo has to be built, or the nature of the work which it is to perform.

But it is obvious what a much greater amount of freedom is given to the designer by the use of commutating poles; he is thereby rendered independent of the main field excitation, so far as sparking is concerned, and also has not to consider the whole of the inductive voltage as setting up the ΔE. The advantages of commutating poles therefore fully warrant their adoption, and when rightly designed they practically remove the commutating difficulty and reduce the output limit to that of heating only.

The toothed armature with which a fixed brush position is to be obtained with carbon brushes but without commutating poles calls, however, for additional consideration, and to this case §§ 37 and 38 are especially directed.

§ 37. **The toothed armature with fixed brush position without commutating poles.**—Taking separately the two items of equation (126), corresponding respectively to the armature cross flux and to the mean inductive voltage, it has been already explained that in generators of fair size $\frac{J\tau}{2p}$ tends towards a constant quantity which is not exceeded, and so also does the normal l_g in multipolar machines; the ratio of pole-arc to pole-pitch upon which l_{gx} depends is also constant. It thus results that the density of the cross flux at the centre of the interpolar gap, or $B_q = \frac{1{\cdot}257 J\tau}{2p \,.\, 2l_{gx}}$, varies but little even in machines widely dissimilar, and is usually about 400 to 500 lines per square centimetre. In a machine of given D and L there is therefore but little scope for the designer to improve the first item when once $\frac{\tau}{N_2}$ has been reduced to its lowest value of 2. In relation to S_k, the width of the brush b_1 may be regarded as open to modification at will, but with the qualification that it must be one or other of a few standard widths, and must be such that the current can be collected without overheating and without an unduly long and expensive commutator. Mechanical considerations of the size of the armature shaft or hub on which the commutator is to be built limit the smallest diameter that it can have; on the score of expense, of brush friction, and of peripheral speed, this minimum diameter will be preferred. If then, with an assumed value for N_2 and a minimum diameter of commutator, β exceeds the minimum practicable thickness of sector, an increase in the number of sectors is accompanied by an increase in S_k when the same brush width is retained, and the quotient $\frac{S_k}{N_2}$ to a great extent remains unchanged.

The gain then lies entirely in the second item, *i.e.* in the inductive voltage by the reduction of $L + \Sigma \,.\, M$. So long as the sector width remains greater than the minimum and can be progressively decreased, although $\frac{S_k}{N_2}$ is unaltered, an increase of N_2 does not cause an equal increase of $\{l(\lambda_1 + \lambda_2) + 2l'\lambda'\}$, so that there is a net gain in the latter part of the expression. But as soon as the point is reached that the width of the sector cannot be further reduced, a further increase of N_2, although not now affecting either S_k or $\{l(\lambda_1 + \lambda_2) + 2l'\lambda'\}$ must be at the expense of a larger diameter of commutator and a higher peripheral speed. $\frac{D_k}{N_2}$ then remains the same, but the balance of advantage still lies with the increased value of N_2, since it occurs again in the denominator outside the square brackets and affects the whole expression.

Thus the control of the various sparking factors in the second item is for a prescribed output in volts and amperes and a given speed closely limited by the various considerations of price and mechanical design. The attention of the designer must be concentrated upon the reduction of $l(\lambda_1+\lambda_2)+2l'\lambda'$ by a careful disposition of the winding and choice of slot-pitch, and lastly upon the value of $\frac{\tau}{N_2}$; beyond this, at best, only a judicious compromise between many conflicting considerations remains open.

§ 38. **Choice of pitch of winding and number of slots.**—In the absence of commutating poles it is evident from § 25 that it is always advisable to adopt such a moderate degree of chord-winding that the two layers of coil-sides short-circuited in each zone do not overlap greatly. At the same time, this shortening of the chord cannot be carried very far without bringing the band of short-circuited coil-sides too near the pole-tips.

The total width of the band from edge to edge, including any intervening slots not filled with short-circuited coil-sides, is in each interpolar zone $\frac{n_s}{2p}-y_R'+\frac{2}{u_n}\cdot\frac{(b_1-m)}{\beta}$ where u_n is the number of coil-sides per slot, and if possible this expression should not exceed 70 per cent. of the number of slots between the pole-tips or $\frac{n_s}{2p}\left(1-\frac{\text{pole-arc}}{\text{pole-pitch}}\right)$; or with a further allowance for the different commuting positions of the short-circuited coils when u_n is large, say *

$$\frac{n_s}{2p}\left(1-\frac{\text{pole-arc}}{\text{pole-pitch}}\right)\geqq 1\cdot 5\left(\frac{n_s}{2p}-y'_R+\frac{2}{u_n}\cdot\frac{b_1-m}{\beta}+\frac{\frac{u_n}{2}-1}{\frac{u_n}{2}}\right). \quad . \quad . \quad (129)$$

A decided check is therefore placed upon the possibility of shortening y_R' considerably. Generally speaking, y_R' should fall short of the pole-pitch by one slot. As soon as the coil-sides short-circuited at adjacent brushes fall in different slots there is no further reduction obtainable in the slot inductance, so that when $\frac{n_s}{2p}$ is fractional, and the remainder exceeds $\frac{1}{6}$, there is little advantage gained by shortening the pitch by more than one slot.

While from the point of view of economy in manufacture a very large number of slots per pole is objectionable, owing to the loss of valuable space in insulation and the reduction in the area of iron at the roots of the teeth through their taper in small armatures, there is, on the other hand, a limit to the minimum number of slots per pole. Apart from considerations connected with the number of commutator parts per slot, a very small number of slots is open to the objection that the possible choice of rear-pitch for the coils reckoned in slots becomes greatly restricted. In order that the span of the short-circuited coil

* Niethammer, *Elektrische Maschinen, Apparate und Anlagen*, vol. i. p. 158.

should not approach too closely to the polar arc by equation (129), it is advisable that y_R' should not be less than say 89 per cent. of the pole-pitch with usual widths of pole-face; while, on the other hand, in order to spread out the short-circuited coils in several slots, y_R' should fall short of the pole-pitch. It therefore usually falls between the limits of 89 and 93 per cent., and the number of slots must not be so far reduced that this condition becomes difficult of attainment. Finally, as already mentioned in Chap. XIII. § 34, with straight-sided open slots, their width of opening should not much exceed $\frac{1}{2}''$ in order that the humming noise may not prove objectionable.

§ 39. **Importance of a large number of sectors.**—The final result of the examination of §§ 36 and 37, both with and without commutating poles, is therefore to bring into especial prominence the value of N_2 or of $\frac{\tau}{N_2}$ as the primary quantity which can be modified by the designer, and it is evident that the extent to which it is advantageous to subdivide a given armature winding into a large number of small sections is only limited by the question of expense in manufacture and the difficulty of dealing with very thin commutator sectors. The armatures of closed-coil machines for high pressures of from 500 to 1500 volts necessarily have a considerable number of turns per section, since they are wound with a large number of active wires; hence, even though the current of such machines may be comparatively small, special care is required to render them sparkless in working. In all cases of coils with different numbers of turns on similar machines, the current J must vary nearly inversely to the square of the number of turns to maintain the same degree of sparklessness.

Generally speaking, we have in practice

with one turn per section, $J \leqq 200$ amperes
„ two turns „ $J \leqq 50$ „
„ three „ „ $J \leqq 22{\cdot}5$ „

In the case of armatures for large currents at low voltages and high speeds, the designer is often met with the difficulty of securing the minimum number of commutator sectors per pole which is advisable, and for which the limiting value may be set at 15. Especially with large bipolar machines and turbo-dynamos does this difficulty arise, since the total number of bars which is required may work out to less than 60; yet even with multipolar designs it may also occur. In such cases recourse will be made to multiplex windings; the commutation of the two or more subdivisions of the winding under each brush do not then exactly coincide, but one is always in advance of the other, so that some advantage is gained in the self and mutual inductance which will be somewhat less than that of the simple undivided loop. Yet against this advantage it must always be borne in mind that the time of commutation is reduced for the same width of brush as explained in § 26. The same reduction in the time of commutation is equally a disadvantage in the adoption of a multiplex wave-wound armature.

Where a duplex lap winding with either independent or re-entrant circuits is to be recommended on account of the paucity of the number of active wires and commutator bars per pair of poles that otherwise results, an additional precaution for securing equal division of the current and sparkless running consists in the use of separate equalising connections for the two windings, one set at each end of the armature, and the final interconnection of the two sets of equalising rings; the farther end of a bar should be at the same potential as the next commutator sector ahead of the one to which the bar is itself attached, and by the above device, due to Mr. F. Punga, this result is automatically attained at a number of points corresponding to the number of equalising rings.*

A more drastic solution of the same problem is the adoption of a single-winding with a commutator at either end, the upper layers of bars being connected at each end to a sector; when the brushes are correctly placed the unit which passes into and out of short-circuit is thereby reduced from a whole to a half loop. Finally, by taking out commutator connections at intermediate points along each bar through the air-ducts, each loop can be positively subdivided into sections (cp. Siemens' patent 11,471, 1904). But care must be taken that such connectors do not themselves add a considerable amount of inductance (cp. Phœnix Dynamo Company's patent 11,701, 1907), and the proper mechanical support of the leading-out wires always remains a difficulty.†

Above the minimum number of sectors per pair of poles, say 30, which is advisable to secure steadiness of voltage, and above the minimum which is necessary to bring the average voltage per sector or $\frac{E_b \cdot 2p}{N_2}$ below, say, 20 to 25 volts, so that there may be no flashing across from sector to sector over the intervening mica, no rational formula for N_2 in terms of τ and J can be given which will supersede as a short-cut the longer calculation of the average inductive volts $\frac{2J(L+\Sigma . M)}{T}$ or of ΔE, of which it forms the chief part.

If the armature be multipolar and parallel-connected, the number of sectors is preferably a multiple of the number of poles, for the reason that it is then easy to attach equalising cross-connections.

But even when N_2 has been provisionally decided upon, there

* *Journ. Inst. Electr. Eng.*, vol. xxxix. p. 600.

† See especially Dr. Pohl, *Journ. Inst. Electr. Eng.*, vol. xl. p. 250, and Mr. Miles Walker's remarks, p. 256.

remains the closely connected question of how many sectors or coils may be assigned to each slot in the toothed armature.

§ 40. **The number of sectors per slot.**—In a toothed armature the concentration of more than two coil-sides in the same slot is, theoretically speaking, wrong, since with a greater number than two the spacial displacement of the sectors is not matched by an equal spacial displacement of the coils. The coils are not therefore precisely similarly circumstanced in their position relatively to the field when short-circuited.

Measured on the circumference of the armature, the maximum displacement of a coil-side within a slot from its correct position for truly uniform distribution corresponding to that of the commutator sectors is

$$\left(\beta \cdot \frac{D}{D_k} - r_s\right)\left(\frac{u_n}{4} - \frac{1}{2}\right) \quad . \quad . \quad . \quad . \quad (130)$$

the assumption being that the centre coil-side is taken as the correct standard, and that the brushes are adjusted to suit this coil-side. D and D_k are respectively the diameters of the armature and commutator, u_n is the number of coil-sides per slot assumed to be arranged in two layers, so that $\frac{u_n}{2}$ is the number of sectors corresponding to a slot, and r_s is the distance between the centres of two adjacent coil-sides in the same slot and in the same layer.

In practice the use of a number of slots equal to the number of sectors usually involves too great a loss of space in insulation and too slender teeth. The wide tooth which results from grouping several coil-sides per layer in the same slot is stronger mechanically, and allows better ventilation through the core by air-ducts. The correspondingly wider slot has the incidental advantage that the slot-inductance of several coils simultaneously short-circuited in the same slot from the very fact of its width is not so much increased as might at first be expected. The practical advantages, therefore, of concentration outweigh the theoretical objections. But such concentration must not be pressed too far, since if certain limiting values are exceeded the hindermost sector of each slot, being the one that is most disadvantageously situated, becomes blackened or eaten away by sparking along its trailing edge. Indeed, this defect is not infrequent in dynamos in which the pressure of economical considerations has led to an undue concentration. The number of sections per slot must, in fact, be considered in relation to the current and number of turns in each coil and other conditions upon which the likelihood of sparking depends.

For ordinary voltages from 100 to 500 the number of ampere-wires per unit area of slot remains very constant in generators of good modern design with toothed armatures, even when of widely different size. It ranges from 800 to 1100 per square inch of slot area, and on an average is 1000. The width of an open slot is practically limited by the necessity of avoiding eddy-currents in the pole-pieces even when laminated, and the permissible depth of slot is limited by considerations of inductance and heating. There is therefore a limit to the permissible size of slot, so that even in large multipolar machines its maximum cross-sectional area is about one square inch. In small machines the size of slot must necessarily be reduced, and the utilisation of space is not so good. It thus results that the curve connecting the ampere-wires per slot with the diameter of armature rises gradually, as shown in Fig. 375, and approaches a maximum of about 1000 ampere-wires per slot.

Practice shows that when such values of the ampere-wires per slot are

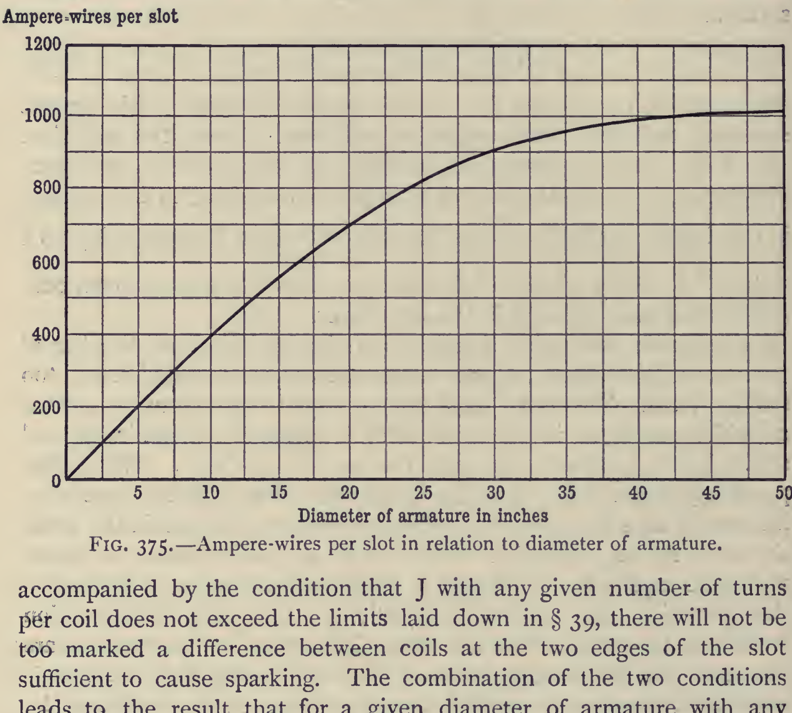


FIG. 375.—Ampere-wires per slot in relation to diameter of armature.

accompanied by the condition that J with any given number of turns per coil does not exceed the limits laid down in § 39, there will not be too marked a difference between coils at the two edges of the slot sufficient to cause sparking. The combination of the two conditions leads to the result that for a given diameter of armature with any particular value of J, and so of the given turns per coil, one, two, or three as the case may be, there is a *minimum* permissible number of sectors per slot. Thus an armature of 20″ diameter.

with J $\leqq$ 22·5 and 3 turns per coil,	must have at least		5	sectors per slot.
,, $\leqq$ 50, and 2 turns per coil,	,,	,,	3	,,
,, $>$ 50 up to 200 and 1 turn per coil		,,	2	,,
An armature of 50″ diameter or over				
with J $>$ 50 up to 200 and one turn per coil		,,	3	,,

It further results from the two conditions that there is in practice a *maximum* possible number of sectors per slot in each case, which is given by the product of the minimum number and the number of turns per coil. It therefore coincides with the minimum number when J is $>$ 50 and there is only one turn per coil, but for small values of J it seldom is reached owing to the consequent size and expense of the commutator. Any number of sectors per slot larger than the minimum up to the maximum possible, being accompanied by the condition that the ampere-wires per slot do not exceed the limits of Fig. 375, is to be regarded not as a concentration of coils into a slot, but rather as a finer subdivision of the winding, and so is to the advantage of the machine. In practice, two sections per slot is the rule for low voltages and high speeds, rising to three as the average, and even to four or five sections per slot in machines of high voltage and low speed.

§ 41. **Methods of reducing the field displacement and angle of lead.**—Many special arrangements have been devised to improve the commutation of continuous-current dynamos, which may be grouped under two heads. Either they are mainly magnetic and directed to the reduction of the armature cross-field B_q, or to the maintenance of the strength of field at the leading pole-corner, so as to minimise the necessary angle of lead; or they aim at annulling as far as possible the self and mutually induced E.M.F. of the short-circuited coil and for what remains at the supply of a reversing E.M.F., so that their action is electrical rather than magnetic.

Under the first heading come all arrangements of the magnetic circuit designed to introduce reluctance into the path of the cross field while leaving that of the main flux as far as possible unaffected. So long as the brushes are to be retained near to the geometrical line of symmetry, but little can be done to reduce the cross flux except to employ a large air-gap which also increases the necessary field excitation; further, the effect of the cross field is then itself of less importance than the mean self and mutually induced E.M.F. But when the brushes are to be advanced, a reduction of the cross field at the leading pole-tip becomes of more value. To secure this, in double horseshoe magnets such as Figs. 240 and 241, the depth of iron across the neck *c* on the line *ab* may be made small; or better still, the fields may be entirely divided along that line. The path of the cross flux is thereby interrupted, while that of the main flux is unaffected, since no lines of the symmetrical field will cross the dividing gap. In the single horseshoe any contraction of the area of the pole-piece will prevent the passage of the lines of the main field into the pole-tip, and their even spreading over the pole-face almost as much as it will throttle the cross flux, so that the method cannot be employed. But in the symmetrical multipolar magnet, such as Fig. 244, analogous methods can again be employed with impunity.

Thus each pole may be divided down its centre by a slot passing right through its axial length, and extending from the pole-face inwards to some little distance or even up to the yoke. The main path of the cross flux is thus bisected and an air-gap of, say, $\frac{3}{4}''$ to $1''$ interposed therein, which will greatly reduce its amount. A combination of a split pole with an unsymmetrical shape for its two halves is employed in the Johnson-Lundell patent dynamo, and has the effect of still further reducing the distortion. Again, if in the ordinary machine the pole-tips are rapidly thinned off to a fine edge, the trailing pole-corner becomes highly saturated, and an appreciable portion of the total cross

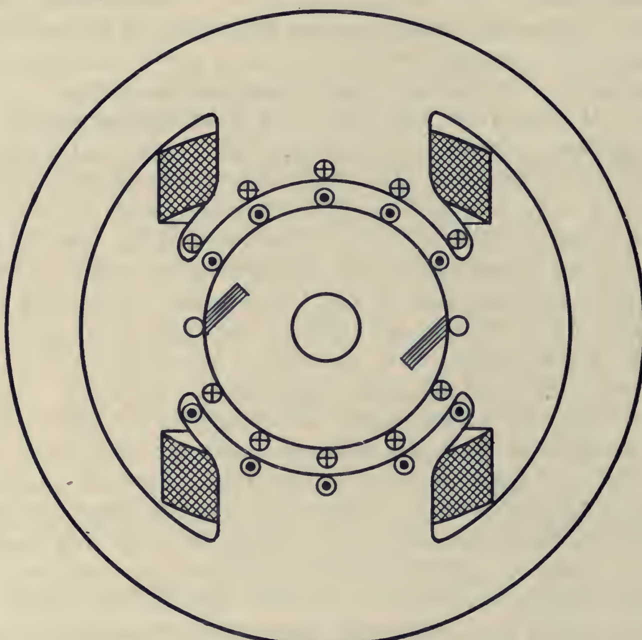


FIG. 376.—Compensating winding on field-magnet.

magneto-motive force is expended therein. The main flux may be slightly reduced thereby, but the distortion is decreased, and the advantage of keeping up the strength of the reversing field at the leading pole-corner may outweigh the disadvantage of the reduction.

With laminated pole-shoes such saturation of the pole-tip may be obtained by cutting away every alternate lamination at the pole-corner, so that the area of metal is only half that of the solid pole-edge of similar shape. An effect on the distribution of the air-gap flux closely analogous to that from the saturation of the trailing pole-tip as described in § 33 (cp. Chap. XVII. §§ 10–12) is produced by the employment of an air-gap tapering gradually from a maximum at the trailing edge to a minimum at the leading edge, the pole-faces being set eccentrically to the armature core so as to give the required tapering air-gap (Fig. 325). A stronger reversing field can thereby be gained under full-load, which is of advantage when the brushes are to be shifted forwards; yet the device is of but little assistance towards obtaining a fixed brush position, since if the brushes are fixed at the position, say, for half-load, the degree of incorrectness of the field at no-load is positively increased.*

Finally, under the same heading comes the complete neutralisation of the cross ampere-turns of the armature by means of a number of *compensating ampere-turns* carrying current in the opposite direction to the turns of the armature and distributed over the pole-face. The compensating turns are in series with the main circuit, so that their neutralising action may be proportioned to the armature current, and

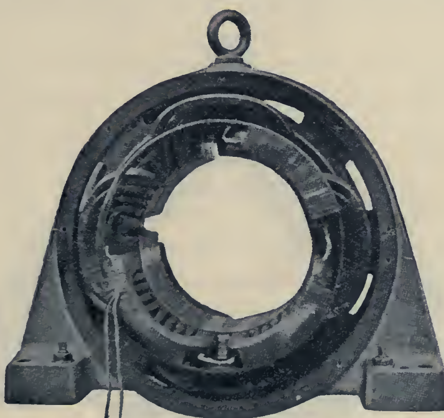


FIG. 377.—Field magnet of 100-kilowatt turbo-dynamo with compensating coils. (Allgemeine Elektricitäts Gesellschaft.)

are wound either in holes pierced through the poles close to their bored faces as first proposed by Professor Ryan, or in slots uniformly disposed over the pole-face. The principle is indicated in Fig. 376, the compensating wires in the 2-pole machine being joined up into a coil enclosing the armature. The ordinary field-winding of the poles may be retained at the back of the laminated pole-shoes which carry the compensating winding. In the multipolar machine the arrangement becomes simpler, since with four or more poles the compensating coils become flatter, and their ends can be more conveniently bent to clear the armature. The two sets of coils, exciting and compensating, are then, so to speak, in quadrature, and the winding becomes very similar to the winding of the stator of a quarter-phase alternator.

Fig. 377 shows a 550-volt magnet for a 100-kilowatt continuous-

* The advantage or otherwise of poles set eccentrically to the armature has been more fully discussed in a pamphlet on "Air-Gap Induction," by C. C. Hawkins.

current turbo-dynamo built by the Allgemeine Elektricitäts-Gesellschaft of Berlin on the Déri system of compensating winding: three of the four large shunt coils are in place, while behind them is seen the compensating winding subdivided between numerous slots. Fig. 378 shows the compensating coils for a 300-kilowatt turbo-dynamo at 230 volts; the lower pressure causes a bar-winding to be adopted. In some Déri compensated machines the field-winding proper is itself distributed among the same slots with the compensating coils, but in quadrature with them. The compensating and field-winding can also be combined with local commutating coils surrounding reversing teeth in the centre of the gaps between the several fields.*

Chord-winding not only has the magnetic effect (already pointed out in Chap. XI. § 22) of neutralising what would otherwise be the back

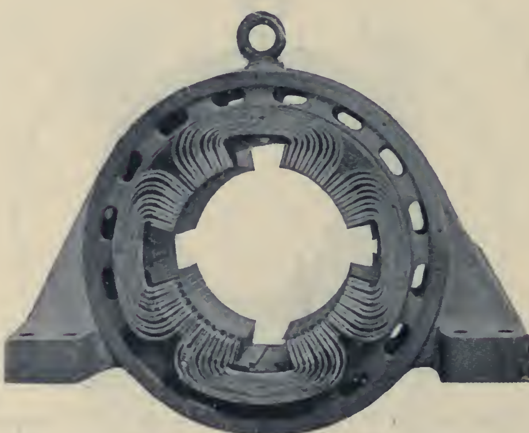


FIG. 378.—Field-frame of 300-kilowatt turbo-dynamo with compensating bar-winding in place. (Allgemeine Elektricitäts Gesellschaft.)

ampere-turns of the armature, and so of assisting in maintaining the strength of the main field, but also, when carried out to a considerable extent beyond the amount required to sensibly diminish the mutual inductance of the simultaneously short-circuited coils, has the electrical effect of reducing the angle of lead. The two sides of a short-circuited coil, when chord-wound, do not at no-load coincide with the interpolar lines of symmetry, but fall short of them, so that they are equidistant on either side of the tips of one and the same pole. They therefore act differentially and not summationally as with diametric winding. A

* For further illustrations of compensated machines, see chap. xxvii. § 3, and W. Hoult, "Direct Current Turbo-Generators," *Journ. Inst. Elect. Eng.*, vol. xl. p. 625; G. Stoney and A. H. Law, "High-Speed Electrical Machinery," *Journ. Inst. Electr. Eng.*, vol. xli. pp. 289–295.

slight movement forwards of the brushes under load then suffices to bring one side nearly up to the leading pole-tip, while the other side advances into a weaker field, and finally may itself cross the neutral line. The *difference* of the E.M.F.'s of the two sides thus rises very quickly in favour of the reversed direction of the current. A small angle of lead is thereby attained, but against this must be set the disadvantage that when the chord-winding is of considerable amount the machine is very sensitive to any changes of the load, owing to the fact that the reversing E.M.F. is the difference of two E.M.F.'s which are nearly in balance; further, the actual E.M.F. given by the armature is appreciably reduced when the chord-winding is carried to an extreme.

§ 42. **Other devices for improving commutation.**—Under the second heading of electrical devices to improve commutation may be mentioned the lamination of the brushes in planes parallel to the divisions between the commutator sectors, the several laminations being insulated from one another except at the end farthest from the commutator surface where they are in electrical connection. By this means, while the load current is free to pass lengthwise through the laminations, any additional current flowing transversely athwart the brush is checked by the artificial resistance which is interposed to its passage across from one side to the other. This idea has been further developed in the "Morganite" carbon brush. In certain forms of this brush, while the true resistivity according to the grade of the carbon ranges from 1850 to 2800 microhms per centimetre cube, the cross-resistance is as much as 6 to 7 times greater than the resistance along the length of the brush; the resistance is also graded from one edge to the other, so that the contact-resistance is low at the trailing edge, and thence rises to a maximum at the leading edge, where sparking is most liable to occur. With a peripheral speed of commutator of 2000 feet per minute the contact-resistance of the highly conductive portion as related to current-density does not differ greatly from the curves of Fig. 343 for soft carbon. The coefficient of friction of the material is low which increases the efficiency of the brush, and may be taken as ranging from 0·1 to 0·15; owing to its lubricating properties, a higher brush pressure than is usual may be employed, and not less than $2\frac{3}{4}$ to 3 lbs. per square inch of contact surface is recommended. The stratification of the brush is, however, somewhat disadvantageous, as the leading edge is liable to break away under high speed of commutator.

A combination of copper and carbon in which the cross resistance is again higher than the longitudinal resistance is found in "Endruweit" brushes (made by the Galvanic Metal-Paper Company of Berlin), which consist of layers of thin copper foil with interleaved paper, the whole being then incandesced until the paper is converted into carbon. In the "copper-carbon" brush of the same firm, although the cross resistance is about five times higher than the longitudinal, the contact-resistance is comparatively small, and the loss over the two sets of brushes averages about 0·6 volt, while in the "metal brush" of the same system a still closer approach is made to pure copper brushes. But in all such stratified brushes it is probable that the bearing surface on the face of the brush becomes of nearly uniform conductivity, and this skin is in parallel with the true cross resistance, so that finally it is the actual contact-resistance upon which we have to depend for the suppression of sparking.

The advantage of high-resistance lugs of manganin connecting the commutator sectors to the armature winding is in continuous-current generators very questionable. By their use the total resistance of a coil and its connections, or $r + 2r_c$, may be trebled or more without appreciably diminishing the efficiency of a high-voltage machine. Their use may therefore be settled entirely by reference to their effect upon the commutation. In themselves they increase the steepness of the correct reversing field which is required for perfectly uniform commutation, and raise the final value which it should have, so that a greater angle of lead would result; again, their effect upon the total resistance of the short-circuit, including the brush contacts, is practically lost in the much greater rise of the resistance of the leaving brush edge, so that the final current-density is not greatly reduced when an additional current is flowing. On the whole, therefore, high-resistance commutator connections do not appear to be of any great service, although they do have the effect of reducing the maximum value of the additional current for a given degree of inaccuracy of the field and of causing this maximum to occur more nearly at the centre of the brush or midway during the period of short-circuit.

In the Sayers dynamo the commutator connections are brought backwards across the armature core so as to be acted upon by the fringe of lines from the trailing pole-tip. A reversing E.M.F. is thus obtained from the difference between the E.M.F.'s set up in the two connections which form the ends of a coil and join it to the commutator, and as the density at the trailing pole-tip increases with the armature current owing to the increasing distortion the reversing E.M.F. automatically rises with the load.

Finally, we come to the use of commutating poles which, from their importance, will be specially considered in the next section.

§ 43. **Commutating Poles.**—In order to obtain complete fixity of brush position under varying loads and sparkless running in difficult cases, such as high-speed dynamos driven by steam turbines, or machines to give a very wide range of voltage, auxiliary *commutating* or *reversing poles*, or as they are also called "interpoles," become almost an indispensable addition. Even in ordinary cases, when the speed is not very high, the improvement which they give in the commutation enables windings which would otherwise be of doubtful success to be more readily employed, and a net saving in cost can be effected; thus the use of wave instead of lap windings can be extended to larger outputs, and the advantages are thereby gained of an armature cheaper to manufacture with a more open and mechanical winding as well as a commutator with open lugs and small diameter and peripheral speed, while the freedom of wave-windings from local currents due to unequal pole-strengths or incorrect centring renders them better adapted to short air-gaps.

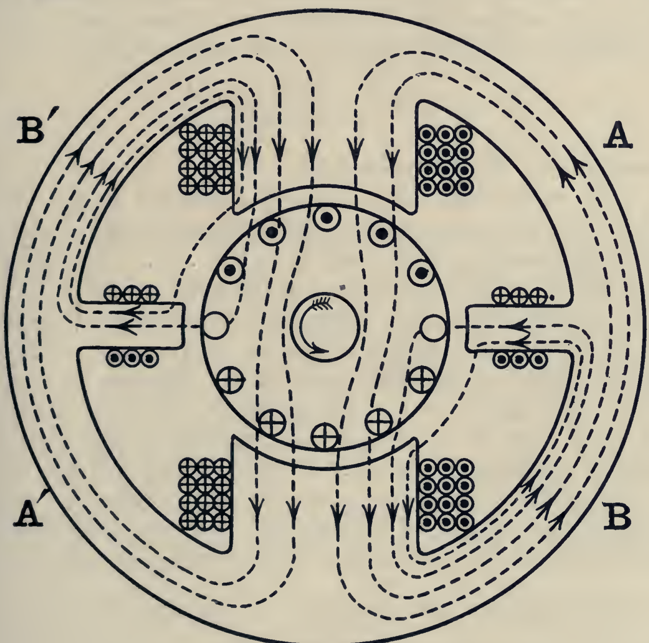


FIG. 379.—Field-magnet with commutating poles.

Such poles are usually made to project from the yoke-ring of the field-magnet in the middle of each interpolar gap, so that their pole-faces are presented to the armature core on the interpolar line of symmetry (Fig. 379), and the brushes are so set that the coils when short-circuited are brought under the influence of the field between commutating pole and armature. To recapitulate the principle outlined in § 30, their function is to supply the right strength of reversing field so as to cause the current-change to follow as nearly as may be a straight line under all conditions of load from zero up to the required maximum overload. They are excited by magnetising coils in series with the armature winding and carrying the full armature current so that their effect may be proportionate thereto; or in some cases the current out of each brush arm of the multipolar machine is taken directly round an adjacent commutating pole before it joins the combined stream. It has been pointed out that it is not necessary that the axial length of the commutating pole-face l_r should be equal to that of the armature core l. Should l_r be less than l, then so far as the short-circuited coil-side is not covered by the commutating pole-face, the reversing density must contain an additional term to compensate for the cross induction B_q over a length equal to the difference between l and l_r. Neglecting, therefore, the ohmic resistance term as of small influence, the full form which equation (122) takes when applied to commutating poles equal in number to the main poles is

$$B_r = -\left\{ B_q(l - l_r) + \frac{N_2 \cdot 2J(L + \Sigma \cdot M)}{\tau \cdot l_r \cdot vT \times 10^{-8}} \right\} \quad . \quad . \quad . \quad (131)$$

and the first term vanishes when $l_r = l$.

In order to produce this flux-density across the air-gap of length l_{gr} under the commutating pole, the effective air-gap ampere-turns on each commutating pole must be $0{\cdot}8B_r \cdot l_{gr}$. But from their position facing the interpolar regions of the armature core it is evident that the commutating poles in themselves afford a direct iron path for the cross flux of the armature issuing or entering along the line of symmetry. The commutating poles must therefore be oppositely excited, so that their B_o would exceed the B_q of the armature which would exist over the length of the commutating pole-faces in the absence of excitation upon them. In other words, the first step must be to furnish the commutating poles with ampere-turns to completely balance the cross M.M.F. of the armature. With the brushes set at the geometrical centre the cross ampere-turns of the armature are equal to the ampere-wires per pole $\frac{J\tau}{2p}$, which act on each magnetic circuit. Hence the compensating ampere-turns on *each* commutating pole must be half this number, or $\frac{1}{2} \cdot \frac{J\tau}{2p}$. Corresponding, therefore, to equation (127), but for one commutating pole, if X_{gr} = the total ampere-turns expended over the air-gap of the commutating pole, we have

$$X_{gr} = \frac{J\tau}{4p} + 0{\cdot}8B_r \cdot l_{gr} \quad . \quad . \quad . \quad . \quad . \quad (132)$$

Two points of importance must now be mentioned. In order to keep the circumferential breadth of the commutating pole within practical limits, the armature winding should be diametric or more nearly concentrated than would otherwise be advisable; the short-circuited sides of a coil will then be similarly situated in reference to the reversing field, and be acted upon summationaliy, the E.M.F.'s impressed on the two sides being in phase and strictly additive. In consequence of y_r' being nearly or exactly equal to the number of slots within the pole-pitch, the $L+\Sigma . M$ is naturally high as compared with the case of the usual windings which are more or less chorded. Secondly, and of even greater importance, owing to the presence of the iron commutating pole immediately above the short-circuited coil-sides, their inductance, so far as the length l_r is concerned, is considerably enhanced; the self-induced flux from the surface of the teeth finds an iron path presented to it, and the air-path is shortened to $2l_{gr}$. Equation (115) thus becomes with commutating poles

$$L+\Sigma . M=\left(\frac{\tau}{2N_2}\right)^2\Big[l\left(k_1' . \frac{h_s}{w_s}+k_1'' . \frac{h_3}{w_3}\right)+(l-l_r)\,k_1'''+l_r . k_1''''$$

$$+l\left(k_2' . \frac{h_s}{w_s}+k_2'' . \frac{h_3}{w_3}\right)+(l-l_r).k_2'''+l_r . k_2''''+2l'\lambda'\Big]\times 10^{-9}$$

$$=\left(\frac{\tau}{2N_2}\right)^2[l(\lambda_1+\lambda_2)+(l-l_r)(k_1'''+k_2''')+l_r(k_1''''+k_2'''')+2l'\lambda']\times 10^{-9} \quad . \; . \; . \; . \; (133)$$

Since the gap between the adjacent edges of a main and auxiliary pole is usually 10 or more times the direct air-gap between auxiliary pole-face and armature, it follows from Fig. 265 that the equivalent strip along *one* side of the pole filled with lines at the normal density under the pole is approximately constant at a width 2·25 times the air-gap. The permeance presented to a coil-side immediately under the centre of a commutating pole is therefore

$l_r\left(\frac{\text{breadth of comm. pole} - \text{slot opening}}{2}+2{\cdot}25l_{gr}\right)\times\frac{1}{2l_{gr}}$, and with diametric winding this is acted upon by $4\pi . j_a . w$, the M.M.F. of $j_a . w$ ampere-wires. The inductance on the assumption that all the flux is linked with all the wires is then

$$\left(\frac{\tau}{2N_2}\right)^2 . 4\pi . j_a . l_r\left(\frac{\text{breadth of comm. pole} - \text{slot opening} + 4{\cdot}5l_{gr}}{4l_{gr}}\right)\times 10^{-9}$$

$$\text{whence } k'''' = j_a . \pi\left\{\frac{\text{breadth of comm. pole} - \text{slot opening}}{l_{gr}}+4{\cdot}5\right\} . \; . \; (134)$$

It remains, however, questionable how far the variations of flux through a solid commutating pole-shoe are damped out by eddy-currents, so that the apparent inductance would be less, and the above figure would have to be multiplied by a damping coefficient appreciably less than unity. In the opinion of some, the increase of inductance due to the presence of the iron poles over the short-circuited coils is only of the order of 10 to 15 per cent.

The effective breadth of the air-gap of the commutating pole with its two strips, one on each side, is analogously = breadth of the pole-shoe $+4{\cdot}5l_{gr}$, and its effective axial width is calculated by Fig. 263 on the same principles as a main pole. Their product multiplied by

B_r gives the total useful reversing flux z_r, to which has to be added the leakage into the adjacent main pole of opposite sign and into the yoke.

The actual distribution of the lines is indicated in Fig. 379, from which it will be seen that the commutating pole is to be regarded as an extension of the leading edge of the adjacent pole into the centre of the interpolar gap, this detached portion being furnished with a separate exciting coil.* Leakage takes place on one side of the commutating pole into a neighbouring main pole under a M.M.F. rising gradually to equality with the sum of the M.M.F.'s of one main exciting coil and one commutating field-coil, and also into the yoke at the root of the commutating pole under a portion of its own M.M.F. Owing to the greater surface of the commutating-pole in proportion to its section and the high M.M.F.'s, this leakage plays a much more important part than in the main poles, and if not properly calculated will lead to the iron of the auxiliary pole being made too small; it will then become very highly saturated, and proportionality of the reversing field to the current will be entirely lost.

With as many commutating poles as there are main poles it will be found from Fig. 379 that for the same useful main flux the density in the main poles is entirely unaffected by the addition of the commutating poles. In the armature the small portion of the path between a commutating pole and an adjacent main pole of opposite sign carries $\frac{Z_a + z_r}{2}$ lines, and the remainder of the path to a main pole of the same sign only carries $\frac{Z_a - z_r}{2}$ lines. Similarly, in the yoke-ring, if Z_r is the total flux of the commutating pole including leakage, this number of lines is added to the flux carried by one section of the yoke (B), and deducted from the other section (A), which completes the circuit from one main pole to another, so that the two densities are proportional to $\frac{Z_m + Z_r}{2}$ and $\frac{Z_m - Z_r}{2}$. If, therefore, the section of the armature core, and similarly that of the yoke, is sufficiently large so that they are far from saturation, in each portion the two changes largely counterbalance one another, or in any case the percentage effect on the *total excitation* required for a given main flux is very small; and this is in practice usually the case. The effect of the commutating poles on the main flux is chiefly to cause it to be somewhat unequally distributed over the main pole-faces, the trailing edge being more crowded, so that if the excitation of the commutating poles is annulled the main flux swings back to an equal distribution, and the amount of throw on a ballistic galvanometer caused by the swinging backwards of the flux across the centre line of the main pole is proportional to the commutating flux.†

* Cp. Dr. R. Pohl, *Electr. Eng.*, 1906, vol. xxxvii. p. 546.

† Professor E. Arnold, *E.T.Z.*, March 15, 1906.

Thus if the ampere-turns necessary to pass the useful reversing flux and the leakage through the commutating pole are calculated, and are added to the ampere-turns expended over the air-gap, their sum is the effective ampere-turns of the pole, or the difference between the number with which it must actually be wound and $\frac{J.\tau}{4p}$.

In the design of commutating poles the aim should be to inject into the armature the necessary amount of reversing flux within the shortest possible axial length of pole-face, so as not to cover with iron more of the coil-sides than is absolutely necessary, and to reduce the percentage of leakage of flux in the commutating pole; any wide departure of B_r from proportionality to the load due to saturation of the iron will tend to set up sparking from the ΔE which results between the brush edges. Hence B_r should be as high as possible, with due regard to the exciting turns being reasonable. On the above accounts commutating poles of less axial length than the gross length of the armature core have many advantages, and have been employed especially by the Phœnix Dynamo Manufacturing Company Limited. In the design employed by this firm, and due to Dr. R. Pohl, a commutating pole with circular core is arranged at one end of the armature, with a trapezoidal pole-shoe of short axial length; the adjacent main pole-shoe of opposite sign is cut away to lengthen the leakage path between the two.

So far the use of as many commutating poles as there are main poles has alone been contemplated. But on the same grounds as mentioned in the last paragraph, it may in some cases be possible and advisable to use only half as many commutating poles. In this case the ampere-turns required on each pole to neutralise the M.M.F. of the armature cross ampere-turns still remain exactly the same, viz. $\frac{J\tau}{4p}$. Equation (131) becomes

$$B_r = \frac{2N_2 \cdot 2J(L + \Sigma \cdot M)}{\tau \cdot v \cdot l_r \cdot T \times 10^{-8}} + B_q$$

But in equation (133) k_2''' and k_2'''' vanish, although λ_2 must be correspondingly increased. Thus as the inductance is reduced the density in the air-gap and the effective ampere-turns over it are not doubled, so that on the whole there is a saving of copper. Yet, on the other hand, there is greater danger from saturation of the magnetic circuit. In the armature and yoke the difference between the highly saturated and weakly saturated portions is again equal to the reversing flux, *i.e.* the densities are severally proportional to $\frac{Z_a + z_r}{2}$, $\frac{Z_a - z_r}{2}$, $\frac{Z_m + Z_r}{2}$, $\frac{Z_m - Z_r}{2}$, where Z_a and Z_m are the normal fluxes before the addition of commutating poles, but z_r and Z_r are greater. More than this, an inequality arises between the total fluxes carried by the magnet-cores of each pair of poles; the one of opposite sign to a commutating pole must carry a total flux roughly proportional to $Z_m + \frac{Z_r}{2}$ and that of the same sign $Z_m - \frac{Z_r}{2}$. Since each armature loop is acted upon by a pair of poles, this does not affect the E.M.F.'s of the various parallel paths, but the difference of density equal to Z_r may lead to an appreciable reduction in the normal main flux owing to the

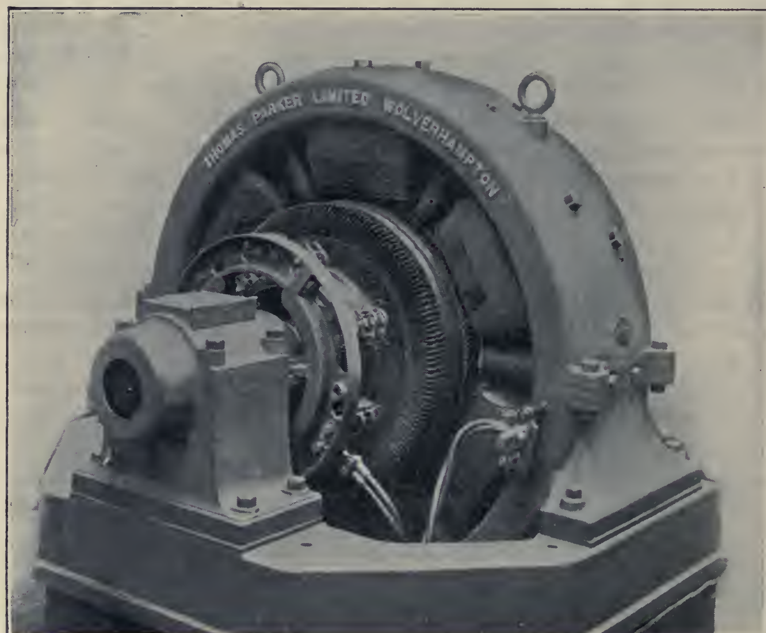


FIG. 380.—210-kilowatt dynamo with commutating poles (Thos. Parker Ltd.).

greater number of ampere-turns required by the highly saturated pole-core. Further, it may become difficult to dispose of the necessary ampere-turns on the single commutating pole without overheating, so that this arrangement requires care in its application.

Figs. 380 and 381 illustrate a dynamo giving 210 kilowatts or 400 amperes and 530 volts at 210 revolutions per minute, manufactured by Messrs. Thomas Parker Ltd., and fitted with commutating poles. Each of these poles being wound with 24 turns carrying the full armature current, while each of the main poles was wound with 2180 turns carrying at full-load 5 amperes, the proportion of the ampere-turns on the commutating to those on the main poles was $\frac{9600}{10,900}$. For further illustrations and details of machines with commutating poles, see Chap. XXVII. § 3 and Chap. XX. § 9, also *Journ. Inst. Electr. Eng.*, vol. xxxix. p. 570 ("Direct-Current Design as influenced by Interpoles," by Messrs. Page & Hiss).

§ 44. **The proportions, etc. of commutating poles.**—The commutating pole must be mechanically strong and well supported in order to prevent its being set into oscillation by the varying drag of the

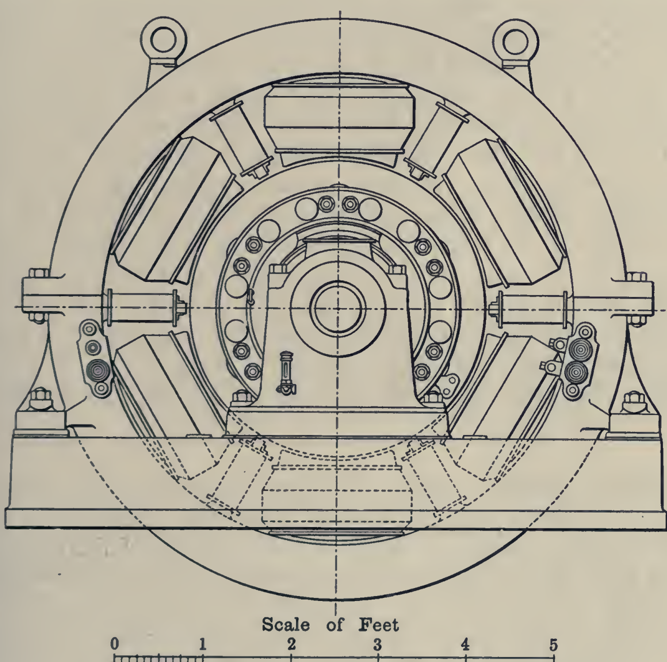


FIG. 381.—210-kilowatt dynamo with commutating poles (Thos. Parker Ltd.).

armature teeth as they pass under it. Its breadth in the direction of rotation must be at least equal to the tooth-pitch in order that the reversing field may not vary very greatly during the passage of a slot under it. As a second condition, if the slots contain several coil-sides in each layer, in order to keep each coil-side under the pole during the whole of the period of commutation, the breadth must with diametric winding be equal to the peripheral speed of the armature multiplied by the time of commutation *plus* the amount by which the hindermost coil-side of a slot is displaced from its position for true uniformity as compared with the foremost coil-side. Hence by addition of twice the expression of (130) to $T \times v$, the value of T being as in equation (121), the breadth of the commutating pole-face would be

$$\left\{b_1+\beta\left(\frac{u_n}{2}-\frac{a}{p}\right)\right\}\frac{v}{v_k}-r_s\left(\frac{u_n}{2}-1\right)$$

$$=\left(b_1-\frac{a}{p}\cdot\beta\right)\frac{D}{D_k}+t_1-r_s\left(\frac{u_n}{2}-1\right) \quad . \; . \; . \; (135)$$

which usually averages about 1·66 to 1·75 times the tooth-pitch t_1. If the winding be long-chord, an additional tooth-pitch should be added, but considerations of leakage usually forbid such an addition in full. Or the pole-shoes can be set aslant to the axis of the armature core, so as to increase the time of their action without increasing the leakage. But in either case it must be remembered that the fringe from the sides has itself considerable effect in extending the time of strong reversing action. In order to accommodate the commutating poles without bringing them too close to the main pole-shoes, the ratio of the pole-arc to the pole-pitch frequently has to be shortened to 0·66 or even less, and there should be at least 3½ to 4 slots in the zone between the main poles. Strictly speaking, in order to secure exact instantaneous balancing of the combined ohmic and inductive voltage in the short-circuited coil, the reversing field in a generator should rise in density from one side to the other by an amount proportional to $R=r+2r_c\,.\,\frac{\beta}{b_1}$, but such refinements are not of value in practice.

With the increased number of ampere-wires per inch length of armature circumference that may be considered possible with commutating poles, a danger arises from the distortion of the main field leading to an undue voltage being generated in the sections under the trailing half of the main poles. Especially is this likely to occur if the main air-gap is also reduced, and the commutator is then liable to "flash over," the sparks under sudden variations of load leaping across from sector to sector and forming practically a short-circuit to the armature. On this account under difficult conditions of machine design as in turbo-dynamos, the combination of a compensating winding with special reversing teeth or poles has much to recommend it, or the main poles must be shaped to retain a fairly even distribution of the field under their faces against the distortion from the armature cross ampere-turns.*

With fairly wide commutating poles and such conditions that the brushes admit of some shifting backwards without sparking, their position may be so adjusted as to produce an appreciable compound-

* *Vide* Chap. XX. § 8 for limiting value of volts per sector, and Chap. XVIII. § 39 for methods of increasing the number of sectors and reducing the inductive voltage per sector.

ing effect. Even when this cannot be secured, the regulation of a shunt-wound generator with commutating poles is usually good, and better than in the ordinary machine.

While the copper on the field winding proper is reduced by commutating poles owing to the shorter air-gaps, the efficiency of the machine is but little affected. But owing to the close proximity of the commutating and main coils, the ventilation of the field-magnet system is to some extent lessened as compared with the dynamo of the same size without commutating poles, and this consideration must be duly allowed for in the design. Especially is it difficult to secure a low rise of temperature on the commutating coils at full load or overloads, and on this account it is becoming increasingly common to wind the commutating poles with bare strip on edge. With large currents at low voltages, and on turbo-generators up to 250 volts, the commutating coil may be formed of a bare copper cylinder divided spirally to form a helix of a few turns.

The air-gap of the commutating poles on large machines with toothed armatures should not be less than $\frac{1}{8}''$ to avoid undue heating of their polar surface by eddy currents.

Experimentally the correct ampere-turns for the commutating poles are best obtained by separately exciting them and altering the excitation until the voltage read between two points touching on the commutator in line with the edges of the brushes, and therefore ΔE, is a minimum or as nearly zero as possible both for light-load and full-load armature currents.

In order to adjust the winding of the commutating poles to the best amount for sparkless and cool running, a diverter may be used, formed of coils on an iron core, and so designed as to have the same time constant as the commutating coils; otherwise with a non-inductive resistance, an undue proportion of the current will be shunted during rapid changes of the load. When the fluctuations of load are very rapid, as in traction work, a difficulty sometimes arises from the inability of the commutating field to follow with sufficient rapidity the change of the armature current which is to be commuted. The diverter can then be adjusted so that a greater proportion of the current is momentarily shunted through the commutating poles so as to accelerate the change of field upon sudden increase of the load.*

Partly on the same account, Messrs. Parsons & Co. have made use in turbo-generators of a special arrangement of compensating coil which also produces a commutating field with almost a complete air-path; not only can a perfect balance up to a heavy overload current be thereby secured, owing to the absence of iron saturation, but also the

* Cp. R. Pohl, *Journ. Inst. Electr. Eng.*, vol. lx. p. 249, and W. Hoult, *ibid.*, vol. lx. p. 630.

absence of hysteresis and eddy-currents enables the quickest changes of current to be simultaneously followed.*

§ 45. **Brushes and brush-holders.**—The width of each brush along the axis of the commutator is usually from $\frac{3}{4}''$ to $2''$, the latter dimension being seldom exceeded, since it then becomes troublesome to maintain proper contact along its entire bearing surface. Hence, to carry any considerable current, two or more brushes are mounted in line on each arm of the rocking-bar, forming in effect one wide brush. This arrangement also renders it possible to adjust each brush separately, or even to remove one temporarily, without interrupting the current; and this advantage is so great that every dynamo which is more than a toy should invariably be furnished with at least two brushes on each arm, each brush being of such width that, if one be removed, the other can temporarily carry the current of both.

FIG. 382.—Carbon brush-holder.

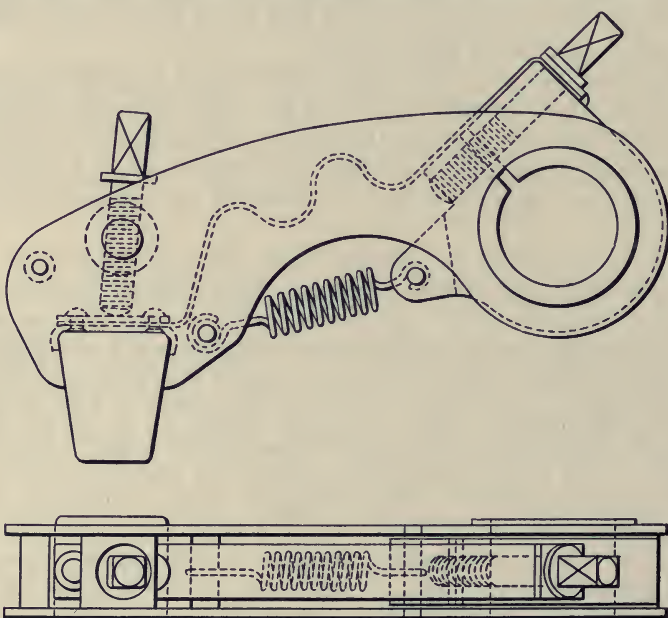


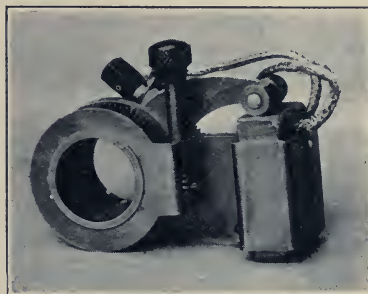
FIG. 383.—Carbon brush-holder (Veritys Ltd.).

Carbon brushes, in order to obtain sufficient contact-surface without unduly increasing the length of the commutator, are usually from $\frac{1}{2}''$ to $1''$ thick in the direction of rotation. They thus cover more than

* G. Stoney and A. H. Law, "High-Speed Electrical Machinery," *Journ. Inst. Electr. Eng.*, vol. xli. p. 291.

one sector, as a general rule 2 or 3, and in cases of very narrow sectors as many as four; but when this is the case a hard variety of carbon is to be recommended rather than a soft graphitic quality. As the conductivity approaches more and more nearly to that of a metal brush, it must be given a somewhat analogous width of, say, $1\frac{1}{2}$ sectors.

Carbon brush-holders may be classified under one or other of two leading types. In the first or pivoted "hammer" type the more or less wedge-shaped block of carbon is fixed rigidly within its box, which forms the farther end of a pair of stamped or cast brass or aluminium cheeks; these latter are pivoted on the brush spindle so as to be free to turn round it were it not for the constraining action of the pressure spring (Figs. 382 and 383). In the second type the brush is a rectangular slab, free to slide radially up or down in a guiding box, but pressed down by a helical or clock spring or springs (Figs. 384 and 385). In both cases the carbon brush is nearly radial, although it may have a slight rake in the direction of rotation, which reduces the tendency to "chattering." In the second type the carbons require some attention, so that they may not become set fast in their boxes through dust and dirt; but on the other hand, they must not be too loose in fit, whereby they tend to take up diffferent positions in the boxes according to the speed when this is variable, with consequent disturbance to their bearing surface. For low speeds the pivoted type is perhaps preferable, and the inclined sliding type for medium and high speeds. In all cases a good electrical connection directly between the carbon and the fixed box or brush spindle is of vital importance. In the first type the



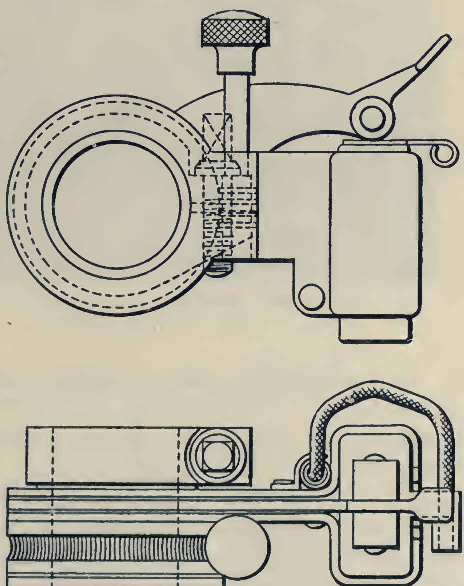


FIG. 384.—"Aston" brush-box, with sliding carbon for small machines (Veritys Ltd.).

brush may be wedged or drawn tight up into its box, with an interposed layer of copper gauze to form a good contact between the two; the box is then joined by a flexible copper connector to the central part of the brush-holder, which is either clamped or screwed to the brush spindle. In the second type the brush is itself drilled with a hole into which is tightly wedged a split pin or rod forming the end of the flexible; or its end may be splayed out into strands which are worked up into the substance of the brush during its manufacture. Metal clamps fitting over the top of the brush are also used, but in any case the use of solder is best avoided owing to the possibility of its melting should the brush become unduly hot. The flexible copper connector is formed up into a twisted pigtail with enough slack to allow of the brush being withdrawn from the box for examination.

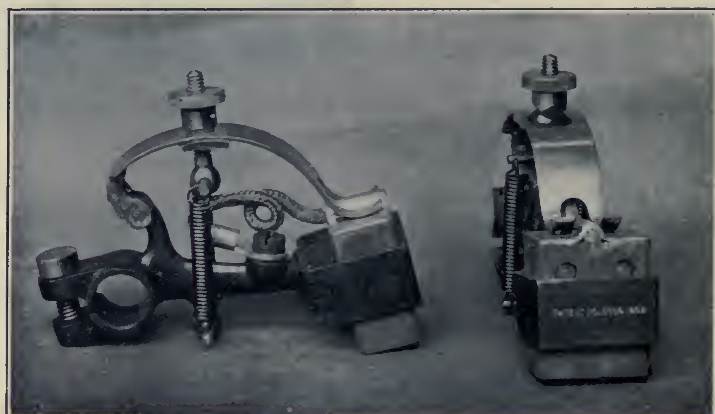


FIG. 385.—Brush-box with sliding carbon for larger machines (W. H. Allen Son & Co. Ltd.).

An intermediate type is found in the parallel-movement brush-holder of the Crocker-Wheeler Co.; the feature of this is that the brush, though rigidly held, does not through wear alter its position relatively to the commutator (Fig. 386).

A combination of a gauze brush with a carbon tip ahead of the copper has been used in many cases where the current is heavy and at the same time variable; the carbon to some extent renders any slight sparking innocuous, but the arrangement has the objection that it is difficult to obtain a proper division of the current between the two heterogeneous substances, owing to the much higher conductivity of the metal.

Copper brushes are most commonly made of gauze of a fine mesh; this is folded over on itself, stitched together with fine copper wire, and pressed flat. The one end is then lightly soldered together, while the other, which is cut diagonally across the mesh to avoid fraying at the tip, is trimmed to an angle so as to bed evenly on the commutator surface over its entire length and thickness. Less frequently brushes are made of fine copper wires or thin sheets of copper soldered together at one end.

As a general rule, with gauze brushes it is best not to short circuit more than one section at a time, unless it be for a mere instant, and therefore (except in the case of duplex- or triplex-wound armatures) the width of brush contact should not exceed the width of one sector and one insulating strip by more than a fraction of a sector. In ordinary cases this is obtained by using a brush of $\frac{1}{4}''$ or $\frac{5}{16}''$ thickness, so set

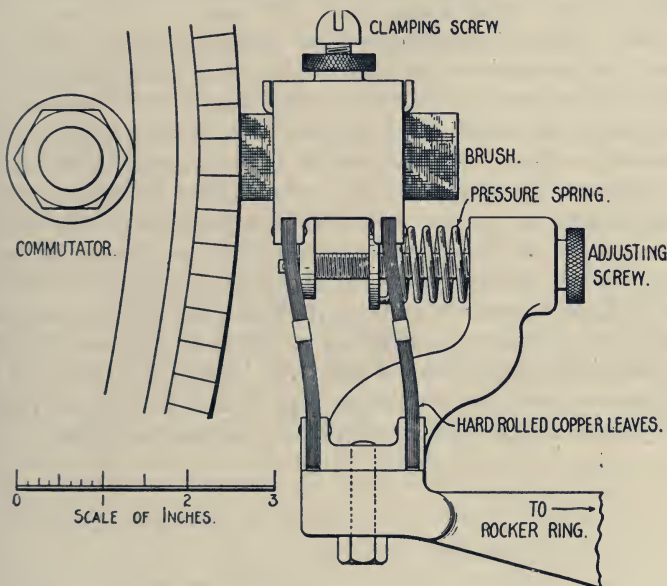


FIG. 386.—Parallel-movement brush-holder (Crocker-Wheeler Co.).

that it makes an angle of about 45° with a line drawn tangentially to the commutator surface.

§ 46. **Causes of local sparking on particular sectors.**—The pressure of the brush-tips on the commutator may be adjusted by altering the tension or pressure of the "hold-on" spring. "Jumping" of the brushes, due to vibration of the machine when running, must be carefully avoided, since it will give rise to sparking, and on this account a substantial brush-carrier with strong but light brush-holders, capable of being firmly fastened, is an essential part of a well designed and well built dynamo. The brushes should then bear lightly and evenly on the commutator. Any pressure beyond this should be avoided, since it will cause increased friction and wear. Occasionally, one or two sectors in a commutator wear down below the general cylindrical surface of the rest, and form what is known as a *flat*; as the brushes pass over the faulty spot the circuit is momentarily broken and sparking occurs, which rapidly increases the evil. The development of a flat is often attributed to inequality in the wear-resisting properties of the sectors, but it is almost always due solely to sparking. Owing to a want of uniformity in the spacing of the winding on the armature surface, a particular section may be short-circuited when in an incorrect position; its passage under the brushes is then accompanied by sparking, and the sector to which it is attached becomes worn. With carbon brushes it is especially important to employ a soft quality of mica having approximately the same rate of wear as that of the metal sectors. Irregularity of the turning moment of the prime mover, if considerable, occasionally causes flats corresponding to the dead centres of the crank-shaft. If an armature wire is broken or its connection to the commutator becomes loose, violent sparking may be set up, and the faulty coil may then be located by running the machine until one sector becomes pitted by the sparks. A complete break may be identified by the greenish colour and snapping sound of the sparks.

The length of the commutator should be such that the sets of brushes can be relatively staggered enough to overlap one another, so as to distribute the wear; otherwise if there is any sideways movement of the armature, sparking may be set up by the brushes striking against the sides of the ridges formed when the brushes are exactly in line. With four or more sets of brushes, they are best staggered in pairs, so that a positive and a negative brush sweep over the same path.

CHAPTER XIX

THE HEATING OF DYNAMOS

§ 1. **Rise of temperature in dynamo at work.** — With the exception of "sparking," no subject is of such importance, alike to the designer, the purchaser, and the attendant, as the question of the heating of dynamos. The continuous generation of heat in the armature and magnet-windings of all dynamos, so long as they are at work, is a necessary consequence of the passage of the current through their coils, and the appearance of this heat implies that a corresponding amount of energy is "lost," in so far as no useful work is derived therefrom. All that can be done from the point of view of economy is to minimise the amount of the heat which is thus generated, so as to obtain a reasonably high efficiency, such as is suited to the circumstances of any given case. Apart, however, from the question of the amount of heat produced every second, or its rate of generation in watts, there is the further and equally important question of the temperature to which any part of the dynamo is thereby raised. Whether it be the field-magnet coils or the armature which is the source of heat in question, when the machine is set to work the temperature of their mass gradually and continuously rises above the temperature of the surrounding air, until, finally, the rate at which the heat is generated is balanced by the rate at which it is carried off by radiation, convection, and conduction; after attaining the temperature which satisfies this condition, no further rise takes place. Evidently, therefore, the *rise* of temperature depends essentially upon the amount of cooling surface provided and its actual effectiveness in dissipating heat, and, this being so, it follows that it may be regulated so as not to exceed a certain maximum, if the amount of cooling surface be duly proportioned to the watts expended. We have, however, first to consider in what way a high temperature is actually injurious to a dynamo, and also the closely connected question, in what way a large range of temperature is detrimental to its working: it will be found that they are so in three ways, through their effect on the efficiency, the regulation of voltage, and the durability of the insulation.

§ 2. **Disadvantages of high temperatures. Increase of electrical resistance.**—In the first place, the higher the temperature of any portion of the electrical circuit of a dynamo, the greater is the loss of energy due to the passage of a given current through it. The limits of temperature within which dynamos are worked under average conditions may be taken as 20° C. and 60° C., or, say, 70° F. and 140° F., the former corresponding to an average value for the temperature of surrounding air in the engine or dynamo room, and the latter to an ultimate temperature which it is usual for the coils to attain when the dynamo is worked continuously, or for many hours together at its normal output. Within a range from 0° C. to 100° C., the increase of resistance to a very close degree of approximation is for each degree the same fraction of the resistance at 0° C.; *i.e.* the resistance may be expressed by the linear formula $R_{t^\circ} = R_o\ (1 + at^\circ)$. For commercial electrolytic annealed copper the Engineering Standards Committee have adopted for the temperature coefficient, α, the value 0·00428 per degree Centigrade, or 0·00238 per degree Fahrenheit. The ratio of the increased resistance for a given rise of temperature to the original resistance evidently, then, depends upon the starting-point, *i.e.* upon the temperature at which the original resistance is measured, and the percentage increase for each degree becomes less as the starting-point is raised, and *vice versâ*. If t° = the initial temperature in degrees Centigrade (or in Fahrenheit degrees above 32° F.), and θ is the further rise of temperature,

$$\frac{R_{t^\circ+\theta}}{R_{t^\circ}} = 1 + \theta \cdot \frac{a}{1 + at^\circ}$$

Thus the increase in the resistance of the copper wire on an armature which rises 70° F. (39° C.) above the temperature of the dynamo-room is 70 × 0·218, or 15¼ per cent. of its initial resistance at 70° F. But if R is calculated during the process of design from a table of resistance at a standard temperature of 60° F., the percentage increase is, as stated in Chap. XV. § 16, nearly 0·225 for each degree Fahrenheit, and R hot at 140° F. is $(1 + 0{\cdot}00225 \times 80) = 1{\cdot}18$ R cold at 60° F.; or again, from a standard initial temperature of 15° C. the temperature coefficient is closely 0·004, and this value which is also the basis of the rules of the Verband Deutscher Elektrotechniker is adopted by many German writers * for practical use.

When a given current is passed through the armature the loss of volts over its resistance when hot is, of course, increased by the same percentages. Not only must this increase be allowed for in the design, but it also involves an equally increased loss of energy. Again, the heating of the series coils on the field-magnets of series- or compound-wound dynamos increases the rate of loss in them for the same output at the terminals; while, in the case of separately excited or shunt machines, the P.D. at the ends of the exciting coils must be raised if

* Cp. *Electrician*, vol. lix. p. 65.

the same number of amperes is to be passed through their turns when hot as when cold, so that usually an adjustable rheostat is necessitated, and this again must be allowed for in the design.

Armature coils seldom have more than three or four layers of wire, and therefore the temperature of the outside, as measured by a thermometer placed in contact with the outer insulating covering of the conductors, may be taken as indicating approximately the actual temperature of the conductors themselves. But in the case of coils with a large number of layers, as already explained in Chap. XV. § 16, the mean temperature as deduced from measurement of the resistance is considerably higher than that of the surface. Thus in our previous calculation (Chap. XVI. § 7), for a depth of winding of $2\frac{1}{2}''$, the mean rise of temperature of a well ventilated coil was taken as 1·6 times the surface rise, and for a surface rise of 45° F. the increase of its resistance will be about $0·218\,(45 \times 1·6) = 15·7$ per cent. of its resistance when at the temperature of the surrounding air, which is assumed to be 70° F. Such considerable percentages show that the effects of heating must on no account be neglected in designing machines or in estimating their efficiency. Even in armatures with a single layer of conductors, if they are the rotating portion, there may be a divergence of some 30 per cent. between their actual temperature as deduced from measurements of their resistance immediately after stopping and the temperature measured by a thermometer laid on their exterior.* In all cases, therefore, measurements of the rise of resistance are to be preferred as giving more information than the temperature of the exterior as measured by the thermometer, although in the case of armatures their very low resistance may necessitate the use of a Thomson double bridge or other suitable method.

§ 3. **Less accuracy of self-regulation.**—In the second place, the rise in temperature of a dynamo when at work produces a disadvantageous effect upon the regulation of its voltage. As already mentioned, the separately excited machine requires the P.D. applied to its exciting coils to be raised if the same number of ampere-turns is to be maintained when the field-winding is hot as when it is cold; while, if the terminal E.M.F. of the dynamo is to be kept constant, its internal E.M.F. must be increased in order to compensate for the increased loss of volts over the heated armature coils, and this necessitates either a further increase in the exciting P.D. or a higher speed of rotation. Similarly, the self-regulation of the compound-wound machine for constant potential is injuriously affected by the differences in the resistances of its shunt, series, and armature coils when hot and when cold; if correctly compounded when cold, the constancy of the potential must necessarily be inferior when it is hot, or *vice versâ*. In fact, in designing compound-wound machines it is

* Wilson, *Electrician*, October 11, 1895, "The Heating of Dynamos."

especially important that the rise of temperature of the field-magnet winding be not so great as to seriously affect the compounding action of the two sets of coils, and it should preferably be limited at the most to about 55° F. or 30° C. on the surface.

§ 4. **Deterioration of insulating materials.**—Thirdly, and of chief importance,—if the temperature of any coil becomes very high, the cotton or other fibrous material commonly used for the insulating covering of the copper wires will be burnt or charred; the insulation between neighbouring turns is thus broken down, and the short-circuiting which ensues is only terminated by complete collapse. A "burnt-out" armature may be the result of an accidental short-circuiting of the machine, the heat from the excessive current almost instantaneously raising the temperature so much as to literally burn the insulation. Quite apart, however, from such accidental heating the result of continually working a machine at a high temperature is a gradual deterioration in the toughness and mechanical strength of the insulating coverings of the wires. All materials in ordinary use for insulating purposes are alike subject to this gradual decay. Slowly but surely they become charred and rotten, the cotton or calico crumbling away when touched, and the blackened paper and press-spahn becoming excessively brittle; so that, although the insulation resistances may still remain very high, the liability to a breakdown is enormously increased. When old ring armatures are taken to pieces for repairs the cotton covering of the internal layers of wire which have been continuously subjected to high temperatures may be found to be a mere charred powder, which can be wiped away with the finger.*

§ 5. **Maximum permissible temperature.**—It will now be sufficiently evident that the ultimate temperature attained by a dynamo when at work is of the utmost importance. If its wires are insulated with the organic or fibrous materials usually employed, the frequent attainment of a very high temperature is incompatible with a long life, and this consideration of durability leads us to fix a maximum temperature, which no part of the machine should exceed in continuous working.

By the use of insulating materials, such as mica and micanite, which may by contrast with cotton and paper be called fireproof, very high temperatures become permissible, such as 250° F., or 120° C. Field-magnet coils of thin and wide copper tape can be successfully insulated with thin mica or asbestos sheet, and drum toothed armatures with barrel winding of bars can be insulated with micanite troughs to take the bars within the slots, although such constructions are not suitable for high-voltage coils and small armature wires, owing to the

* Cp. Chap. XIII. § 17, and E. H. Rayner, *Journ. Inst. Electr. Eng.*, vol. xxxiv. p. 656.

room which they occupy. But even though the insulation may be quite satisfactory to withstand high temperatures, the useful field for a "fireproof" construction, at least in the case of dynamos which are normally worked in reasonable temperatures, is very limited. The heat gradually spreads to the commutator and brushes, and impairs the commutation, while the second consideration of § 3, namely, the great difference in the electrical resistances when hot and when cold, makes the regulation of the voltage more difficult. Returning, therefore, to the more usual case of armatures and rotating coils insulated with cotton, linen, tape, fibre, or paper, the maximum temperature which they should be allowed to attain in continuous work cannot be set higher than about 190° F., or say 90° C. at the most, and even then it must be remembered that, if a coil consist of a large number of layers, the limiting temperature of the surface must be appreciably lower.

§ 6. **Maximum permissible rise of temperature as limiting output.**—Having thus fixed upon a maximum permissible temperature, it is evident that the number of degrees by which a dynamo may be allowed to rise in temperature without exceeding the limit depends upon the starting-point from which the rise takes place; in other words, upon the temperature of the surrounding air during the working of the dynamo. Thus, in the case of dynamos working in hot atmospheres, for instance, in the engine-room of a steamer in the tropics, where the normal temperature may be, and frequently is, as much as 115° F. (46° C.), the permissible rise is much smaller than in the case of a dynamo working in a well ventilated central station on land, where the temperature will seldom exceed 70° F. (21° C.). Since the maximum current of a dynamo is dependent upon the *rise* in temperature which is permitted, it follows that the output is indirectly limited by the normal temperature in which it is to work, and from which the rise is reckoned. It is seldom, however, that in specifications of dynamos the actual limits of temperature are stated, inasmuch as they must necessarily be somewhat vague, and it is more frequent to find the maximum *rise* of temperature alone specified, this rise being such as will not be likely to endanger the insulation under ordinary conditions. It is generally stipulated that the machine must run satisfactorily during a test of six hours' duration at full-load without undue heating of any part, and that at the end of the run the surface temperature of the armature or field-winding must not exceed the temperature of the surrounding air by more than 70° F. For short periods of, say, 1 to 2 hours, the normal full-load current may usually be exceeded by some 20 to 30 per cent. without raising the temperature of an armature in an excessive degree, and such a permissible overload enables the dynamo to deal with a large demand for current lasting a comparatively short time. The rise of temperature is usually obtained by comparison of the readings of two thermometers, the one registering the temperature of the room within a few feet of the dynamo and the other placed in contact with the exterior of the winding, and therefore indicating the temperature of the surface.

Assuming a normal temperature of 70° F. (21° C.) for the surrounding air, it will be seen that the maximum temperature which the surface of the coils may attain in continuous work is 140° F. (60° C.), and this limit is found to give thoroughly satisfactory results in practice. It further results from such a rule that the temperature of an armature, as measured by rise of resistance, may differ by 32 per cent. from its temperature as measured by thermometer, before a limit of 185° F. is exceeded. With stationary field-magnet coils having considerable depth of winding it might be advisable to fix an even lower limit of surface rise, such as 54° F. or 30° C., in order that the centre layers may not exceed the permissible maximum temperature, but the absence therein of the vibration and mechanical stresses to which the rotating armature is subjected permits of a close approach to 190° F. without impairing the durability of their insulation.

Thus the great importance of the safe *rise* of temperature lies in the fact that it limits the maximum current that may be passed through a given armature in continuous working, and so determines its normal output. Of course, the permissible armature current may be determined in the case of continuous-current dynamos by the question of sparking as explained in Chap. XVII., or in the case of alternators by the self-induction of the armature causing too great a drop in the terminal volts (Chap. XXIII.); but in the majority of continuous-current dynamos the full normal output is decisively fixed by the serious heating which would occur in everyday working if the armature current were increased.

§ 7. **Testing dynamos for rise of temperature.**—The time taken for the armature and field-coils of a dynamo to attain their ultimate temperatures is dependent upon their size. At the commencement of the run the rate at which heat is produced is almost as great as when the machine has attained its maximum temperature, but part of this heat is absorbed in raising the temperature, not only of the winding, but also of the core or magnet on which it is wound. The larger this mass which has to be heated, or the greater its specific capacity for heat, the longer will be the time taken in raising its temperature, until the final state is reached in which the heat has to be dissipated almost entirely by radiation and convection-currents in the surrounding air. A certain difference of temperature between the cooling surfaces and the surrounding air must then have been established sufficient to enable the heated masses to part with their heat as fast as it is generated. Strictly speaking, the temperature approaches its final value asymptotically, the rate of increase being a maximum at starting and thence gradually falling off; but if a machine be run with constant load for several hours, and the rise of temperature of armature or field-winding, as taken at intervals of, say, one hour, be plotted as ordinates to a horizontal axis of time, the curve so obtained will be found to gradually bend over and become more and more flat; finally, the readings will fall almost in a straight and horizontal line, showing that a steady temperature has then practically been attained (cp. Fig. 388). Such an experiment enables us to be certain that the final state has been reached, and the time which we find that a dynamo of given size takes to attain its maximum temperature will serve as a clue to the number of hours for which a machine of similar size should be run at full-load in order to test it thoroughly. While an armature, of which the core dimensions are 9″ diameter × 6″ length, will attain its final temperature after about four hours' run at full load, an armature 15″ diameter × 8″ long will barely reach its maximum rise in six hours, and larger machines will require to be run for still longer periods. Even, however, in large machines, since they are multipolar and are usually barrel-wound or in other ways have their windings well exposed to the cooling effect of the air, there is but little rise of temperature after the first 8 or 10 hours. The thermometer employed to measure the temperature of the surfaces should preferably be of a sensitive chemical type, the graduated glass stem having a very fine bore, and the small cylindrical bulb containing but little mercury; after being laid or held in close contact with the winding, it should be covered with some material (such as a piece of rag) which is a bad conductor of heat, and then allowed to remain undisturbed for several minutes until the mercury entirely ceases to rise. When a rotating armature is stopped at the end of a run or at any time for the purpose of taking thermometer readings, the temperature of its exposed surface continues to rise for some minutes after the rotation has ceased. The generation of heat ceases with the rotation, but the simultaneous cessation of the air-currents set up by the revolving parts virtually amounts to a large reduction in the cooling power of the surfaces, and in consequence the fall of temperature between the inner or hottest parts and the external surface is reduced; in other words, the outside rises by conduction to a higher temperature, more nearly the same as that of the centre of heat. Both the cotton insulation of the wires and the shellac or other varnish with which they are coated are bad thermal conductors, and it therefore takes an appreciable time for the heat to penetrate through them to the outside of the armature. It may further be mentioned that, owing to the low thermal conductivity of insulating materials in general, if a heated armature be cursorily felt with the hand the bare metal of the binding-wires, or even of the commutator, will appear hotter than the insulated wires, and these latter may seem comparatively cool to the touch: any such conclusion is, however, entirely illusory, and the continued application of the hand to the conductors will usually suffice to correct the error.

§ 8. **The growth of the temperature rise.**—If $R_{max.}$ = maximum rise, *i.e.* the final excess of the temperature of a homogeneous body above the temperature of the surrounding air, the rise after any time t is given by the equation

$$r_t = R_{max} \cdot \left(1 - \epsilon^{-\frac{t}{T_c}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (136)$$

where ϵ is the base of natural logarithms, and T_c is the "time-constant" of the body which is being heated. The curve of the rise of temperature in relation to time is therefore an exponential curve, exactly analogous to that of the rise of current in an inductive circuit. The "time constant" is the time in which the total mass which is being heated would reach the final excess-temperature if the initial rate of increase was maintained and there was no cooling action from radiation, convection, or conduction due to the rising temperature. It is therefore simply the ratio $\frac{\text{final excess-temperature}}{\text{initial rate of temperature-increase}}$, and is given in minutes or seconds, according as the initial rate of rise is given in minutes or seconds. It is also from the nature of equation (136) the time in which the rise actually reaches $\frac{\epsilon - 1}{\epsilon} = \frac{1{\cdot}718}{2{\cdot}718}$, or 63·3 per cent. of the final excess temperature, as is evident when t is made equal to T_c. The cooling equation is conversely $r_t = R_{max.}\ \epsilon^{-\frac{t}{T_c}}$.

With an exact knowledge of the total watts dissipated under any given conditions and also of T_c, and assuming the heat to be generated fairly uniformly throughout the body under consideration, any question as to the heating or cooling of a dynamo as a whole or of any part could be solved. One factor in the determination of T_c, namely, the initial rate of rise, is dependent upon the specific heat of the material in question, and is therefore known as a fact of physics: the initial rise per second is equal to the quotient of the rate at which heat is generated in watts per unit volume divided by the specific heat per unit volume, the latter being expressed in electrical terms as the joules which will raise the temperature, *e.g.*, of a cubic centimetre or a cubic inch 1° C. Taking the specific heat of copper per unit mass as 0·095 (calories required to raise 1 gramme of mass 1° C.), and the weight of a cubic inch as 0·32 lb., then since 1 calorie = 4·19 joules, the joules required to raise one cubic inch 1° C. are $0{\cdot}095 \times 4{\cdot}19 \times 0{\cdot}32 \times 453{\cdot}6 = 57{\cdot}6$, or 3·5 joules per cubic centimetre. Taking the specific heat of iron or steel per unit mass as 0·113, and the weight of a cubic inch as 0·282 lb., the joules required to raise one cubic inch of iron 1° C. are 0·113 ×

$4.19 \times 0.282 \times 453.6 = 60.4$, or 3.68 joules per cubic centimetre. There is therefore but little difference between iron and copper, and an average figure of 59 joules per cubic inch, or 3·6 per cubic centimetre may be taken for a composite mass such as an armature. The total watts of the armature are then to be averaged over its entire volume, and the truth of the above approximation for the joint specific heat is of course dependent upon how far the heat is actually developed in the core and winding in proportion to their respective volumes.

In the case of field-magnet coils, with numerous turns of cotton-covered wire, allowance must be made for the higher specific heat of the cotton. The specific heat of cotton is about four times that of copper, say, 0·38, and the joint specific heat of the copper and cotton will depend upon the ratio of the copper to the total. The heat is only developed in the copper, and for calculating the initial rate of rise, if the watts are averaged over the entire mass of copper and cotton, their joint specific heat per unit mass is higher, but if the watts are averaged over the entire volume, it is lower than that of solid copper. With closely bedded rectangular wire the ratio of the copper volume to the total volume is the "space-factor" σ, and the weight of a cubic inch of cotton is 0·0323 lb., so that the ratio of the weights of unit volume of copper and cotton is 9·9; the specific heat per gramme of the combined copper and cotton may then be calculated from the formula $0.095\left(\dfrac{4 + 5.9\sigma}{1 + 8.9\sigma}\right)$. Or analogously to the above expressions, the joules required to raise a cubic inch of cotton 1° C. are $0.38 \times 4.19 \times 0.0323 \times 453.6 = 23.3$, and the joules for any ratio σ of copper volume to total volume are

$$23.3 + 34.3\,\sigma$$

as plotted in Fig. 387.*

The result is that the joint specific heat of field-magnet coils per unit volume is appreciably less than that of solid copper, and if the wire is round, the air-interstices will tend to decrease it still further. On the other hand, the thermal capacity of the adjacent metal of the magnet which is heated by conduction has above been neglected, and this always tends to increase the specific heat. Since T_c is usually required in minutes, we therefore have initial rate of temperature increase in degrees Centigrade per minute

$$= \frac{\text{watts per cubic inch of total volume} \times 60}{\text{joules to raise 1 cubic inch 1° C.}}$$

the denominator being 59 for armatures, and being taken from Fig. 387 for field coils in relation to the space-factor.

It is, however, equally necessary for the determination of T_c that the

* C. W. Hill, "Crane Motors and Controllers," *Journ. Inst. Electr. Eng.*, vol. xxxvi. pp. 294 and 307.

final excess-temperature should be known, and it is, in fact, mainly to check this that the dynamo is tested by a run of several hours duration. Experiment cannot therefore be dispensed with unless from previous cases the final excess-temperature and the total watts dissi-

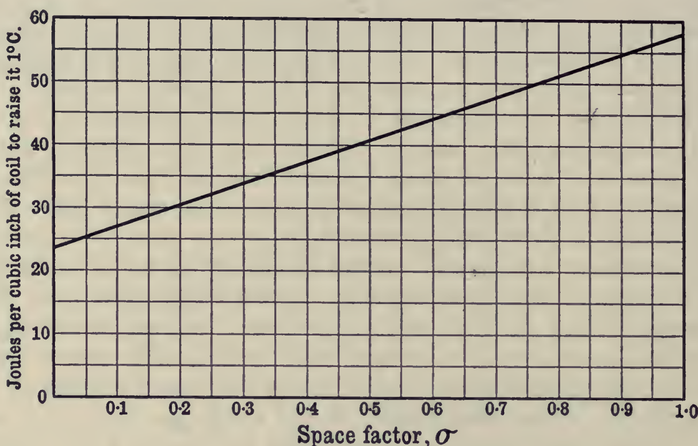


FIG. 387.—Specific heat of cotton-insulated coil in relation to space-factor.

pated as heat can be regarded as known, and this is more usually the case with field-coils than with armatures. Apart from such knowledge, it can only be said that the curve of rise in relation to time will be an exponential curve, and therefore will have to some scale the shape of

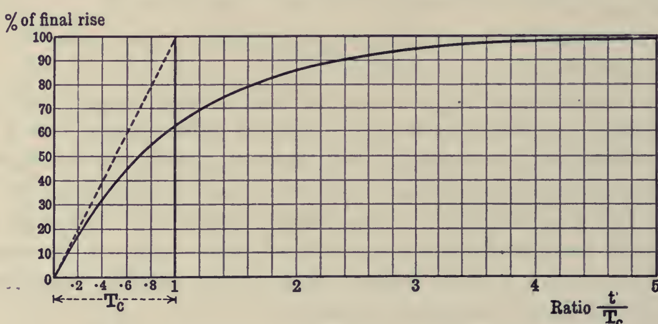


FIG. 388.—Curve of temperature rise in relation to time.

the curve in Fig. 388. At the end of time T_c the rise will be 63·3 per cent. of the maximum, at $t=2T_c$ 86·5 per cent., at $t=3T_c$, 95 per cent., and at $t=4T_c$ will be 98·2 per cent., so that it then only differs by 1·8 per cent. from the final. An approximate calculation can, however, thence be made as to the time for which a given machine must be run in order to reach practically its final temperature or any particular percentage of it. The longer period for which large machines must be run in order that they may attain their final temperature is obviously due to the ratio of their volume to their surface, which increases with an increase in size, although this tendency is partially counteracted by the accompanying increase which is usual in the number of poles.

While the exponential curve reproduces the facts sufficiently closely for most practical purposes,* it is not strictly true when, as is usually the case, the total watts being dissipated in a dynamo are not constant, but are varying as it gradually warms up.

§ 9. **Predetermination of temperature rise of stationary field-magnet coils.**—The final rise of temperature of a field coil is primarily determined by the ratio of the total watts to its cooling surface S_c, and under given conditions is approximately proportional thereto; or $t^\circ = k \cdot \frac{W}{S_c}$. The value of the "heating coefficient" k, or the rise of temperature for a ratio of one watt per unit area of cooling surface, is not really a constant, but itself depends upon the final temperature which is reached. This is due to the increased effect from radiation, as the difference of temperature of the surface from that of the surrounding air rises;† if the whole cooling effect resulted from radiation, the reduction in the heating coefficient when a high final temperature was attained would be considerable, but in the dynamo at work it is of less account owing to the preponderating effect from conduction and convection.

But apart from this variation of the heating coefficient, and when it is assumed that the final rise is strictly proportional to the watts per square inch of cooling surface, the value of k is dependent upon a number of conditions of which the chief are as follows:—

(1) The construction of the field-coil, whether a solid mass or divided by ventilating spaces into sections as described in Chap. XVI. § 7, and whether wound on spools or otherwise. Sheet-metal spools or bobbins assist in conducting heat to the iron core if closely fitting. When furnished with wooden end-flanges, the latter act as partial heat insulators and raise the temperature. On the other hand, brass or iron end-flanges assist considerably in conveying away the heat. Thick insulating bobbins, such as those moulded from vulcanabest or isolit, although convenient for the winder, have the disadvantage of increasing the heating coefficient.

(2) The nature and thickness of the insulating wrappings, especially

* Cp. R. Goldschmidt, "Temperature Curves and the Rating of Electrical Machinery," *Journ. Inst. Electr. Eng.*, vol. xxxiv. p. 672 ff., where curves of actual machines are given, and the amount of their divergence from the exponential shape is noted.

† G. A. Lister, "The Heating Coefficient of Magnet Coils," *Journ. Inst. Electr. Eng.*, vol. xxxviii. p. 402.

in such coils as are self-supporting, and after winding are slipped directly over the pole. Thus with a coil of double-cotton-covered wire which is merely varnished on the outside, the mean rise may be only about $1\frac{1}{4}$ or $1\frac{1}{3}$ times the surface rise; if wrapped with thin tape, the mean rise will be increased to $1\frac{1}{2}$ or $1\frac{3}{4}$ times, while if the coil be again overlapped with canvas and string to a considerable thickness the mean rise will be further increased by some 25 to 50 per cent.* Thus any insulating wrappings, owing to their low thermal conductivity, have a prejudicial effect upon the mean rise and also upon the maximum internal rise, even though they may tend to lower the rise of the outside. The bare edges of copper tape wound on edge in a single layer, as in the coils of alternators, add considerably to their power of conducting the heat immediately to the outside and there radiating it away.

(3) The peripheral speed of the armature, which causes a more or less efficient circulation of the air round the field-coils. This fanning effect is especially marked in multipolar machines with armatures of large diameter.

(4) The load of the armature; as this is increased its temperature rises, and the air which is thrown off from its surface and circulates round the stationary coils becomes hotter, so that the dissipation of heat from the field is checked.

(5) The type of machine,—whether bipolar or multipolar, semi-enclosed or open. In the single horseshoe machine the opposing inner faces of the coils have less cooling effect than the outer faces, whereas in the multipolar machine the whole of the external surface is of much more nearly equal value. But this presupposes that the higher number of poles is found in the larger machine; in a small machine with a great number of poles the opposing faces of the coils are again brought close together, and this is especially true with commutating poles which nearly fill up all the space between the main poles.

And lastly, (6) the position of the coil, whether at the bottom of the machine or at the top, to which the air heated by the armature ascends.

The basis of calculation for the cooling surface of the magnet-coils varies in the experiments of different observers, so that in comparing them it must be noted whether only the external cylindrical surface is employed or whether the exposed surface of one or both end-flanges is added thereto, or lastly, whether the whole of the surface, external and internal, is adopted.

Probably the best basis is to be found in the outside surface plus

* Cp. throughout E. H. Rayner, "Report on Temperature Experiments carried out at the National Physical Laboratory," *Journ. Inst. Electr. Eng.*, vol. xxxiv. p. 628 ff.

both the end-flanges, as in Chap. XIV. § 5. In the long coils of the bipolar single horseshoe magnet, which was formerly employed, especially when wound between wooden end-flanges, the exterior cylindrical surface only afforded a good basis, but with the shorter coils of modern multipolar machines having a considerable depth of winding, the cooling effect from the ends plays a more important part, and the combination of the external cylindrical surface with that of both end-flanges is here adopted.* In the case of coils divided into sections, the same basis may be taken, the total axial length of the coil being reckoned without exact calculation of the air-ducts, but in combination with a low value for the heating coefficient.

The rate at which heat is developed in the magnet-coils admits of easy measurement and calculation as the product of the square of the current into the resistance. The ratio of the watts to the area of cooling surface is therefore much more accurately known than in armatures; and further, as the field-magnet coils are usually stationary in continuous-current dynamos, and therefore less affected by currents in the surrounding air than is the rotating armature, their temperature rise can be predicted with greater certainty.

In the values of the "heating coefficient" the distinction must further be carefully noted between the rise of the exterior surface of the coil and the mean rise of the whole of the copper wire, and again between these and the maximum temperature rise at any spot within the coil.

Taking any one layer, as we proceed from the centre along the axial length of the coil towards either end, the temperature always declines towards either end-flange, which shows the influence of the end-flanges in assisting the dissipation of the heat. On the other hand, as we pass from the outside radially through the layers the temperature rises, reaches a maximum, and again falls, but to a lesser degree, as we approach the inner layer next to the iron of the poles. The innermost layers thus part with some of their heat by conduction to the iron of the magnet, so that their temperature falls about midway between that of the outer and that of the hottest layer. The hottest spot in the coil is approximately in the centre of the axial length, and about three-quarters of the total depth away from the outside, and the maximum temperature rise at this spot exceeds the surface rise by an amount which is from 50 to 60 per cent. higher than the difference

* Mr. Lister (*Journ. Inst. Electr. Eng.*, vol. xxxviii. p. 401) advocates the adoption of the entire surface, external and internal, and the iron of the pole undoubtedly assists in conducting away the heat. Yet since, when the machine is at work under full load, the pole gradually becomes heated by the hot air and radiation from the armature and also by any eddy-currents in the pole-face, this action is to some extent checked, so that an air-space between coil and pole may even become of value. In ordinary cases the external surface is at least as good if not a more reliable guide to which the cooling effect will be proportional.

between the mean rise and the surface rise; thus if the temperature rise, measured by change of resistance, is 20° higher than the surface rise as measured by a thermometer, the maximum temperature rise will be about $1\frac{1}{2} \times 20 = 30°$ higher than the surface rise.* On the average the maximum temperature rise is about 18 to 20 per cent. higher than the mean rise, when the machine is fully loaded. While the highest temperature attained has a practical bearing upon the durability of the insulating materials, the mean rise throughout the coil has the greatest influence upon the regulation of the dynamo and its general working, so that heating coefficients which give the mean rise have perhaps the greatest scientific value and are frequently adopted. On the other hand, so long as in the majority of specifications it is the rise of the exterior surface as measured by a thermometer for which alone a limit is laid down, this must continue to possess the greatest interest for the designer; also, it remains a rough and ready test upon which immediate agreement can be come to between purchaser and manufacturer. Measurement of the mean rise of temperature by the observed increase of resistance requires great accuracy in the readings of the voltmeter and ammeter, as it is only a difference of which use is finally to be made.

In degrees Centigrade the mean rise of temperature $\theta°$, reckoned from an initial temperature $t°$, may be calculated from the observed resistance $R_{t°+\theta°}$ and $R_{t°}$ by the formula,

$$\theta°\text{C.} = \frac{R_{t°+\theta°} - R_{t°}}{R_{t°}} \times (234 + t°) \quad . \quad . \quad . \quad . \quad (137)$$

On an average, perhaps, it may be said that the mean rise is as much as $1\frac{3}{4}$ to 2 times the surface rise, so that to calculate the latter the heating coefficient for mean rise is reduced in this proportion.

It is evident that no exact figures can be given which can be applied without consideration of the particular conditions of the design and type of machine, but that the influence of peripheral speed, etc. can to some extent be taken into account by the use of such curves as those of Fig. 389, which indicate approximately the limits which usually occur in practice. The lower dotted curve for the mean rise of well ventilated coils may approximately be expressed in terms of square inches and degrees Fahrenheit as $k = \dfrac{260}{1 + 0{\cdot}35 \cdot \dfrac{v}{1000}}$ where v is the peripheral speed of the armature in feet per minute. The mean temperature rise is also affected by the amount of cotton in coils which are of the same size and otherwise alike, and is less with a larger size of wire and smaller amount of cotton. Shunt coils are appreciably benefited by subdivision into sections by ventilating

* R. Goldschmidt, *Journ. Inst. Electr. Eng.*, vol. xxxiv. p. 720.

spaces; by their use the mean rise is made to approach more closely to the surface rise, so that it may nearly coincide with the highest of the curves given in Fig. 389 for the surface rise. Such subdivided coils are slightly detrimental to the efficiency of the machine, since, if the in-

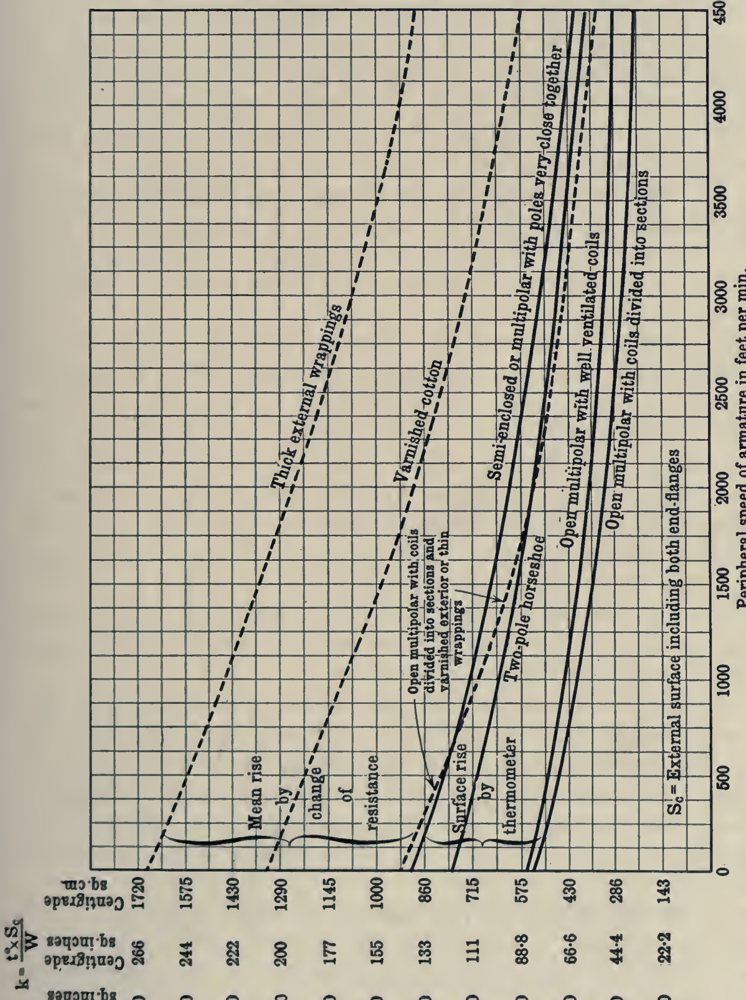


FIG. 389.—Heating coefficient of field magnet coils.

tervening gaps were filled with copper, the watts would be reduced, but in a large machine this has only a very small effect on the total efficiency.

The mean rise should preferably not exceed 60° C. or 108° F., so that the maximum temperature at any spot may not exceed about 90° or 195° F.

If the permissible rise of surface temperature be taken as about 50° to 60° F., it will be seen that from 2 to $2\frac{3}{4}$ square inches of cooling surface must be allowed per watt with solid coils, and $1\frac{1}{2}$ to 2 with coils in sections. A greater rise, although perhaps admissible in the matter of the final temperature reached, is seldom advisable owing to the difference which it causes in the exciting power according to the time during which the machine has been at work. If a number of separate bobbins are revolved at a high peripheral speed, as in alternators of the type shown in Fig. 84, a much higher ratio of watts to square inches is permissible, owing to the great cooling effect from their rapid movement through the air.

§ 10. **Sources of heat in the armature, and their relative importance.**—The final rise of temperature in an armature is a much more complex problem, and this for two reasons. In the first place, it is difficult to calculate with accuracy the watts actually expended in heating the armature.

The sources of the heat are threefold, namely, (1) the loss of electrical energy due to the passage of the total current through the armature resistance; (2) eddy or Foucault currents set up in the copper winding, and in the armature core; and (3) the magnetic loss due to hysteresis. The first and third of these can be calculated with a close degree of approximation, but the second loss is very largely indeterminate, inasmuch as it is dependent upon a number of obscure conditions, such as the width of the armature wires, the proportions of teeth and slots, and the strength of field.

To show how variable is the relative proportion of the three losses, the opposite Table gives a few results of actual tests made on machines of different construction and size.

The figures show also that the eddy-current loss is usually of considerable importance, and, as often as not, is greater than the $C_a^2R_a$ loss. The precautions taken to minimise eddies have been already described in Chapter XIII., but their complete elimination cannot be attained, and the probable loss due to these must be closely known from previous experience if the rise of temperature of the armature is to be predicted with any certainty.

§ 11. **Effect of peripheral speed.**—In the second place, even when the rate at which heat is developed in the armature is accurately known, the rate at which it can be dissipated is most materially affected by the speed of rotation, and also by the shape and number of the pole-pieces which surround the armature, in so far as they allow of or actively promote a more or less free circulation of the air over the surface of the winding. The exact effect of either of these causes does not admit of very accurate generalisation, since it necessarily varies under different conditions. How important a part is played by the peripheral speed of a rotating armature in increasing its cooling

	Dimensions of Armature Core in Inches.		KW.	Volts.	Amps.	Revs. per Min.	Peripheral Speed in Ft. per Min.	Watts.			B_g	B_a	Uncorrected B_{t2}
	Diam.	Length.						$C_a^2 R_a$	Hyst.	Eddies.			
4-pole drum, slotted . .	$10\frac{1}{2}$	6	$9\frac{1}{4}$	140	66	780	2140	350	100	340	7000	10,300	22,620
,, ,, . .	12	7	$13\frac{1}{2}$	105	130	780	2450	440	107	574	7200	14,400	21,800
,, ,, . .	14	8	27	220	123	780	2860	640	210	425	7180	12,300	23,000
,, ,, . .	21	11	50	550	91	400	2200	1360	320	600	7500	14,650	24,200
6-pole drum, slotted . .	19·3	8·85	45	120	375	550	2780	687	660	1240	7160	9,000	20,700
2-pole drum, smooth . .	$7\frac{1}{4}$	15	11·2	140	80	880	1830	330	100	400	5200	9,100	...
,, ,, . .	$9\frac{5}{8}$	20	35	110	320	780	2150	1160	215	585	6800	13,500	...
,, ,, . .	10	12	15	130	115	865	2420	520	187	530	4470	17,500	...
,, ,, . .	20	36	120	135	900	375	2100	2300	694	2730	5300	11,400	...

power is at once evident from a comparison of the watts which it can dissipate with the watts which a stationary field-coil can dissipate: their exterior surfaces being reckoned in each case as effective in cooling, if the rotating armature have a peripheral speed of 2000 feet per minute, it will be found that, roughly speaking, it can dissipate at least twice as many watts per square inch of cooling surface as the stationary field-coil for the same rise of temperature. If the surface of the armature winding be broken up, so that the air can play upon more than the outer face of the conductors, and can reach freely to the core, the cooling action is much assisted. Again, the influence of eddy-currents in the pole-pieces, when these latter are not laminated, reacts upon the heating of the armature. It is therefore impossible to lay down any general formula, expressing the cooling power of a given surface at different speeds, which will meet the case of entirely different types of machines, and all curves connecting rise of temperature with a certain ratio of watts per square inch must necessarily be only approximate.

§ 12. **The loss over the ohmic resistance of the armature.** —Considering in detail the three sources of heating in an armature, the first, or the electrical loss over the copper resistance of the winding, is entirely independent of the speed, and is simply equal to the product of the square of the total armature current and the resistance of the armature from positive to negative brushes, *i.e.* $C_a^2R_a$ watts. In the process of designing a dynamo the resistance of the armature can easily be calculated, and from it the loss of watts due to the passage of the normal armature current. It is only necessary to remark that if the resistance of an armature from brush to brush be actually measured it will usually be found to be slightly higher than the resistance as calculated from the length and area of copper used. This discrepancy is due to the inferior conductivity of the soldered joints, even when carefully made. In this respect armatures wound with former-shaped coils have the advantage that in them the soldered joints are reduced to a minimum, and need only occur at the unions with the commutator sectors. Owing to the low resistance of a large armature with few bars, the most convenient method of measuring its actual value is by soldering leading-in strips on to two commutator sectors at such a distance apart that they divide the winding into two parallel paths, passing through them a known current of considerable strength, and measuring the difference of potential between the points, where the current leaves and enters. From the quotient obtained by dividing the measured difference of potential by the known current, the resistance of the armature can be calculated according to the nature of the winding, and from our knowledge of this resistance, after making due allowance for its increased value when hot, the $C_a^2R_a$ loss can be accurately determined.

§ 13. **The eddy-current loss as depending on speed.**—It is in the second or eddy-current loss that most difficulty lies. With a given core, a given winding, and a given field this loss will be directly proportional to the square of the speed; since, if the speed be increased x times, the E.M.F. of an eddy will be equally increased, and this will increase the current of the eddy x times, so that the product of eddy E.M.F. $\times$ eddy current will be increased x^2 times. The rate of loss in watts may therefore be expressed by a coefficient F multiplied by the square of the speed, *i.e.* by FN^2, where $N =$ revolutions per minute, the value of F being different in different armatures and being also dependent upon the particular excitation which is normal for a given armature. During the process of designing, the value of F must be estimated from the coefficient of machines previously tested. A brief description of the experimental methods by which the eddy-current loss in a given dynamo when run at different speeds can be measured with considerable accuracy will be found in Chap. XX. §§ 10, 11. Two points alone require to be further mentioned. In the first place, the value of the eddy-current loss, as thus determined, includes any eddies set up in the pole-pieces of the magnets by the rotation of the armature. Such loss of power in the pole-pieces will be considered in § 23 of the present chapter, and does not here concern us. In continuous-current machines, having conductors wound uniformly over a smooth core, the loss in the pole-pieces is quite inappreciable, and it may therefore be assumed that in such cases all the eddy-currents which the above methods measure are generated in the armature itself; but in toothed armatures some deduction may require to be made for the loss of watts in solid pole-pieces. In the second place, during the experiments the distribution of the field is approximately the same as that at no-load, and it has already been shown, in Chapter XVII., that this distribution may be, and usually is, considerably modified at full-load. Hence the eddy-current loss at full-load may be greater than the experiment would show, and when the results of such tests have been checked by other methods it has been found in practice that this difference in the distribution of the field does sensibly increase the amount of the eddy-currents.*

We are, however, justified in assuming that the methods above alluded to furnish evidence for determining, at least approximately, the eddy-current loss at full-load. In applying such results to new machines, not only must any alteration in the length and volume of the core be taken into account, but, as previously stated, it is equally important to estimate the probable effects of alteration in the value of the flux-density in the air-gap or armature, the degree of lamination of

* In a test on a Siemens 1500-kilowatt dynamo the losses deduced from a Hopkinson test showed an increase of 15 per cent. at full-load as compared with the sum of the no-load and C^2R losses (*Electr. Eng.*, vol. lxii. p. 15).

the core, the width of the bars in smooth-core armatures, and the density in the teeth of slotted armatures. An alteration which at first sight might seem unimportant in any one of a number of conditions may radically affect the amount of the eddy-currents. The subject needs therefore to be discussed in further detail.

§ 14. **Eddy-currents in the armature core.**—The eddy-current loss may be divided into (*a*) that within the armature core, (*b*) that in the copper winding, and (*c*) that in the binding wire on the circumference of the armature. To allocate to these sources their respective shares in the total loss is an extremely difficult matter, as any change in one of the conditions such as can be tried in a practical machine produces but a small change in the total which can scarcely be isolated; yet the combined effect of all the losses adds up to an amount which seriously reduces the possible output of an armature from the heating point of view.

Taking them seriatim, we have first the armature core with its supporting framework and surroundings. The eddy-current loss produced by an alternating field in thin iron plates such as are used in transformers and laminated armature cores is proportional to the square of the maximum induction and also to the square of the thickness of the plates, and the same law has been experimentally established by Mr. Holden * when the field rotates about a stationary armature, or *vice versâ*. The energy consumed in thin plates or discs in an alternating field per cycle and per cubic centimetre of iron is $\frac{1\cdot645 t^2 . f . B_a^2}{\rho} \times 10^{-16}$ joules, where t is the thickness of the plates in centimetres, f is the periodicity or frequency, and ρ is the resistivity of the iron. The latter quantity may be taken as from 1×10^{-5} to $1\cdot2 \times 10^{-5}$ ohms with wrought iron or thin sheet steel,† whence the loss of energy per cycle and per cubic centimetre is approximately $1\cdot645\ t^2 . f . B_a^2 . 10^{-11}$ joules. The loss of power at any frequency $f=\frac{pN}{60}$ in a total volume of V_c cubic centimetres of iron is therefore in an alternating field

$$1\cdot645 t^2 . f^2 . B_a^2 . V_c \times 10^{-11} \text{ watts}$$

The same form of expression may also be applied to the case of a rotating field, but owing to the E.M.F.'s being on the whole greater in the rotating field, and the average length of path and its resistance smaller, the first numerical constant requires to be increased. For circular plates of large diameter as compared with their thickness the best approximation gives for a rotating field ‡

$$2\cdot78 t^2 f^2 B_a^2 . V_c \times 10^{-11} \text{ watts} \quad . \quad . \quad . \quad (138)$$

* *Electrician*, vol. xxxv. p. 327.

† For high-resistance alloys, see Chap. XII. § 13.

‡ See Prof. E. Wilson, *Proc. R. S.*, vol. lxx. p. 359, and *Electr. Eng.*, vol. xxx. p. 226; and Prof. F. G. Baily, *Phil. Trans.*, 1896, vol. clxxxvii. A. pp. 715–746.

But no rigid mathematical solution can be given, since the numerical constant is dependent to a certain extent upon the distribution of the field in the air-gap, and further, as has been pointed out by Professor W. M. Thornton, such curves as those of Fig. 141 show that between B and A the flux at one depth may be increasing while at another depth it is decreasing, so that the rate of cutting lines is not simply proportional to the distance from the centre, as it would be if the induction was uniform over any radial section. For the volume of the teeth the coefficient would again be higher.

When the above formula is worked out, the watts, even in the case of a high-speed multipolar dynamo, are found to be small in comparison with other more serious losses, and in practice the actual loss from eddy-currents in the discs would be even smaller, owing to any magnetic screening. The reduction in the eddy loss through the increase of resistance when the iron becomes heated is about 5·5 per cent. for each 20° F. rise of temperature.

It must not, however, be assumed that the loss in the armature core as a whole is by any means negligible. The end-fringe from the flanks of the pole-pieces is quite appreciable, and unless the end-plates which clamp together the discs are kept well outside the edges of the poles the lines which curve round into the ends of the armature will set up considerable eddy-currents. There is too a loss from each ventilating air-duct, wherein the lines bend into the flat surface of the discs at the sides of the duct. Again, in barrel-wound armatures the cylindrical structure which supports the end-connections of the bars should not be a solid sheet of metal, but as far as possible should be cut away near to the armature core so as to remove the iron into which the lines of the fringe might stray (cp. Figs. 149 and 151).

Next, the operation of turning the surface of a smooth armature-core increases the eddy-loss by burring over the edges of the discs, and requires to be carried out with considerable judgment. Analogous to this in a toothed armature core is the operation of drifting or filing out the slots so as to remove any roughness or inequality of their sides. Any such mechanical treatment requires to be both lightly and cleanly executed with the minimum amount of injury to the laminations. On this account it is not advisable to mill the longitudinal grooves on the assembled core, and even when the notches are stamped in the discs before assembling they must be carefully inspected to see that they are free from burrs, by reason of which the rough edges of neighbouring discs would be driven into good contact with one another.*

§ 15. **Eddy-currents in bolts through armature core.**—If the body of an armature core is traversed by a number of bolts, and these, although insulated from

* For the eddy-loss specially in the laminated teeth of a toothed core, cp. F. E. Meurer, *Electr. World*, vol. xlix. p. 792.

the discs through which they pass, are in metallic connection with the end-plates at either end of the core, not only is the effective area of the core periodically reduced by the presence of the bolts, but there results a conducting system analogous to a squirrel-cage rotor in an induction motor. Each bolt becomes the seat of an alternating impressed E.M.F. proportional to the flux which it cuts, the latter depending upon the radial depth within the core at which it is situated; this E.M.F. of periodicity $f=\frac{pN}{60}$ will pass through zero midway in the interpolar gaps, and will have a double-peaked maximum under each pole. The resistance of the portions of the end-plates corresponding to each bolt, and forming part of the circuit, will be low owing to their large area, and may be neglected. When the resistance of their contact-area with the head of the bolt is also neglected, and the specific resistivity of wrought-iron bolts is taken to be 1×10^{-5} ohm, the resistance of each bolt of diameter d centimetres and length l centimetres is $\frac{l \times 10^{-5}}{\pi d^2/4}$ ohms.

If the bolts pass nearly through the centre of the radial depth of the core, and so are completely surrounded by iron, the inductance of each will be so high as compared with the above resistance that its self-induced E.M.F. will be nearly equal to its impressed E.M.F.; the resultant E.M.F. will be low, and the current will lag behind the impressed E.M.F. by some angle approaching 90°. Hence even if the current be large, the watts $E_t I \cos \phi$ will be so small as to be negligible by comparison with other more important losses.

But this condition also implies that the self-induced flux due to the current through the bolt is almost equal to the original field which it cuts; the current, lagging nearly 90 electrical degrees, reaches its maximum magnetising effect in the centre of each interpolar gap in the position best calculated to oppose the passage of all lines below the bolt circle and to concentrate them above the bolt circle. Almost all the flux is driven into the outer layers of the core, and only a small percentage is left within the bolt circle sufficient to supply the loss of volts over the ohmic resistance of the bolt. The area of the core is thus in effect reduced, and the current reaches such a magnitude that its maximum ampere-turns in combination with those of the field winding are sufficient to drive the flux through the armature path of which the permeability is reduced by reason of the density being nearly doubled. The eddy-currents in the bolts will supply half the total number which are required, but in order to retain the same total flux the field ampere-turns must themselves be raised as compared with the number which would be required over the armature path if the bolts were absent. Thus the correlative of the negligible expenditure of watts in the bolts is an appreciable increase in the loss from eddy-currents, and possibly also from hysteresis, in the discs.*

On this account either the bolts passing through the body of the core must be insulated from the end-plates, as shown in Fig. 161, or the effective area of the core must be reckoned practically as that between the bottom of the slots and the bolt circle.

As the bolt circle is brought nearer to the inner edge of the discs their inductance and magnetising effect decrease, but on the other hand the expenditure of watts in the bolts themselves increases. If the bolts are immediately under the inner edge of the discs we approach to the condition of a conductor lying on the surface of an iron core as in Fig. 347, and the inductance of each bolt is so low as compared with its resistance as to be negligible. If for a rough estimate the distribution of the lines which leak out of the core is assumed to be sinusoidal, with an average induction B, the effective E.M.F. of each bolt would be $2{\cdot}22\,(\mathrm{B} \times \text{pole-pitch})\,f \,.\, l \times 10^{-8} = \mathrm{E}$, the pole-pitch being of course measured on the inner circumference of the discs. The

* Cp. G. Kapp, *Dynamos, Alternators, and Transformers*, 2nd. edit. p. 279.

lag of the current being negligible, the expenditure of power in each bolt is then $\frac{E^2}{R}$, or by substitution of the above expressions is $3{\cdot}9\,(B \times \text{pole-pitch})^2 f^2 \,.\, d^2 l \times 10^{-11}$ watts. If in practice such a figure as $B = 100$, and a pole-pitch of 20 centimetres in a multipolar machine of fairly large size is assumed, the loss in each bolt of diameter 2·54 centimetres for a frequency of 30 works out to 0·9 watts per centimetre of its length. The interest of such a case lies in the fact that similar or larger losses may be debited to an armature core supported on a central hub, since in effect each arm of the hub corresponds to a bolt lying immediately under the discs.

§ 16. **Eddy-currents in the copper winding of armatures.**—(*b*) The origin and nature of the eddy-currents which occur in solid copper conductors embedded in slots during their passage through the excited field have been thoroughly investigated by S. Ottenstein,* and from his experiments the following conclusions are mainly derived. Since the eddies are due to the alteration in the number and direction of the lines within the slot as its position relatively to a pole is altered by rotation of the armature, the total loss can be divided into two components, (1) due to the rate of alteration of a supposed longitudinal field passing straight down the length of a slot, and (2) due to the rate of alteration of a supposed transverse field passing straight across the slot. To correspond with this and to isolate each effect separately, the actual field within the slot whatever its nature must be resolved into two components at right angles, their combination at any moment and at any particular spot within the slot reproducing the actual slanting direction of the lines together with their number. The E.M.F.'s are at right angles to the direction of each component field, and the eddies are reduced by subdivision of the copper into a number of thin strips side by side in the case of the longitudinal field and into a number of layers in the case of the transverse field.

The maximum value of either the longitudinal or the transverse induction will be different for each bar within the slot according to its position; thus the former will be small near the middle of the slot, greater at the bottom due to lines leaking out again from the teeth when highly saturated, and greatest immediately at the top where the lines enter from the air-gap. With normal proportions of slot the maximum value of the longitudinal induction at the top of the slot is twice or thrice that of the cross induction, but it very greatly diminishes both from the top and bottom of the slot towards its middle, so that its influence on the whole is not so preponderant. The cross field does not show nearly so much difference in its values for the upper and lower halves of the slot. Also, the surface exposed to the cross-induction is about three times the width of the bar in an armature with narrow deep slots and two layers of winding. Hence in such armatures a further transverse division into more than two layers might give a lower total loss than an equal number of longitudinal subdivisions; in other words, four bars one above the

* "Das Nutenfeld in Zahnarmaturen und die Wirbelstromverluste in massiven Armatur-Kupferleitern," *Sammlung Elektrotechischer Vorträge*, vol. v. (Ferdinand Enke, Stuttgart).

other would be preferable to two layers of two thinner bars abreast. On the other hand, with wide shallow slots, longitudinal subdivision into several sections abreast would be the more advantageous, as the longitudinal induction preponderates. In either case soldering of the bars together at their ends nullifies all the advantage of the division, so that it is useless to divide a thick bar into several laminæ in parallel which are immediately united outside the limits of the core length. Lastly, the top of the bars should be kept well below the level of the top of the slot, where there is a strong field of curving lines; the use of hard-wood wedges for securing the bars in a slotted armature finds in this an additional recommendation. Shading off the field at the pole-tips, as by cutting away half the pole-tip laminations, is found to produce little or no effect on the total loss.

As an empirical approximation, the loss per slot filled with a single solid bar with normal insulation from the walls and distance from the top may be said to be proportional to the depth and width of the slot, and when the conductor is subdivided, the loss, roughly speaking, varies inversely as the number of layers and as the number of conductors abreast, *i.e.* inversely to their product $= \frac{\tau}{n_s}$. The two fields co-exist, and as a practical clue to the amount of either field within the slot may be taken the difference between the apparent induction at the roots of the teeth and a fixed induction of, say, 17,000, at which but few lines spread into the slot except at its extreme top. The total loss in the n_s slots is so far

$$\propto n_s^2 \frac{w_s \,.\, h_s}{\tau} \mathrm{L} \,(\text{app. } \mathrm{B}_{t_2} - 17{,}000)^2$$

The longitudinal field declines from a maximum under the centre of the pole to zero on the interpolar line of symmetry, while the transverse field, at least so far as the centre of the slot is concerned, shows the exact opposite. But in either case the rate of change or the rise and fall of the field is largely localised in close proximity to the pole-tips. An examination of the gradient of the field as dependent upon the length of air-gap and of the interpolar zone shows that with a usual ratio of pole-arc to pole-pitch the loss does not increase so fast as the diameter of the armature, and only increases in proportion to the ¼th power of the air-gap; further, that it becomes proportional to the 1·25th power of the number of poles. The conclusion that the length of the air-gap has but little effect, although an increase in it tends to increase the loss, is borne out by experiment.

In default, therefore, of more complete knowledge, the eddy-current coefficient, which must be multiplied by the square of the number of revolutions per minute to determine the loss in the bars of a slotted armature may be given as

$$k_3 \,.\, \left(\frac{\text{apparent } \mathrm{B}_{t_2} - 17{,}000}{1000}\right)^2 \times n_s^2 \,.\, \frac{w_s \,.\, h_s}{\tau} \,.\, \mathrm{L} \,.\, \mathrm{D}^{\frac{3}{4}} \,.\, l_g^{\frac{1}{4}} \,.\, p^{\frac{5}{4}} \quad . \quad . \quad . \quad (139)$$

In the smooth-core drum the field is that of the air-gap, and is more nearly a purely longitudinal one which would cause a loss proportional to the cube of the thickness t of a solid bar. There is still, however, considerable curving of the lines close beyond the pole-tips which would give importance to the height h. The cube of the thickness probably overrates its influence, so that the loss may perhaps be estimated as proportional to $t^2 h$, and the complete expression becomes with solid bars

$$k_4 \left(\frac{\mathrm{B}_g}{1000}\right)^2 \times t^2 h \mathrm{L} \tau \mathrm{D}^{\frac{3}{4}} l_g^{\frac{1}{4}} p^{\frac{5}{4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (140)$$

The bars of the smooth-core armature will, however, be twisted or stranded if their width exceeds the limit named in Chap. XIII. § 20, which introduces great uncertainty owing to the indefiniteness of the compression and amount of contact between the separate strands. It can only be assumed that the stranding has caused an equal increase in the resistance in all directions across the bar, and the constant factor k_4 will be assigned a lower value agreeing with the results of experiment in average cases.

Under load when the armature is carrying current, the distortion of the field by armature reaction increases the loss. The maximum value of either the longitudinal or the transverse field at the trailing pole-corner is increased and at the leading pole-corner is decreased, so that these two changes largely counterbalance one another and the loss in the interpolar bars is but little increased. But the fields now vary almost continuously under the poles, and the loss in the slots under the poles may be regarded practically as an entirely additional quantity. Finally, the copper bars of a section during the period of its commutation are traversed by an alternating flux self-induced by the armature current and crossing the slot from side to side. The current is then no longer uniformly distributed over the cross-section of a large solid conductor, and since this effect produces a loss proportional to the square of the load-current, it may also be derived by assuming a virtual increase in the resistance of the conductor. The energy lost thereby, although assisting to heat the armature, is however rather to be considered under the question of commutation, being closely connected with the apparent inductance of the section during short-circuit.

Comparison of the results of direct eddy-current tests at no-load, with the efficiency measured by Hopkinson's method, shows that the increase at full-load is of the order of 10 per cent.

It is not only along the active length of the bars that eddy-currents are set up. Where the solid bars project beyond the core of the toothed armature, or where the stranded bars are soldered to their end-connectors in the bar-wound drum with evolute connectors, the fringe from the flanks of the poles contributes a small quota to the total loss.

(*c*) The bands of binding-wire, especially where soldered together, are a further source of loss. On smooth-core armatures they are of minor importance, but when employed in connection with toothed armatures they may give rise to considerable losses. By the use of a material of high specific resistance such losses can, however, be reduced to an almost negligible quantity even in toothed armatures.

§ 17. **Approximate formulæ for eddy-current loss in armatures.**—Enough has been said to show the great difficulty of expressing in any simple formula the combined effect of the numerous variables which enter into the question of the amount of the eddy-current loss. The law which governs any one constituent may be unknown, or if known the magnitude of its effect may be quite uncertain owing to its being dependent upon varying conditions in practice, such as the turning of the core or the stamping of the discs. By some writers the total eddy-current loss has been placed on the same footing as that by hysteresis, and has been expressed as a multiple of the latter. Or again, it is regarded as determined by the pounds of iron in the armature core, the flux-density and the number of cycles per second, just as is the hysteresis, although not necessarily following the same law of rate of increase with increase of the induction. The basis of either method is, however, so far from the truth that any correspondence of their results with facts must remain entirely empirical, and when applied under changed conditions they can lay no claim to reliability. Yet the need of some such formula to the dynamo designer is so imperative that the required form of expression may at least be tentatively suggested as a starting-point for purposes of comparison, even if the constants require to be afterwards corrected as experience accumulates.

The problem, therefore, is to construct a formula for F, or the watts lost in eddy-currents at one revolution per min., which will in some degree differentiate between the different localities of eddy-currents and the different causes of the loss of energy by them. The losses other than those in the active length of the copper winding may be divided into a portion due to the active length of the iron under the poles, and a portion due to the end-fringe from the flanks of the poles. The first for a given linear speed is probably best regarded as proportional to the cylindrical surface of the core, *i.e.* to DL, since it is largely due to the extent of the surface contacts between the core laminations caused by mechanical processes of manufacture. Indeed, in toothed armatures a good deal of evidence might be brought to show that this portion of the loss is proportional to the number of slots, each as causing continuity of surface, or to their own area of surface. Next, this loss will vary as the square of the density on the surface at different parts of the teeth, yet since this diverges from proportionality to the air-gap induction only when the apparent density in the teeth has been pressed very high, the term B_g^2 may be retained approximately. Lastly, it will vary as the square of the linear speed, and therefore for a given number of revolutions per minute as D^2, so that it finally becomes $= k_1 \cdot N^2 \cdot D^3L \left(\frac{B_g}{1000}\right)^2$.

In the second portion due to the end-fringe may be grouped the losses in the end-connections and solid ends of the bars where they project past the core, and the whole is roughly proportional to the circumference, or to D. All these losses will be proportional not simply to B_g^2, but to X_g^2, or in the toothed armature, to $(X_g + X_t)^2$ as causing the stray lines between the pole and the body of the armature core or supports of the end-winding. They will also vary as the square of the linear speed, and so become $= k_2 \cdot N^2 \cdot D^3 \left(\frac{X_g + X_t}{1000}\right)^2$.

For the toothed armature k_1 varies between 4×10^{-10} and 6×10^{-10}, k_2 is about 3×10^{-10}, and k_3 the factor in equation (139) for the copper winding has such a value as 80×10^{-10}, all dimensions being reckoned in inches, and flux-densities being measured in lines per square centimetre. An equation of the form

$$F = \Big[D^3 \Big\{ 5 \Big(\frac{B_g}{1000}\Big)^2 L + 3 \Big(\frac{X_g + X_t}{1000}\Big) \Big\} + 80 \Big(\frac{\text{app. } B_{t2} - 17{,}000}{1000}\Big)^2 n_s^2 . \frac{w_s h_s}{\tau} LD^{\frac{3}{4}} l_g^{\frac{1}{4}} p^{\frac{5}{4}} \Big] 10^{-10} \quad . \ . \ . \ (141)$$

may thus be sugggested for the toothed drum. In practical cases the first term is by far the greatest, the second about 5 per cent. of the first, while the third term never reaches any appreciable amount until the uncorrected B_{t2} exceeds 22,000 with fairly narrow slots and deep bars in two layers, or 24,000 with four layers of shallow bars. A wide slot should only be used in combination with thin bars and with low values of B_{t2}. Analysis of numerous experiments on toothed armatures shows that the total loss is not far from proportional to B_g^2, and when the total measured loss in both armature and pole-pieces does not rise so rapidly the reason may be traced to magnetic screening reducing the loss in the poles (cp. § 23).

Smooth-core drums show an even closer proportionality of the total loss to the square of the air-gap induction. The first coefficient k_1 is lower, since the core surface is not so much worked, and may be taken as 4×10^{-10}, while k_4 in equation (140) has such values as $20{,}000 \times 10^{-10}$ for solid bars and 2000×10^{-10} for stranded bars. Thus for the smooth armature with stranded bars

$$F = \Big[D^3 \Big\{ 4 \Big(\frac{B_g}{1000}\Big)^2 L + 3 \Big(\frac{X_g}{1000}\Big)^2 \Big\} + 2000 \Big(\frac{B_g}{1000}\Big)^2 . t^2 h L \tau D^{\frac{3}{4}} l_g^{\frac{1}{4}} p^{\frac{5}{4}} \Big] \times 10^{-10} \ldots (142)$$

In conclusion, it may again be repeated that further direct experiment in the laboratory as to the various sources of eddy-currents and their suppression is much needed, and the lack of such a firm experimental basis must be the excuse for the approximate nature of the formulæ which have been above hazarded.

In commercial work machines can seldom be designed so as to test separately the influence of the several varying factors, yet it is only to the more complete elimination of eddy-currents that we must look if the efficiency and output of dynamos for a given mass of iron and copper are to be appreciably increased in the future.

§ 18. **Hysteresis loss in armatures.**—Returning to the third source of heat, or the "magnetic" loss by hysteresis, the amount of power spent in changing the direction of magnetisation of the core must be calculated, as explained in Chap. XII. § 14. In the core below the teeth the density and hysteresis loss is greatest in the outer layers. The nominal maximum density when the flux is averaged over the whole of a cross-section midway between the poles is only about 82 per cent. of the true maximum near the bottom of the teeth. But since the curve of Fig. 139 for rotating magnetisation is based on the nominal maximum B_a averaged over an interpolar cross-section, the values of h for other armatures with the same nominal maxima B_a may thence be obtained. The loss in the core is then for any given maximum induction simply proportional to the number of complete cycles per second, and to the volume of the core; it may therefore be expressed by a coefficient H multiplied by the number of revolutions per minute, or in terms of the symbols of equation (26), $H = h \cdot \frac{p}{60} V_c$ and $HN = H_w$. The cross-magnetisation of the armature under load here also increases the loss as compared with that at no-load. The total volume of the teeth must be calculated separately from the rest of the armature core, and in default of experimental figures strictly applicable to their case h must again be taken from Fig. 139, as explained in Chap. XII. § 14, *qu.v.*

§ 19. **Predetermination of rise of temperature in armatures.**—Combining the three losses together, we obtain as the total watts, expended in heating the armature

$$W = C_a^2 R_a + FN^2 + HN \quad . \quad . \quad . \quad . \quad . \quad (143)$$

and it is the ratio of this to the cooling surface of the rotating armature that determines its rise of temperature.

In estimating the relative cooling value of any surface the general principle must be to allow approximately for differences in their degree of exposure to the outer air, and also for differences in their peripheral speed. Thus the end-windings of multipolar barrel-wound armatures are of more importance than the centre portion of the core which is directly under the poles. Again, in large multipolar armatures in which the discs are supported on the arms of a hub, not only is a clear air-space secured between the internal circumference of the core and the cylindrical hub, but the percentage difference between the peripheral speeds of the inside and outside grows less as the diameter of the armature is increased. Finally, the number and width of the radial air-ducts through the armature core will exercise a considerable effect, and may require to be taken into account in the calculation. It is, however, useless to adopt any great degree of refinement in the calculation of the cooling surface owing to the number of secondary conditions which affect the result. Different types of machines will require different methods of reckoning the cooling surface, but in general the curve connecting the heating coefficient with the peripheral speed will have the shape of those in Fig. 390 when the cooling surfaces are reckoned on some consistent basis. Practical experience will determine a method of calculating a figure to which the cooling effect may be regarded as proportional, even though all the factors which enter into it may not be strictly taken into account. The general effect of taking close account of the surfaces of the inside of the core and of the air-ducts will merely be to raise the value which must be assigned to the coefficient when the armature is stationary.

The two cases of smooth-core drum armatures and toothed barrel-wound multipolars in which the armature core is well ventilated by air-ducts at intervals of 4 or 5 inches may, however, be distinguished. In some designs the ventilating ducts have been made so numerous

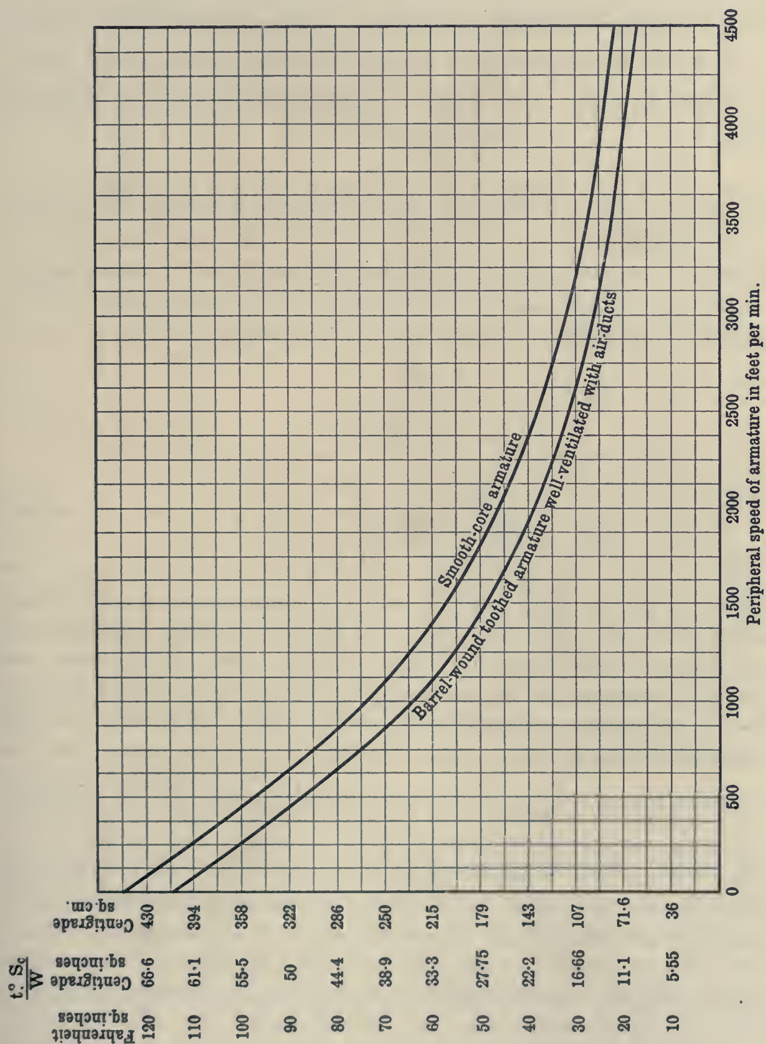


FIG. 390.—Heating coefficients of armatures.

and so wide that the effective length of the iron is only two-thirds of the gross length of the core. In small machines less than 15 inches in diameter, owing to the confined nature of the internal apertures into the core, the effect of such ducts is not so marked as in large machines, although they still remain of value when made of considerable width and accompanied by a high peripheral speed. On an average the curves given in Fig. 390 have been found by the writer to give good results in the two cases of continuous-current bi- or multipolar armatures of usual construction up to peripheral speeds of 4000 feet per minute. The value of the heating coefficient $k = \frac{t^\circ \times S_c}{W}$, or the temperature rise in degrees Fahrenheit per square inch per watt is for barrel-wound armatures $\frac{115}{1 + 0{\cdot}8\left(\frac{v}{1000}\right)^{1{\cdot}3}}$ and for smooth-core armatures $\frac{125}{1 + 0{\cdot}7\left(\frac{v}{1000}\right)^{1{\cdot}3}}$, where v is the peripheral speed of the external surface of the armature in feet per minute, and S_c is the cooling surface in square inches. The final maximum rise of temperature of the outside of the armature as measured by thermometer is thus

$$t^\circ F = \frac{115W}{S_c\left\{1 + 0{\cdot}8\left(\frac{v}{1000}\right)^{1{\cdot}3}\right\}} \quad . \quad . \quad . \quad . \quad . \quad (144)$$

or

$$\frac{125W}{S_c\left\{1 + 0{\cdot}7\left(\frac{v}{1000}\right)^{1{\cdot}3}\right\}} \quad . \quad . \quad . \quad . \quad . \quad (145)$$

The two curves draw together at high speeds, but the percentage effect from the ventilating ducts and open winding of the barrel multipolar remains greater as the peripheral speed is increased, and in the expression for the curves it is seen that in the one case the factor 0·8 appears instead of 0·7 in the other. The effect of the peripheral speed has in many formulæ been given by a factor of the form $1 + 0{\cdot}5 \frac{v}{1000}$, but the use of the first power of the peripheral speed appears to underestimate its great influence. The constant value when $v = 0$, or the armature is at rest, namely, 115 or 125, is lower than the average value for stationary field-magnet bobbins owing to the lesser thickness of the layers of copper in the armature; for any given type of machine it may be determined by measuring the rise of temperature for a given number of watts when the armature is placed in its appropriate field-magnet and is stationary.

The cooling surface to be used in connection with the curves of Fig. 390 is in the case of toothed armatures with barrel winding the external cylindrical surface, plus the internal cylindrical surface of the winding at both ends so far as it projects beyond the core; the latter must be again reduced in proportion to its lower speed, when there is considerable difference between the external and internal diameters.

The former surface is $\pi D_1 L_1$ where L_1 is taken from the outer edge of the commutator lug to the extreme opposite end of the armature, *i.e.* $= L + 2l_c$ nearly. The latter is approximately $\pi D_2(L_1 - L) = \pi D_2 \cdot 2l_c$, or when reckoned as of less value in proportion to its lower peripheral speed, $\pi \cdot \frac{D_2^2}{D_1} \cdot 2l_c$.

In smooth-core bipolar armatures, whether ring or drum, S_c is reckoned as the external cylindrical surface $\pi D_1 L_1$ plus the area of the two ends $2 \times \frac{\pi D_1^2}{4}$. In the case of small bipolar rings in which the wire practically fills the internal space, and also in small hand-wound drums in which the winding is wrapped closely round the core without any means of ventilation, the cooling surface must not be reckoned as so great, and the area of only one end should be taken. In large multipolar drums with smooth core and involute end-connectors, the area of one entire end and the annular ring occupied by the connectors at the other end $\left(= \frac{\pi D_1^2}{4} + \frac{\pi(D_1 - D_2)^2}{4}\right)$ should be added to the external cylindrical surface.

In all cases the question of the heating of the commutator must be also considered in relation to that of the armature. The above figures assume that the temperature of the commutator is lower than that of the armature, and this is in practice always secured with a smooth-core and copper gauze brushes, so that the temperature rise of the far end of the armature is usually some 5° to 10° F. higher than that of the commutator end. With carbon brushes, however, the final temperature of the commutator may exceed that of the armature. Hence the commutator, so far from helping to dissipate the heat of the armature, positively assists in raising the temperature of the near end by conduction, unless there are long radial commutator connections which exert a powerful fanning action.

For a peripheral speed of 2000 feet per minute, a rise of 65° F. is approximately obtained if the watts per square inch or $\frac{W}{S_c} = 1{\cdot}41$ to $1{\cdot}5$, or *vice versâ* if $S_c = 0{\cdot}71$ W to $0{\cdot}66$ W; in other words, about three-quarters or two-thirds of a square inch of cooling surface, as above reckoned, must be allowed per watt expended over the armature.

It may here be remarked that a cool machine is by no means necessarily efficient; although in most cases these two desirable qualities are attained by the same means, still it should be remembered that, while the efficiency is dependent on the ratio of the lost watts to the useful output, the rise of temperature is determined by the ratio which the lost watts bear to the cooling power of the surfaces.

Finally, with a given armature, since the eddy-current loss is dependent upon the square of the revolutions, the amount of current that can be taken out of it at different speeds for a fixed rise of temperature depends largely upon the proportion of the copper to the eddy-loss, and upon the way in which the effectiveness of the cooling surface is modified by alterations of the peripheral speed.

§ 20. **Heating of the commutator.**—The heating of the commutator with carbon brushes has an importance second only to that of the armature. The sources of heat within the commutator itself are fourfold, namely—

(1) The loss of energy due to the passage of the armature current over the contact resistance of the brushes, this current being assumed to be divided between the several sets of brushes of the same sign and between the portions of any one brush in strict proportion to their areas; *i.e.* on the supposition of a uniform current-density under the brushes.

(2) The additional loss due to the unequal division of the current over the surface of the brush contact, and to sparking if commutation is not properly performed.

(3) The loss from the mechanical friction of the brushes.

And (4) the loss from eddy-currents in the sectors which are at any moment carrying the armature current and the adjacent sectors. As the current flows along the sectors which are undergoing commutation, and is gradually tapped off into the brushes, these sectors are practically situated in a stationary magnetic field, while they themselves are moving forwards. Eddy-currents are thereby set up in the mass of the copper plates forming the commutator.

When copper brushes are employed, these several losses are not sufficient to cause any great rise of temperature in the commutator, even though there may be undue sparking. But with carbon brushes the great increase which is possible in the first and second items renders it imperative for the designer to carefully consider the heating due to the combined effect of the four causes, and this is especially the case in machines of low voltage and large current.

§ 21. **Calculation of commutator losses.**—The commutator losses are calculated as follows. From Chap. XVIII. § 16 the loss of volts at a row of brushes is $P = s_{u\,eff.} \cdot R_k = f_u \cdot s_u \cdot R_k$. Let $p_1 =$ the number of rows of brushes of one polarity; then, assuming that the total armature current C_a is equally divided between them, the average current-density is $s_u = \dfrac{C_a}{p_1 \cdot F_u}$, and the current to be collected at one row is $\dfrac{C_a}{p_1}$. But so far as the brushes themselves are concerned, the effective current that is passing through their contact-surface is $f_u \cdot \dfrac{C_a}{p_1}$. Therefore the watts in any one row are $\Delta P \cdot \dfrac{C_a}{p_1} \cdot f_u$, and since there are $2p_1$ rows in the two sets of brushes, the total watts over both sets, positive and negative, are

$$2\Delta P \cdot C_a \cdot f_u = 2C_a \cdot s_{u\,eff.} R_k \cdot f_u = 2C_a^2 \frac{R_k}{p_1 \cdot F_u} (f_u)^2$$

The effective current-density or the form-factor can only be determined for each particular case, if all data as to the shape of the short-circuit current curve, etc. were known, and upon $s_{u\,eff.}$ also depends the value to be taken for R_k. Further, ΔP should, strictly speaking, be determined for the positive and negative brushes separately, since R_k varies so appreciably according to the direction of the current and temperature.

(1) In the absence of such complete data, R_k must be taken from the curves of Chap. XVIII. § 14, on the assumption of a uniform current-density s_u corresponding to $\frac{C_a}{p_1 \cdot b_1 \cdot b}$, where b is the length of the brush surface in one row measured parallel to the axis of rotation, and b_1 is the width of contact in the direction of rotation. The total loss of watts over the brushes is then

$$2C_a^2 \cdot \frac{R_k}{p_1 b_1 b}$$

where $\frac{R_k}{b_1 \cdot b}$ is the resistance of one row, and $\frac{R_k}{p_1 b_1 b}$ is the resistance of the p_1 rows of one polarity. In the bipolar machine or the wave-wound machine with two sets of brushes, $p_1 = 1$; in the lap-wound multipolar with as many sets of brushes as there are poles, $p_1 = p$, but in the wave-wound multipolar, even though there are several rows of brushes of one polarity, some rows may be omitted, and p_1 need not be equal to p.

(2) The additional loss due to want of uniformity of current-density cannot reach any great amount with copper brushes owing to the sparking that would result, but with carbon brushes it may form a very considerable item without evident overheating of the brush edges. Since the increase depends upon the square of the form-factor, it may easily amount to 30 per cent. of the normal loss, or in extreme cases may double it. The advantage of carbon in reducing the sparking is, in fact, secured at the expense of the efficiency of the machine.

(3) If p be the pressure on the brushes in lbs. per sq. inch, and a be the total area of surface of all the brushes on the machine, the total pressure is $P = pa$, and the loss in watts from the mechanical friction is

$$\frac{\mu P \times v_k' \times 746}{33{,}000}$$

where v_k' is the peripheral speed in ft. per min., and μ is the coefficient of friction

$$= \mu P \times v_k' \times 0{\cdot}0226 \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad (146)$$

The experiments of Prof. F. G. Baily and Mr. W. S. H. Cleghorne* show that the coefficient of friction of a dense electrographitic carbon brush of low specific resistance (Le Carbone quality X) on a perfectly dry commutator is 0·45, and that with fairly hard carbon brushes of coarser grain an average figure would probably be 0·4. At high peripheral speeds, where some vibration is likely to be present, this is again reduced to 0·3. Assuming $p = 1\frac{1}{2}$ lb. per square inch, this leads to limiting values of approximately 15 to 10 watts per square inch of brush surface per 1000 feet per minute. In general it may be said that the higher the contact-resistance of the carbon the lower is its coefficient of friction.

The above-cited paper of Prof. Baily and Mr. Cleghorne also shows that the application of a small amount of paraffin wax as a lubricant reduces the coefficient of friction of the electrographitic brush as low as one-fifth of its former value, *i.e.* to 0·09 without increasing the electrical losses.†

The coefficient of friction of copper gauze brushes may be taken as 0·3.

(4) Definite experiments as to the magnitude of the fourth loss from eddy-currents are wanting, but probably they are but small, although increasing with increased thickness of the commutator plates.

§ 22. **Temperature rise of commutator.**—The rise of temperature of the commutator surface should preferably not exceed 70° F., or as a maximum may be allowed to reach 80° F. in continuous running at full-load. It may be calculated by a formula similar in its construction to that for the rotating armature. The constant of the numerator which determines the rise per watt per sq. inch when the commutator is at rest is lower than in the armature, owing to the better exposure of the former to the air. On the other hand, owing to its smooth surface, the influence of the peripheral speed is not so marked as in the case of armatures, although still considerable if there are separate connectors to the armature winding. If W = the total watts expended over the commutator, and the cooling surface be reckoned in sq. inches as the external cylindrical surface plus the area of one side of the radial connectors up to a limiting length of say 3″ from the commutator surface

$$t°\text{F.} = \frac{100\text{ W}}{\text{S}_c\left\{1 + 0{\cdot}3\left(\frac{v_k'}{1000}\right)^{1.3}\right\}} \quad . \quad . \quad . \quad (147)$$

where v_k' is the peripheral speed of the commutator in feet per minute.

* *Journ. Inst. Electr. Eng.*, vol. xxxviii. p. 157.

† But it must be very sparingly used, cp. Chapter XXI. § 9.

Any such formula is, however, liable to many disturbing conditions, among which especial importance must be given to the number and shape of the connections which lead from the armature winding to the sectors. If these are numerous and are thin but wide blades of copper, they have a powerful fanning action, which very greatly assists in dissipating the heat of both the commutator and the armature.

A cool commutator is of great assistance in any case of difficult commutation, and on this account the commutation of turbo-generators is much improved by special means for ventilation both inside and outside the commutator.*

§ 23. **Eddy-currents in pole-pieces.**—It remains to consider the question of eddy-currents as set up in the pole-pieces when a rotating toothed armature causes the density of the field over their bored face to be rhythmically varied. Such currents do not spread to any great depth within the metal mass, but whirl round near the surface facing the armature. One complete cycle of varying induction corresponds to the passage of one tooth and one slot past a fixed point on the pole-face; or, in other words, a period corresponds to the time taken by the armature in moving through the pitch of the teeth ($t_1 = w_{t1} + w_3$). If n_s = the total number of teeth, the periodicity is $f = \frac{N}{60} . n_s$. If the local paths had no inductance the current would run along the pole-face opposite to each projecting tooth, and then dividing would curve round to complete their circuit opposite the slots. Owing, however, to the action of the inductance which causes the current to lag behind the impressed E.M.F., the positions of the eddies are displaced relatively to the teeth; if the reactance ($2\pi f$ L) were very high, or the resistance very low, the angle of lag would approach a quarter of a period, so that the currents would embrace the teeth and openings of the slots. The self-induced flux would then act to reduce the density opposite the teeth, and to raise it opposite to the slots. Thus by the effect of magnetic screening the distribution is rendered more uniform, and the eddy-currents are prevented by their own inductance from reaching any great amount.

The amount of the eddy-current loss depends entirely upon the nature of the curve of the non-uniform distribution. The extreme limit is reached if we assume a uniform current to flow across the pole through a strip of width equal to a quarter of the pitch or $\frac{t_1}{4}$ centimetre, and to complete its circuit by returning through another adjacent strip of the same width, and further if we assume the maximum B_g to hold throughout the first strip and the minimum B_g throughout the second strip. The E.M.F. in the small strip is proportional to the peripheral speed of the moving teeth, to the difference in the inductions, and to the length L of the pole-piece axially.

* Cp. Messrs. Siemens Bros.' patent commutator, described chap. xx. § 9.

If v = the peripheral speed of the armature, or $\frac{\pi DN}{60}$ in centimetres per second, the E.M.F. acting round the entire circuit is then

$$v(B_{max.} - B_{min.})\ L \times 10^{-8} \text{ volts,}$$

or half this value acting along the one strip. The resistance of the single strip to a distance h centimetre within the pole-piece is $\frac{L \cdot \rho}{\frac{1}{4}t_1 \cdot h}$, where ρ is the resistivity in ohms of the material of which it is composed; the resistance of the end or cross portions of the circuit is regarded as negligible as compared with that of the length. The loss of power in the one strip is, therefore in watts

$$\frac{E^2}{R} = v^2\left(\frac{B_{max.} - B_{min.}}{2}\right)^2 L^2 \times \frac{\frac{1}{4} \cdot t_1 h}{L \cdot \rho} \times 10^{-16}$$

and as there are $\frac{A}{\frac{1}{4}t_1}$ strips in the width of the pole, where A is the pole-width in cm., the total loss in the pole for the depth h is

$$v^2\left(\frac{B_{max.} - B_{min.}}{2}\right)^2 \frac{ALh}{\rho} \times 10^{-16} \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad (148)$$

ALh being the volume of the portion under consideration (Fig. 391).

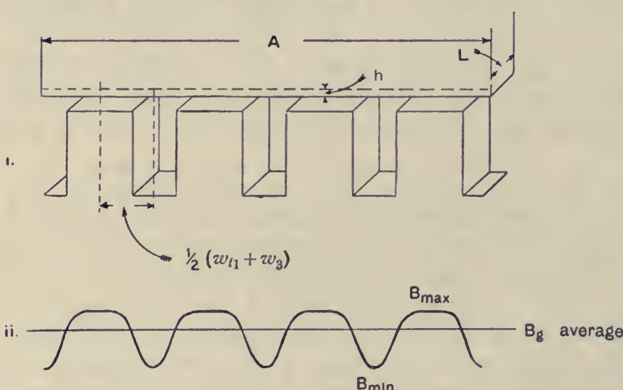


FIG. 391.—Eddy loss in pole-pieces.

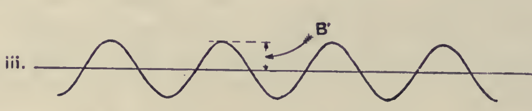


FIG. 392.—Eddy loss in pole-pieces.

While the above may be regarded as giving an extreme limit for the eddy-current E.M.F. and loss in a layer h cm. thick, the formula is easily extended to the case where the fluctuation of the density is assumed to be sinusoidal. The range of induction being, as before, from $B_{max.}$ to $B_{min.}$, the maximum E.M.F. acting round the edges of the circuit corresponding to half the pitch of the teeth is $\propto B_{max.} - B_{min.}$, or along the one edge $\propto \frac{B_{max.} - B_{min.}}{2}$; thence it gradually diminishes after a sine law to zero midway between the two edges. The loss of power is proportional to the square of the E.M.F., and the average value of the squares of the ordinates of a sine curve is half the square of the maximum value (Chap. VI.

§ 17). Thus the total loss is exactly half of that given by equation (148). Finally, a still closer approximation may be made by again returning to the actual curve of distribution and drawing through it a straight line (shown in Fig. 391, ii.) corresponding to the average value of B_g as calculated from the formula at the end of Chap. XV. § 7. The actual flux may then be regarded as produced by the superposition upon the straight line of an alternating flux. This alternating flux may be replaced by an equivalent sine-wave (Fig. 392) having the same effective value B_e, and giving a certain maximum value B' which is not quite the same as $\frac{B_{max.}-B_{min.}}{2}$; the value of B_e^2 or, which amounts to the same, of $\frac{1}{2}(B')^2$ must then be substituted in equation (148) for $\left(\frac{B_{max.}-B_{min.}}{2}\right)^2$. Since AL=the area of the pole-face, the loss per sq. cm. of pole-face in the layer of thickness h cm. is thus

$$\tfrac{1}{2}v^2(B')^2 \frac{ALh}{\rho} \times 10^{-16} \text{ watts,}$$

or since v may also be expressed as $n_s \cdot \frac{N}{60} \cdot t_1$, is

$$\tfrac{1}{2}\left(n_s \cdot \frac{N}{60}\right)^2 (B' \cdot t_1)^2 \frac{h}{\rho} \times 10^{-16} \text{ watts} \quad . \quad . \quad . \quad (149)$$

Since the eddy-currents curve round in the mass of the pole, and especially at the ends of the pole-face, they may be partially reduced by axial slits along the pole-face, the thickness of the subdivisions being less than $\frac{1}{2}t_1$; but any such subdivision cannot be so effective as the more usual plan of laminating the pole in a direction at right angles to the axis of rotation just as the armature core is itself laminated. Taking a single thin lamination and a length of one-quarter of a tooth-pitch, let it be mentally divided into a number of much thinner longitudinal strips. From the proportions of the original lamination all transverse resistance to the actual circulatory currents as they cross the lamination may be neglected in comparison with the longitudinal resistance down one side and up the other. On this assumption the E.M.F.'s generated transversely give immediately without deduction the P.D.'s corresponding to the longitudinal components of the actual current, *i.e.* to the varying currents along the strips, through which alone energy is lost and which alone need to be considered. The central axis being an equipotential line and being regarded as at zero potential, the potential along the length of any elementary strip, distant x cm. from the centre line and dx wide is

$$V = n_s \cdot \frac{N}{60} \cdot t_1 \cdot B' \sin \alpha \cdot x \times 10^{-8}$$

one end of the strip being located at the line where the induction has its normal value. The spacial rate of change of V in a longitudinal direction or

$$\frac{dV}{\frac{t_1}{2\pi} \cdot d\alpha} = n_s \cdot \frac{N}{60} \cdot B' \cdot 2\pi x \cdot \cos \alpha \times 10^{-8}$$

is also the longitudinal current-density Δ at any point × the resistivity ρ, and the watts per unit volume $=\Delta^2\rho$. The watts at the point in question, or $\Delta^2\rho$ × the infinitesimal volume are then

$$\left(n_s \cdot \frac{N}{60}\right)^2 (B')^2\, 4\pi^2 x^2 \cdot \cos^2 \alpha \times \frac{h}{\rho} \times \frac{t_1}{2\pi} \cdot d\alpha \cdot dx \times 10^{-16}$$

The integral $\int x^2 dx$ between the limits $x = -\frac{t}{2}$ and $x = \frac{t}{2}$, where t is the thickness of one lamination, being $\frac{t^3}{12}$, and the integral $\int \cos^2 \alpha \cdot d\alpha$ between the limits $\alpha = 0$ and

$a=\frac{\pi}{2}$ being $\frac{\pi}{4}$, the loss in the thickness of one lamination and in one length corresponding to a quarter of the tooth-pitch is

$$\frac{\pi}{4} \cdot \frac{\pi}{6} \cdot \left(n_s \cdot \frac{N}{60}\right)^2 (B')^2 t_1 \cdot t^3 \cdot \frac{h}{\rho} \times 10^{-16}$$

Multiplying by the number of laminæ $=\frac{L}{t}$, and by the number of lengths $\frac{A}{\frac{1}{4}t_1}$, the total loss in the volume of a layer h cm. deep is

$$\frac{\pi^2}{6}\left(n_s \cdot \frac{N}{60}\right) (B')^2 \cdot t^2 \frac{V}{\rho} \times 10^{-16}$$

or the same as the expression of § 14 for the loss in an alternating field; and the loss per sq. cm. of pole-face is

$$1{\cdot}645\left(n_s \cdot \frac{N}{60}\right)^2 (B')^2 \cdot t^2 \frac{h}{\rho} \times 10^{-16} \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad (150)$$

so that the effect of the tooth-pitch has disappeared.

But in either equation (149) for solid pole-pieces, or equation (150) for laminated pole-pieces there remains the great difficulty underlying the value to be given to h. As we pass the outer skin of the pole-piece, and proceed farther into its mass, the flux rapidly becomes more uniform, and the watts, being proportional to the square of the difference of the induction, diminish still more rapidly. It is, in fact, very difficult to determine the exact depth of the fluctuations, and the values which are to be assigned to $B_{max.}$ and $B_{min.}$ for each successive layer. With ordinary speeds and a considerable number of teeth, as in practical cases, the periodicity of the alternating currents set up—from 500 to 2000 cycles per second—is so high that the screening action largely reduces the loss, and has the effect of rendering the general law of the proportionality of the eddy-current E.M.F. to the speed, and of the loss to the square of the speed, no longer true. Especially is this the case with solid pole-pieces, to which the case of a magnetic brake becomes more nearly analogous. In such a brake, in which all the work is expended in producing eddy-currents in a solid mass, it is found* that the loss or the energy absorbed is proportional to a very low power of the speed, such as the 1·2 power. Approximate calculations based on equation (149) for solid pole-pieces thus give values far exceeding those that are found in practice, and the preceding formula can only be regarded as illustrating some of the elements of the problem. The complex effects of screening can hardly be taken into account in an elementary treatment of the subject,† but more reasonable figures are obtained on the assumption ‡ that the induced currents are damped out or extinguished within the mass of metal after the

* Dettmar, *E.T.Z.*, vol. xxi. pp. 947, 948.

† See Niethammer, *E.T.Z.*, vol. xx. p. 767, and (1900) p. 549.

‡ Due to Picou after Potier in *Industrie Electrique*, 1905, p. 35; and also to R. Rüdenberg, *E.T.Z.*, February 23, 1905, p. 181.

same law as that by which the intensity of an electromagnetic wave of high frequency decreases when meeting the surface of a metal. The resulting formula is then that the loss per square centimetre of a pole-face in watts is

$$\frac{1}{8\pi} \cdot f^{1.5}(B't_1)^2 \sqrt{\frac{1}{\kappa \cdot \mu}} \times 10^{-7}$$

$$\text{or} \quad 4\left(n_s \cdot \frac{N}{60}\right)^{1.5} (B't_1)^2 \frac{1}{\sqrt{\kappa \cdot \mu}} \times 10^{-9} \quad . \quad . \quad . \quad . \quad (151)$$

where B' is reckoned at the surface, f is the frequency, the wave-length or tooth-pitch t_1 is in centimetres, and κ is the resistivity of a centimetre cube of the metal in absolute units. For annealed wrought iron ρ may be taken as 10^{-5} or $\kappa = 10^4$, and for cast steel as 2×10^{-5} or $\kappa = 2 \times 10^4$, while for cast iron ρ rises to 10^{-4} or $\kappa = 10^5$. The high electrical resistivity of cast iron is therefore decidedly advantageous in reducing the eddy-loss, but since both μ and κ or ρ appear together in the denominator, the gain is partly counterbalanced by the low permeability when the density of the flux at the pole-face is high. Since the variation of the flux-density is never very high at the surface, μ may be taken as corresponding to the mean air-gap density. The thickness of the layer by which the fluctuation is damped to 1 per cent. of its amount at the surface is now $h = 0.73\sqrt{\frac{\kappa \cdot t_1}{v \cdot \mu}}$, and is thus very small. Further, the loss becomes proportional to the 1·5th power instead of to the square of the frequency. According to the experiments of Herr Dexheimer, cited by Professor Arnold,* the loss was proportional to the 1·7th power, but the later experiments of Messrs. T. F. Wall and S. P. Smith † again confirm the term $\left(n_s \cdot \frac{N}{60}\right)^{1.5}$ which is thus to be adopted. The formula given by Mr. Carter ‡ in centimetre measure becomes $3.77\left(n_s \cdot \frac{N}{60}\right)^{1.5} (B't_1)^2 \frac{1}{\sqrt{\mu}} \times 10^{-11}$, or if $\kappa = 10^4$ is introduced as for wrought iron,

$$= 3.77\left(n_s \cdot \frac{N}{60}\right)^{1.5} (B't_1)^2 \frac{1}{\sqrt{\kappa \cdot \mu}} \times 10^{-9} \text{ watts per square}$$

centimetre, so that it may practically be taken as equivalent to formula (151).

The chief difficulty remains in the term $(B't_1)^2$, *i.e.* especially in the value to be assigned to the amplitude of the variation of the induction, B', at the surface. According to the formulæ of Mr. Carter, $\frac{B_{max.} - B_{min.}}{2}$

* *Die Gleichstrommaschine*, (2nd. edit.) vol. i. p. 648.

† *Electr.*, vol. lvii. p. 568, and *Journ. Inst. Electr. Eng.*, vol. xl. p. 577.

‡ *Journ. Inst. Electr. Eng.*, vol. xxxiv. p. 49.

is equal to $m\left(\frac{1}{2}-\frac{1}{\sqrt{4+\left(\frac{w_3}{l_g}\right)^2}}\right)$ times the average induction, where m has the same meaning as in Chap. XV. § 7. This agrees very well with the experimental values which can be deduced from the curves of Mr. T. F. Wall* and Mr. Matthews, and in normal cases gives figures of about 0·2 B_g on an average. If the curves of flux distribution on the pole-face were strictly sinusoidal, $\frac{B_{max.}-B_{min.}}{2}$, as above calculated, could be at once substituted for B′ in equation (151), and when the ratio of slot-opening to air-gap is not very large, the curves of flux distribution are in fact more or less sinusoidal, especially if the slots are slightly overhung. Yet equation (151), in contrast to equations (149) and (150), gives actually too low values for the loss at least with solid pole-pieces, and it is therefore best suited to the calculation of the loss in laminated pole-shoes with laminations about $\frac{1}{12}$th of an inch thick. With thinner laminations about $\frac{1}{40}$th of an inch thick, the constant of equation (151) may be reduced from 4 to 2·5.

But when the ratio $\frac{w_3}{l_g}$ is large, the flux curve is flatter above each tooth with pointed depressions between the teeth. The form-factor is then higher than for a sinusoidal curve upon which equation (151) is based, but on the other hand the amplitude of the equivalent sine wave may be less than half the difference between the actual maximum and minimum values. Since these effects counteract one another, the simple substitution of Mr. Carter's expression for $\frac{B_{max.}-B_{min.}}{2}$ in place of B′ would again become roughly correct.

For a given value of $\frac{w_3}{l_g}$, if the tooth-pitch t_1 is much increased, the zones of iron through which the currents flow past the slots become small as compared with the width of the teeth over which the induction is practically uniform, so that the eddy-current loss over the pole-face as a whole might be but small. Formula (151) takes this into account, since for a given armature on increasing t_1 although the product of t_1^2 with m^2 itself increases, yet the number of slots and frequency are reduced. The loss is however probably then underestimated, so that when $t_1 > 2w_3$, it is perhaps better to assume the flux-curve to be made up of depressions opposite each slot in length equal to $2w_3$ with the crests joined by straight lines of length $(t_1 - 2w_3)$ over which $B_{max.}$ holds. We then have to consider only $\frac{2w_3}{t_1}$ of the total pole-face, but the frequency is increased to $n_s \cdot \frac{N}{60} \cdot \frac{t_1}{2w_3}$, and in place of the wave-length

* *Journ. Inst. Electr. Eng.*, vol. xl. pp. 555 ff. and 573.

t_1 we have the shortened wave-length $2w_3$. Formula (151) for the loss per sq. cm. of the *total* pole-face then becomes * when t_1 exceeds $2w_3$

$$4\left(n_s \cdot \frac{N}{60} \cdot 2w_3\right)^{1.5} (B')^2 \sqrt{\frac{t_1}{\kappa\mu}} \times 10^{-9} \quad . \quad . \quad . \quad . \quad (151a)$$

The method devised by Messrs. Wall and Smith for measuring the eddy-loss in the pole-faces consisted in comparing the temperature which they reach with that due to embedded heating coils; if the tooth-pitch and the approximate values of $\kappa = 10^4$ and $\mu = 2400$ for wrought-iron are inserted for comparison, the experimental formula deduced by these observers becomes

$$12\left(n_s \cdot \frac{N}{60} \cdot 2w_3\right)^{1.5} \left(\frac{w_3}{l_g}\right)^{3.5} B_g^{2.1} \sqrt{\frac{t_1}{2w^3}} \; \frac{1}{\sqrt{\kappa \cdot \mu}} \times 10^{-12} \text{ watts per sq. centimetre} \quad . \quad . \quad . \quad (151a)$$

Although this gives lower values than (151) for solid poles with small values of $\frac{w_3}{l_g}$, it gives much higher values, up to three times as much, when $\frac{w_3}{l_g} = 5$. It would appear, therefore, that with a slot of great width of opening as compared with the single air-gap the increased form-factor causes the loss to be greater in the case of solid poles; or when $\frac{1}{2}(B_{max.} - B_{min.})$ is substituted for B′, the constant of (151) must be raised, say, to 10 when $\frac{w_3}{l_g} = 4$.

The general conclusion is that the loss increases very considerably when the ratio $\frac{w_3}{l_g}$ exceeds 2, and the usage of practice which calls for laminated poles when this value of the ratio is exceeded is entirely justified by experiment. Yet in order to keep the loss in solid poles within reasonable limits, the true statement of the case would probably make the loss depend upon more complex conditions than a simple power of the ratio $\frac{w_3}{l_g}$.

Further experiment is required to show more certainly the effect of a change of the normal induction in the air-gap. While the earlier experiments of Messrs. Wall and Smith showed a loss not quite proportional to the square of the induction, their later experiments, as shown above, give a power slightly higher than the square. In the case of a magnetic brake, the total work with a constant speed and varying induction is proportional to the 1·6th or 1·8th power of the induction or of the total flux, and since the hysteresis loss is but a small proportion of the whole, it is evident that the loss from eddy-currents in a solid pole increases by some power of the induction less than the

* Cp. O. S. Bragstad and A. Fraenckel, *Electr.*, vol. lxii. p. 967.

square. But all apparent variations from proportionality to the square of B_g are probably to be ascribed to changes in the value of μ. Experiment * may be made on this point by winding a number of coils across the pole-face at such distances apart as to correspond to the pitch of the teeth; the alternating voltage set up in the exploring coils for different values of the induction can then be measured. When tested in this manner, a machine in which the air-gap was very little smaller than the width of opening of the slot showed that the eddy-current loss in the pole varied apparently as the 1·7th power of the induction, while in the case of a machine in which the air-gap was 1·86 times the width of the slot, the distribution, as might be expected, was so nearly uniform that no effect could be detected except by a telephone.

In every case when the field is distorted by armature reaction under full-load, the eddy-current loss is increased, not only in the armature but also in the pole-pieces; the amplitude of the variation of the field is increased in the case of a dynamo at the trailing edge, and decreased at the leading corner, but as the loss is proportional to the square of the amplitude, there must be a net increase in the loss. With a short air-gap and wide open slots the effect is so great that the increased heating of the trailing corners is distinctly marked. Messrs. T. F. Wall and S. P. Smith † found an increase in the eddy-currents in the pole-pieces of as much as 50 per cent. between no-load and full armature current.

§ 24. **Eddy-currents due to flux-pulsation.**—So far the eddy-currents due to the "flux-swing" across the face of the pole caused by the teeth of a slotted armature have been alone considered, and this phenomenon must be distinguished from "flux-pulsation" or a pulsation in magnitude of the main flux throughout the entire magnetic circuit, also due to the teeth.‡ The phenomenon of "flux-swing" occurs, strictly speaking, when the area of the air-gap through which virtually the whole flux of one pole enters the surface of the armature remains constant in magnitude, but oscillates in its spacial position relatively to the pole-face in small swings to and fro. "Flux-swing," therefore, is a maximum when with a small air-gap the polar arc is some multiple of the tooth-pitch plus one-half of a tooth-pitch.

But if with a small air-gap, *i.e.* with a large ratio of $\frac{w_3}{l_g}$, there is a decided difference between the number of teeth, the whole or any portions of which are at any time actually under the polar arc, the virtual area of the air-gap alternately expands and contracts, and the actual magnitude of the flux periodically alters in consequence of the variation of the reluctance of the air-gap. Obviously this effect reaches its maximum when the polar arc is an exact multiple of the tooth-pitch, since then at the ends of the pole-face there are at one time two slots and at another time two teeth, the latter case amounting virtually to an addition to the air-gap area of

* Dettmar, *E.T.Z.*, vol. xxi. p. 948.

† *Journ. Inst. Electr. Eng.*, vol. xl. pp. 579 and 593.

‡ For the distinction, see G. W. Worrall, *Journ. Inst. Electr. Eng.*, vol. xxxix. p. 217, and vol. xl. p. 413.

the surface of an entire tooth. As there is no definite boundary line to the field, both phenomena always coexist to some extent, but even when the flux-pulsation is at its maximum it only has a very small percentage effect as compared with the flux-swing across the armature.

Dr. W. M. Thornton has also shown that there are distinct pulsations produced in the field by commutation in a direct-current machine *; their frequency corresponds to the speed and number of commutator sectors, and their amplitude is increased by any causes which assist in producing sparking, as by commutation in an unduly strong field when the short-circuit current set up in the loops under commutation powerfully affects by its magnetising ampere-turns the value of the main flux. An additional cause of pulsation in the value of the flux throughout the entire magnetic circuit is found in the case of an armature core which is not truly cylindrical, or of which the shaft shows a tendency towards whirling; in the former case the eccentricity of the armature causes a double frequency pulsation of the magnetic flux in the main magnetic circuit.† But Mr. Field has shown that in such cases of pulsation throughout the entire magnetic circuit the loss by eddy-currents is not likely to be large, ‡ even in a solid pole or yoke; the magnetic effect does not extend to any great depth from the surface of the iron, so that only a thin skin is affected, and the loss is not proportional to the volume and to the square of the frequency, but rather to the area of surface acted upon and to the square root of the frequency.

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* *Journ. Inst. Electr. Eng.*, vol. xxxiii. pp. 547 and 556.

† *Ibid.*, vol. xxxii. pp. 596 and 599, and vol. xxxiii. p. 547.

‡ *Electrician*, vol. lii. pp. 598 and 704. Cp. also *Journ. Inst. Electr. Eng.*, vol. xxxiii. pp. 564 and 568, and especially p. 1125.

CHAPTER XX

THE DESIGN OF CONTINUOUS-CURRENT DYNAMOS

§ 1. **Range of speeds in practical use.**—The practical art of designing is a matter of striking a balance between a variety of conflicting considerations, all of which are of importance in different degrees, and each of which will vitally affect the entire design of the machine. Thus a dynamo must be efficient, yet at the same time it must not be too costly to manufacture; it must be compact, yet well ventilated; thoroughly strong, yet not too heavy. Any one feature, however desirable in itself, will, if carried to excess, have some disadvantageous consequence in another direction, and he is the best designer who can effect a series of compromises such that, while each consideration is given its proper weight, none are forced into undue prominence, and a design well balanced as a whole results from his practised judgment.

Since the dynamo is primarily a generator of electric pressure, the first consideration must be the production of the requisite volts. This, being a question of the rate of cutting lines, will depend on the speed, the number of active wires, and the flux passing through the armature. In the majority of cases, for a given output, the speed may be taken as fixed: either it is directly specified, or it is to a great extent settled by recognised practice or questions of mechanical strength and durability. Thus, to take the case of an ordinary continuous-current dynamo, it will be driven by belt or ropes, or it will be directly-coupled to the prime mover. In the former case it will usually fall into one or other of two series, running at speeds which may by comparison be regarded as respectively high and low. In the latter case the prime mover may be either a steam-engine, steam-turbine, or less frequently a water-turbine or gas engine. When driven by a steam-engine the speed of the dynamo may again be classified as high or low, according as the engine is of the enclosed class with forced lubrication, or of the open type so largely used for marine or mill work. Or again when driven by a steam-turbine, although the speeds then reach their maximum they fall into two or three marked lines.*

* W. J. A. London, "Mechanical Construction of Steam Turbines and Turbo-Generators," *Journ. Inst. Electr. Eng.*, vol. xxxv. p. 163; A. G. Ellis, "Steam Turbine Dynamos," *ibid.*, vol. xxxvii. p. 307.

In the De Laval type two dynamos, each of half the output, are driven through toothed gearing at about one-tenth of the turbine speed, and the combination is not made for outputs above 200 kilowatts, while the Parsons and Curtis turbines are seldom made for continuous-current outputs above 500 kilowatts, and in the largest sizes are practically confined to alternators. Thus, although makers vary considerably in the speeds which they select for different outputs, yet there is sufficient agreement to enable a table to be drawn up giving average values for the speeds of the different classes.

Output in Kilowatts.	Speed in Revolutions per Minute.								
	Belt-driven.		Engine-driven.			Steam-Turbine.			
	High.	Low.	Enclosed.		Open.	De Laval geared dynamo.		Parsons' type.	Curtis type.
5	1200	750	800	Single-crank.	330	3000			
10	1000	700	750		300	2400			
15	900	650	700		275				
20	800	600	650		250	2000			
30	700	500	650	Two-crank.	330				
40	600	450	600		300	1500			
60	550	420	550		275				
100	480	...	475		250	1250			
150	430	...	450		225				
200	...	...	400		200	775			
300	...	...	350	Three-crank.	150	...			
500	...	...	300		120	...		2500	1750
750	...	...	275		100	...		2250	1400
1000	...	...	250		90	...		1900	1150
1500	...	...	200		85	...		1600	900
2000	...	...	...		...	...		1400	750
3000	...	...	...		...	...		1250	600
4000	...	...	...		...	...		1000	550
5000	...	...	...		...	...		900	500

§ 2. **Determination of length and diameter of armature core.**—Given the required output and speed for which a dynamo is to be designed, the determination of the best diameter and length of armature core, without reference to other designs or machines previously built, can only be effected by a method of trial and error. The revolutions per minute being fixed, the peripheral velocity which is usually regarded as permissible in the desired type of machine gives a rough idea of a reasonable diameter of core; at least, it fixes an upper limit which should not be exceeded without due consideration. The nature of the output will determine the number of poles that is likely to prove best. Assuming, then, a certain diameter of core D and an average value for the polar angle ϕ, we can estimate the polar arc, and, adopting a usual proportion of length of core in relation to the number of poles, we can calculate the effective area of the air-gap s_g, making allowance for the fringe (Chap. XV. § 6). With a normal value B_g for the air-gap density, $Z_a = B_g \cdot s_g$ is thus arrived at, and thence the requisite value of τ for the given speed follows of necessity. Allowing a density at the root of the teeth not exceeding 22,000 lines per square centimetre, a preliminary division of the armature circumference into slots and teeth can be made, and from the dimensions and number of the slots the size of wire that can be accommodated is found. The resistance of the armature from brush to brush can then be calculated. At this point the full-load current is a decisive factor, the output of any continuous-current dynamo being limited by the two considerations of heating and sparking, either or both. If, therefore, either the total watts lost in the armature or the ampere-wires per pole are found too great, the only course is to increase either the length or diameter of core, or both, the value of Z_a being increased, and the value of τ being proportionately reduced. By a series of trials such a mutual relation between τ and Z_a is reached as will give a suitable size of armature and a winding of practicable nature which will not overheat nor give rise to sparking.

But without passing through such a prolonged series of trials, a preliminary division of the product $\tau \cdot Z_a$ into two suitable factors may be obtained from the following considerations.

When the total electrical watts, $E_a \cdot C_a$, which are to be generated, are divided by the given speed expressed in revolutions per minute (the latter being strictly an angular velocity), the quotient $\frac{E_a \cdot C_a}{N}$ virtually gives us the total electrical resisting torque that has to be overcome. This torque is conveniently retained in the form of watts per revolution per minute, *i.e.* it is expressed in terms of a unit of torque which develops a power of one watt for every revolution per minute. This particular unit is easily found to be equal to $\frac{60}{2\pi} \times 10^7 = 9{\cdot}55 \times 10^7$ dyne-centimetres $= 0{\cdot}974$ kilogrammetres, or in British units $7{\cdot}05$ lb.-ft., so that when a machine is said to develop, *e.g.* 10 watts per revolution per minute, we mean that its torque has a value of ten of such units.

From equation (10) a second expression for the total electrical torque is obtained, namely, $\frac{2p \cdot Z_a \cdot A_w}{2\pi} \times 10^{-1}$ dyne-centimetres where $2p \cdot Z_a$ is the total flux into and out of the armatures at all the $2p$ poles, and A_w are the total ampere-wires $(\tau \cdot J)$ of the armature. In terms of the unit of torque described above, its numerical value is

$$\frac{2p \cdot Z_a \cdot A_w}{2\pi \times 10} \times \frac{2\pi}{60 \times 10^7} = \frac{2p \cdot Z_a \cdot A_w}{60 \times 10^8}$$

Equating the above two expressions for the torque,

$$\frac{E_a \cdot C_a}{N} = \frac{2p \cdot Z_a \cdot A_w}{60 \times 10^8}$$

The value of $\frac{E_a \,.\, C_a}{N}$ may at the outset be obtained to a close degree of accuracy from the known value of the *useful* electrical torque or watts *of output* per revolution per minute $=\frac{E_e \,.\, C_e}{N}$. The armature E.M.F. which must be induced at full-load bears a more or less constant relation to the terminal voltage E_e which is known. The loss of volts over the resistance of armature and brushes decreases in the shunt machine from about 10 per cent. of the terminal E.M.F. in small low-speed machines to 3 per cent. in large machines, and averages about 5 per cent., while at the same time the shunt current decreases similarly from 8 to 2 per cent. of the external current C_e, and averages about 3 per cent. Hence $E_a \,.\, C_a = 1{\cdot}05\, E_e \times 1{\cdot}03\, C_e = 1{\cdot}08\, E_e \,.\, C_e$. In the compound-wound or series-wound machine the additional loss of volts over the series winding counterbalances the reduction in or entire absence of the shunt current, so that in general $E_e \,.\, C_e = \frac{E_a \,.\, C_a}{1{\cdot}08}$ approximately, and

$$\frac{E_e \,.\, C_e}{N} = \frac{2p \,.\, Z_a \,.\, A_w}{1{\cdot}08 \times 60 \times 10^8}$$

From the results of many actual machines, curves connecting the values of the two factors, *i.e.* the total flux $2p \,.\, Z_a$, and the total ampere-wires A_w, respectively, with the watts per revolution per minute, may be plotted, and such curves are found to possess a very considerable degree of uniformity.* In practice, the total flux and the total ampere-wires both increase more or less similarly, as might naturally be expected, *i.e.* each increases as the square root of the torque. When plotted as ordinates against the machine torque as abscissa, each curve would then be a parabola with its axis on the abscissa axis,† and in fact such is roughly their shape. By the use of such curves, then, a provisional settlement of Z_a and of $\tau = \frac{A_w}{J}$ where $J = \frac{C_a}{q}$ can be made immediately.

§ 3. **The output per revolution per minute as proportional to the length of core and the square of its diameter.**—But a further step may also be made, which supersedes the necessity even for the above provisional determination. The calculations are simplified and shortened by an approximation which at once gives a preliminary guide to the size of machine as fixed by the two leading dimensions, length of core and diameter.

* For such curves, see R. Goldschmidt, *Journ. Inst. Electr. Eng.*, vol. xl. p. 457.

† *Electr. World*, vol. li. p. 419.

For Z_a may be substituted the equivalent expression

$$\beta \cdot \frac{\pi D_{\prime\prime}}{2p} \cdot L_{\prime\prime} \cdot B_g \times 6{\cdot}45$$

where $D_{\prime\prime}$ and $L_{\prime\prime}$ are in inches, and since $A_w = \pi D \cdot a_w$, where a_w = the ampere-wires per unit length of circumference, we have

$$\frac{E_e \cdot C_e}{N} = \frac{\pi^2 D_{\prime\prime}^2 L_{\prime\prime} \cdot \beta \cdot a_w \cdot B_g \times 6{\cdot}45}{1{\cdot}08 \times 60 \times 10^8}$$
$$= \beta \cdot a_w \cdot B_g \cdot D_{\prime\prime}^2 \cdot L_{\prime\prime} \times 10^{-8} \text{ nearly}$$

or since β, the ratio of the polar arc to the pole-pitch, varies but little from an average value of 0·735,

$$\frac{E_e \cdot C_e}{N} = 0{\cdot}72 \times 10^{-8} \times a_w \cdot B_g \cdot D_{\prime\prime}^2 L_{\prime\prime} \quad . \quad . \quad (152)$$

It has been already stated in Chap. XVIII. § 35 that a_w, as fixed by the limits of heating and sparking, tends to a more or less constant value, or at least does not vary greatly in armatures of somewhat similar size, so that it may be treated as constant over small ranges of variation. The same is equally true of B_g, so that finally the watts of output per rev. per min. are proportional to the square of the diameter and to the length of the core, or

$$\frac{E_e \cdot C_e}{N} \propto D^2 L$$

This important result requires a little further explanation. With an armature core of given diameter, if its length and that of the pole are similarly altered, all else remaining unchanged, the volts are altered in the same proportion. The ratio of the heat generated per unit length of the core to the cooling surface along that length is unaffected for the same current; yet as the end-connections which are best exposed to the air and play the greatest part in the cooling have not been altered, the amperes for the same rise of temperature might be slightly increased if the core has been shortened or may require to be decreased if the core has been lengthened. It is also evident that if commutation is chiefly dependent upon the presence of a reversing field, the ratio of B_c to the inductive voltage per unit length of core is unaltered, but in this case the fixed amount of inductance from the end-connections forms a gradually decreasing percentage of the whole as the core-length is increased, so that sparkless commutation would be benefited by increased length. If, on the other hand, the voltage is to remain the same, the number of active wires will be altered inversely to the length of core, and their area can then be either increased or decreased, so that their current-carrying capacity varies directly as the length, save in so far as there is a gain or loss in the space taken up by their insulation. Except, therefore, for secondary and indirect effects, the output per rev. per min. is proportional to the length of the core.

If now, while the length of the core is unaltered, its diameter is increased, more wires of the same size in similar slots can be wound on without affecting the ratio of the heat generated in them to the cooling surface; the increased cooling effect from the higher peripheral velocity is mainly offset by the greater eddy-current loss. But the flux will be equally increased in proportion to the greater diameter, so that the voltage is proportional to the square of the diameter. If the ampere-wires per pole have been thereby increased beyond the sparking limit, the increase in diameter will require to be accompanied by a further increase in the number of poles. The same general principle applies to an increase in the diameter of the smooth-core

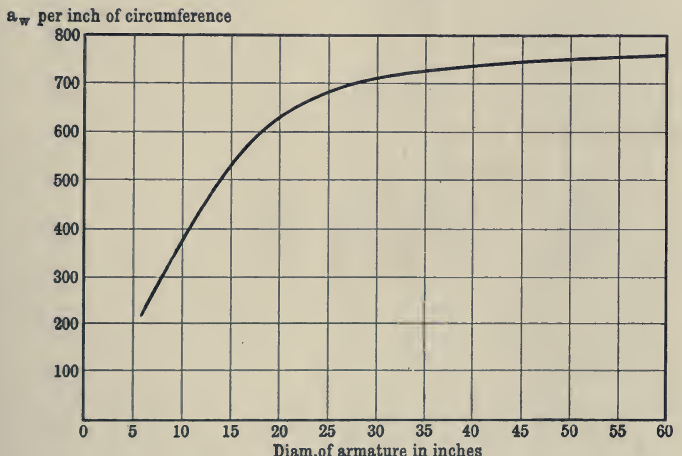

FIG. 393.—Normal values of a_w.

armature; if the number of active wires and the flux are correspondingly increased, the voltage will vary as the square of the diameter, but the increase in the ampere-wires per pole will necessitate an increase either in the number of poles or in the length of the air-gap. The latter change will again allow the radial depth of the copper to be increased, so that the current might from the heating point of view be increased still further. This is, however, only another way of stating the fact that a_w is in practice not really constant, but increases in large machines, and an exactly analogous influence is traceable in the case of toothed armatures. With larger diameters it becomes possible to increase the depth of the slots without reducing the section at the root of the tooth too much; the possible volume of armature copper can thus be improved even more than in proportion to the increased diameter. Thus as a preliminary generalisation it may be held that in all cases the watts per rev. per min. are $\propto D^2L$, but only so far as a_w is constant. The average values which a_w assumes, according to the diameter of the armature, are indicated in Fig. 393,* from which it will be seen that

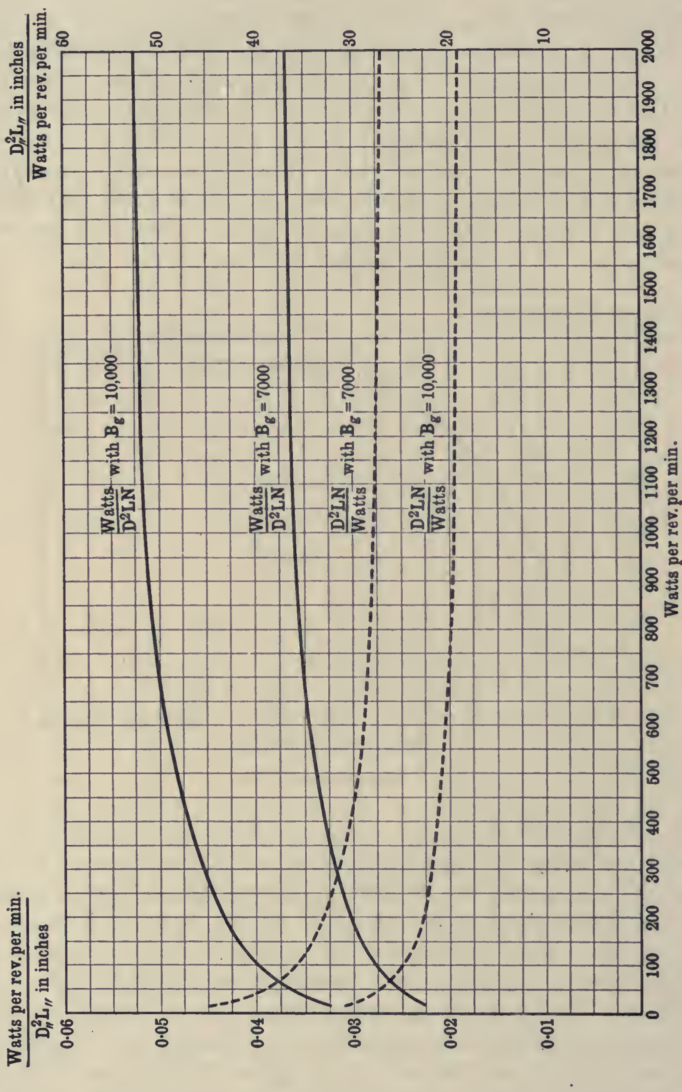

FIG. 394.—Torque constant and size constant of continuous-current dynamos.

in large machines the value of a_w reaches practically a constant maximum value. Moreover, it is not feasible to increase the depth of the slot

* Cp. an analogous curve giving the estimate of Dr. R. Pohl, *Journ. Inst. Electr. Eng.*, vol. xl. p. 242.

beyond certain limits, and for each value of B_g there must be a corresponding width of tooth to receive the flux. The two values of a_w and B_g then become simply dependent upon one another, since, if the flux-density is made very high, the slot must be made narrow and the area of copper is reduced, with consequent heating; if the density is reduced and the slot is made wide, the heating may be kept within bounds, but the tendency to spark is increased. We thus have

$$\frac{\text{Watts of output}}{D_{\prime\prime}{}^2 L_{\prime\prime} \cdot N} = 0{\cdot}72\, a_w \cdot B_g \times 10^{-8}$$

To express the meaning of the quantity $\frac{\text{watts}}{D^2LN}$ it may be described as the "dimensional torque constant," since it is the specific torque that can be obtained in relation to the dimensions of the machine, or indirectly to the cubic volume, *i.e.* it is the watts per rev. per min. that can be obtained per cubic inch (not strictly of the volume of the armature but) of the product of the square of its armature diameter and its length of core. The reciprocal of the dimensional torque constant or $\frac{D^2LN}{\text{watts}}$ is also often sued, and may be described as the "size constant" of a machine in relation to its specific torque, since it is the number of cubic inches that the D^2L of the armature must give for each watt per rev. per min. With definite conditions of rating as fixed by rise of temperature and sparklessness, both quantities are dependent upon the total watts per rev. per min., the one increasing as the size of dynamo increases owing to the better utilisation of space which is possible in large machines, and the other conversely decreasing. With good design under favourable conditions the values shown in Fig. 394 may be reached, although in small machines at voltages above 250, especially when the speed is low, it is not possible to obtain such high values for $\frac{W}{D^2LN}$, or such low values for its reciprocal.

§ 4. **Formulæ for approximate size of armature.**—With toothed drum armatures B_g ranges from 6500 in small to 9500 in large machines, while a_w without special commutating poles rises from 400 per inch in a small machine of 5 to 10 kilowatts up to 700 or 780 per inch in large machines with thoroughly good ventilation and favourable commutation conditions. The limiting values within this range are therefore approximately

$\frac{E_e \cdot C_e}{D_{\prime\prime}{}^2 L_{\prime\prime} N} = 0{\cdot}019$ up to $0{\cdot}052$ watts per rev. per min. per cubic inch of D^2L;

or $\frac{D_{\prime\prime}{}^2 L_{\prime\prime} N}{E_e \cdot C_e} = 52$ to 19 cubic inches of D^2L per watt per rev. per min.

If λ be the ratio $\frac{L}{D}$ for which suitable values have been given in Chap. XIV. § 18

$$\frac{D_{\prime\prime}^3\lambda N}{E_e \cdot C_e} = 52 \text{ to } 19 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (153)$$

$$\text{or } D_{\prime\prime} = 3{\cdot}73 \sqrt[3]{\frac{E_e \cdot C_e}{\lambda N}} \text{ to } 2{\cdot}67 \sqrt[3]{\frac{E_e \cdot C_e}{\lambda N}} \quad . \quad . \quad . \quad . \quad . \qquad (154)$$

By choosing such values between these two limits as will correspond to the capacity of the machine or its output per rev. per min., the designer can obtain a fairly accurate idea of the diameter and length of core which will be necessary. An average value for a_w in moderately large toothed armatures is about 600 per inch or 240 per cm. of circumference, but with commutating poles may be taken as high as 900 per inch or 350 per cm. of circumference.*

In such provisional determinations of the diameter and length of core considerable latitude may be permitted to the judgment and discretion of the designer; for within certain limits the relation between D and L may be varied without greatly affecting the cost or efficiency of the machine, although, as a general rule, as small a diameter as is practicable is to be preferred. In the technical office of a dynamo factory, however, the result is in most cases quickly arrived at by reference to a list of standard armatures, or at least of standard diameters of core discs, from which armatures of different lengths may be built up. After a number of machines have been standardised a table can be drawn out showing the maximum value which the useful torque expressed as $\frac{\text{watts of output}}{\text{revs. per min.}}$ has for each of the standard armatures, in continuous working under average conditions of voltage and speed.

§ 5. **Watts per rev. per min. for each size of armature core.**—In the use of any such table care is required to guard against the disturbing effect of certain secondary conditions which prevent the output per rev. per min. of any size of core from being strictly constant. These conditions are firstly the speed, secondly the permissible temperature, and lastly the voltage and nature of the winding, whether with one, two, or more turns per commutator sector, lap or wave. With regard to the first, it is evident that if the revolutions are exceptionally high, the eddy loss will assume such large proportions as to decisively limit the possible current that can be carried by a given winding on the armature. Although this is to a certain extent counterbalanced by

* For values of the product $a_w \cdot B_g$ in relation to D^2L, see the curves in the paper of Messrs. J. C. Macfarlane and H. Burge on "Output and Economy Limits of Dynamo-Electric Machinery," *Journal Inst. Elec. Eng.*, vol. xlii. p. 235. For average modern machines of large size, they give $a_w \cdot B_g = 8 \times 10^6$, whence the dimensional torque $= 0{\cdot}058$ watts per rev. per min.

the increased cooling due to the high peripheral speed, yet on the whole if the speed be high the watts per rev. per min. must be slightly reduced, or conversely with very low speeds the output, *ceteris paribus*, may be appreciably increased. Next, the permissible rise of temperature must be taken into consideration, and this rise may itself be greatly affected by an alteration of the nature or disposition of the winding, and its relative exposure to the air; or by a change from copper to carbon brushes on the commutator. But most important of all is the influence of the E.M.F. A high voltage implies a large number of active wires, with an increased thickness of insulation, the percentage loss of space in insulation is therefore much greater than in low-voltage machines, and the watts per rev. per min. are reduced. If the amperage is small, round wires may be necessitated, by which the ratio of the copper to the available area is very largely reduced as compared with the same armature wound with rectangular bars. In a small multipolar toothed machine with round wires the ratio of the copper area to the area of the slot may sink to as low as 0·25. In a low-voltage machine, say for 110 volts with rectangular bars, the ratio rises from 0·35 for small outputs at low speeds to 0·55 for large outputs; but here again, if the bars become very thick, it may be necessary to limit the number of bars per slot to two only in order that the width of opening may not be too great, when the ratio again sinks to 0·45. At 250 volts the ratio for normal speeds and outputs of 40 to 200 kilowatts ranges from 0·4 to 0·52; while at 500 volts it rises from 0·25 in very low-speed small machines with round wire to 0·3 in small machines of 50 to 80 kilowatts at moderate speeds, and to 0·5 in machines of 500 to 1500 kilowatts.

Obviously, the three conditions above summarised affect any rules as to the output of a machine such as those of eqs. (153–4), so that they can at best only be rough guides. Still, within wide limits each size of armature core may be regarded as corresponding to a certain "specific useful torque," and by the value of this its cost to manufacture is almost immediately determined with very fair accuracy. The value which the output watts per rev. per min., or the kilowatts at an assumed speed of 1000 revs. per min. have in any particular machine may therefore be regarded as determining its weight, cost, and commercial value in general.

§ 6. **Design of 4-pole dynamo with toothed armature for 80 KW.**—Suppose that it is required to design a dynamo with toothed drum armature to give 80 kilowatts or 350 amperes at 230 volts when directly coupled to a steam-engine running at 500 revs. per min. If the magnet has four poles, and a simple lap winding is adopted, the amperes to be collected at each brush arm will be $\frac{350}{2} = 175$, and in each parallel branch of the armature winding

$J = \frac{C_a}{q} = \frac{350}{4} = 87{\cdot}5$ amperes (cp. Chap. XI. § 23). A 4-pole field with simple lap-wound armature will therefore be very suitable.

The watts per rev. per min. are $\frac{E_e \,.\, C_e}{N} = \frac{80{,}000}{500} = 160$, and for ordinary values of B_g, say about 7500, the specific size constant will fall between the two curves for $\frac{D^2LN}{\text{watts}}$ in Fig. 394. A value of thirty may therefore be taken, whence

$$D_{\prime\prime}{}^2 L_{\prime\prime} = 30 \times 160 = 4800$$

or if, in consideration of the 4-pole field, we take for the ratio, $\lambda = \frac{L}{D}$, a value of 0·54,

$$D_{\prime\prime} = 3{\cdot}1 \sqrt[3]{\frac{160}{\lambda}} = 20{\cdot}6 \text{ inches, and } L_{\prime\prime} = 11{\cdot}1 \text{ inches.}$$

Thus in the present case the designer would look out in his table of standard machines the size of armature core suitable for a 4-pole machine, and corresponding most closely to 160 watts per rev. per min. We will assume that this leads him to a core of 21″ external diameter and 11″ long, or $D_{\prime\prime}{}^2 L_{\prime\prime} = 4850$.

The ratio of the polar arc to the pole-pitch, or β, being 0·735, the effective area of the air-gap will be approximately

$$\beta \,.\, \frac{\pi D_{\prime\prime}}{2p} \,.\, L_{\prime\prime} \times 6{\cdot}45 = 860 \text{ sq. cm.}$$

which with an air-gap density $B_g = 7500$ gives a total useful flux per pole $Z_a = 6{,}450{,}000$ lines.

The loss of volts over the resistance of the armature winding will be about $2\frac{1}{2}$ per cent., say 6 volts, over the positive and negative brushes 2 volts, and over the series winding of the field-magnet about 1 volt, so that the total induced armature E.M.F. must be $E_a = 239$ volts.

By equation (22)

$$E_a = 239 \text{ volts} = \tau \,.\, \frac{500}{60} \,.\, 6{\cdot}45 \times 10^{-2}$$

whence $\tau = 444$.

The total ampere-wires on the armature would then be $A_w = J\tau = 87{\cdot}5 \times 444 = 39{,}000$ approximately, and per inch of circumference $a_w = \frac{39{,}000}{\pi D_{\prime\prime}} = 590$, a suitable value.

The ampere-wires per slot by Fig. 375 should not exceed 720, giving $\frac{39{,}000}{720} = 54$ slots, and as the number of wires per slot must be a multiple of 2, the nearest combination would be 55 slots with 8 wires in each, in two layers of four abreast. At the same, since $J = 87{\cdot}5$, there should not be more than one turn per sector, so that the total number of commutator sectors N_2 would be $\frac{440}{2} = 220$, which does not give too thin a width of sector, and the number per slot would be 4. The width of the slot if open at the top will, however, approach closely to the maximum given in Chap. XIII. § 34; moreover, the choice of the rear pitch in slots is then limited to 13 or 12, of which the former is rather too long and approaches too nearly to the pole-pitch, and the latter is rather short in relation to the polar arc. It will therefore be preferable to increase the number of slots and reduce the number of sectors per slot. The next stage is 3 sectors per slot, or $\frac{444}{6} = 74$ slots, and six wires in each, three abreast. In designing-office practice the number of sections may have to be arranged to suit a list of standard commutators, and still more often standard slottings are in use for each size of core. An uneven number of slots is more commonly found with 4-pole dynamos, so that they may also be adapted for wave winding, and it will therefore be assumed that in the present instance a notching-wheel for 75 slots is available.

The total number of active wires will then be $75 \times 6 = 450$ with a commutator of 225 parts. The required flux will in correspondence thereto be reduced to $Z_a = 6{\cdot}38 \times 10^6$.

As a general rule the width and depth of slot, the air-gap, and all the main dimensions of the magnetic circuit are standardised and known to the designer, who from the drawings will have before him such details as are given in Fig. 279. But a few preliminary calculations will be worked out to indicate the method by which the main constructional data are reached. By Fig. 274 the depth of the slot will be, say, 1·4″, and the diameter at the bottom of the slot 18·2″. The number of teeth through which the flux of one field passes is $\frac{75}{4} \times 0{\cdot}735 =$ say 14, and allowing for three air-ducts each $\frac{1}{2}''$ wide in the core, and also for 10 per cent. of insulation between the core-discs, the net length of iron across the core is $9{\cdot}5 \times 0{\cdot}9 = 8{\cdot}55''$. A maximum value for the apparent induction at the root of the tooth being $B_{t2} = 23{,}000$, the necessary area is $\frac{6{,}380{,}000}{23{,}000} = 278$ square centimetres = 43 square inches. The width of 14 teeth at the bottom of the slots must therefore be $\frac{43}{8{\cdot}55} = 5{\cdot}02''$, and the width of a tooth $w_{t2} = \frac{5{\cdot}02}{14} = 0{\cdot}358''$. The width of the tooth-pitch at this diameter being $w_s + w_{t2} = \frac{18{\cdot}2 \times 3{\cdot}14}{75} = 0{\cdot}762''$, the width of the slot will be $0{\cdot}762 - 0{\cdot}358 = 0{\cdot}404''$, so that a value of 0·4″ will be adopted for w_s.

The armature discs will be mounted on the arms of a cast-iron hub, and their inside diameter may be made $10\frac{3}{4}''$, giving a radial depth *a* below the slots of 3·725″, and allowing free ventilating apertures between the arms of the hub. The net area of iron in the double cross-section which carries the flux Z_a is therefore $2ab = 2 \times 3{\cdot}725 \times 8{\cdot}55$ square inches = 412 square centimetres, and the maximum induction midway between the poles is $B_a = \frac{6{,}380{,}000}{412} = 15{,}500$.

The final design of the armature winding may now be proceeded with. By the rules of Chap. XIII. § 18, from the width of the slot must be deducted say $0{\cdot}075 + (0{\cdot}038 \times 3) = 0{\cdot}19''$, and the remainder $0{\cdot}4 - 0{\cdot}19 = 0{\cdot}21''$ divided by the number of bars abreast gives the permissible thickness of copper $= \frac{0{\cdot}21}{3} = 0{\cdot}07''$. From the depth of the slot is to be deducted $0{\cdot}160 + (0{\cdot}034 \times 2) = 0{\cdot}228''$, and the remainder divided by the number of layers, *i.e.* $\frac{1{\cdot}4 - 0{\cdot}228}{2} =$ say $0{\cdot}585''$. The area of the copper is therefore $0{\cdot}585 \times 0{\cdot}07 = 0{\cdot}041$ square inch, with a resistance $\omega' = 0{\cdot}0586$ ohm per 100 yards at 60° F.

Next must be calculated the amount by which the end-connections will project beyond the core at either end, and the mean length of a half-loop. The pole-pitch corresponds to $\frac{75}{4} = 18{\cdot}75$ slots, and in order to economise in the length of copper and to distribute the short-circuited sections among different slots, a moderate degree of short-chord-winding effect will be advantageous. The pitch at the engine end in slots will therefore be made nearly two slots short of the pole-pitch, or $y_R' = 17$ slots. Hence the half slot-pitch will be $\frac{y_R'}{2} = 8{\cdot}5$, and $w_s + w_{t2}$, measured at the bottom of the slots, is $0{\cdot}762''$, while $w_s + d =$ say $0{\cdot}4 + 0{\cdot}06$. The total axial projection at either end is therefore by equation (52)

$$l_e = \frac{8{\cdot}5 \times 0{\cdot}762 \times 0{\cdot}46}{\sqrt{(0{\cdot}762)^2 - (0{\cdot}46)^2}} + \tfrac{1}{2} + 1{\cdot}25 \times 0{\cdot}4 + 0{\cdot}7$$
$$= 4{\cdot}9 + 1{\cdot}7 = 6{\cdot}6''$$

The total length of one end-connector from slot to slot is by equation (53)

$$l' = 2(\sqrt{6{\cdot}95^2 + 4{\cdot}9^2} + 1.7)$$
$$= 2(8{\cdot}5 + 1{\cdot}7) = 20{\cdot}4''$$

m, reckoned on the mean circumference becoming 6·95″. The length of the core being 11″, the total length of a half-loop is $11 + 20{\cdot}4 = 31{\cdot}4$ inches $= 0{\cdot}872$ yard, and by equation (59) $R_a = \frac{450}{16} \times \frac{0{\cdot}872 \times 0{\cdot}0586}{100}$ $= 0{\cdot}0144$ ohm at 60° F.

Assuming that the final rise of temperature of the armature is about 60° F. above the temperature of the engine-room, and that the latter is

70° F., $70 \times 0{\cdot}225 = 15{\cdot}75$ per cent. must be added to the resistance of the armature at 60° F., or R_a hot $= 1{\cdot}157 \times 0{\cdot}0144 = 0{\cdot}0166$ ohm.

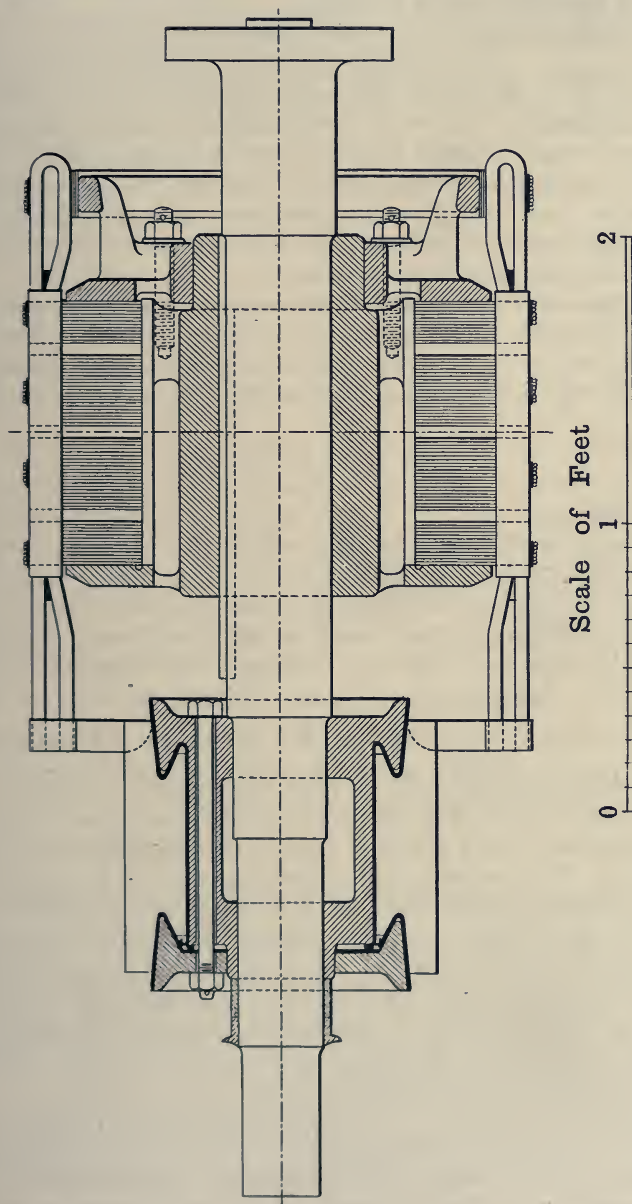

FIG. 395.—4-pole armature 21″ diameter × 11″ core length.

When an assumed value for the shunt current of 5 amperes is added to the external current, the loss of volts over the armature is $C_a R_a =$

$355 \times 0.0166 = 5.89$ volts, which is within our original allowance, and the loss of watts is $C_a^2 R_a = 355 \times 5.89 = 2090$. A sectional view of the completed armature is shown in Fig. 395.

The further design of the magnetic circuit can now be continued. With an air-gap of $\frac{5''}{16}$ since $\frac{w_{t1}}{w_s + w_{t1}} = \frac{0.48}{0.88} = 0.545$, the ratio $\frac{w_{t1}}{l_g} = \frac{0.48}{0.3125} = 1.52$ will permit of the use of solid pole-shoes (Chap. XIX. § 23). Their axial length is made $\frac{1}{2}''$ less than that of the armature core, in order to prevent lines from curving round into the flat surface of the core-discs and there generating eddy-currents, and the interpolar edges will be rounded off so as to give an effective polar arc of 63°, or rather less than has been before assumed.

The ampere-turns required over the armature core, teeth, and air-gaps are calculated, as in Chap. XV. § 9, where all the details have been worked out for the particular machine now in question. The back ampere-turns for an assumed angle of lead of 5° will be by eq. (87)

$$X_b = \frac{355}{4} \cdot 450 \times \frac{10}{360} = 1110$$

and $X_P = 13{,}600$.

On the assumption of a leakage coefficient $\nu = 1.16$, the number of lines passing through the magnet will be $Z_m = 1.16 Z_a = 7{,}405{,}000$; or from previous designs it may already be definitely known that $\mathfrak{S}_l = 60$. Any such preliminary estimate would in the actual course of design require to be subsequently verified. Taking a high value for B_m, say 17,000 in cast steel, so as to secure thorough stability, an area $\frac{7{,}405{,}000}{17{,}000} = 436$ sq. cm., or 67·6 sq. ins. is obtained. A solid cast-steel circular magnet-core will be most economical; a diameter of say $9\frac{1}{4}''$ may be taken, and will give a suitable amount of overhang of the pole-shoe in each direction beyond the circumference of the magnet-core. The necessary length and thickness of coil can then be determined from general principles, as has been done in Chap. XVI. § 7, and, as there pointed out, if the heating coefficient and mean rise are taken from the curve of Fig. 389 for undivided coils, practically the same result is reached for the total axial length and depth of winding as when the coils are divided into sections, but the latter will have the advantage of a lower mean rise for the same surface temperature and of greater economy in copper.

The general dimensions of Fig. 279 are thus determined upon, a rough sketch being made of the magnetic circuit; or the corresponding data may be already known from the standard magnet frame which is to be employed. The calculation of the ampere-turns required over the magnet-cores and yoke can then be completed, as in Chap. XV. § 9.

In the present case from Fig. 389 for the peripheral velocity of 2750 ft. per min. with coils divided by air-channels, the heating coefficient will be 75, or for a surface rise of 50° F., $\frac{50}{75} = 0{\cdot}66$ watt per sq. in. of cooling surface may be allowed.

The complete winding of shunt and series coils for the compound-wound dynamo on this basis has been worked out in detail in Chap. XVI. § 18. The loss of volts over the series winding is 1·19, which, together with the loss of 5·89 over the armature resistance, only slightly exceeds the assumed amount of $6 + 1 = 7$ volts.

Returning to the armature, its heating must be finally checked, and in order to calculate this, to the loss of watts over its resistance must be added the hysteresis and eddy losses. The mean diameter of the core below the teeth being $14{\cdot}475'' = 36{\cdot}8$ cm., the net volume of iron in the body of the core is $36{\cdot}8 \times \pi \times 206 = 23{,}800$ c. cm. For the density of $B_a = 15{,}500$ the joules per c. cm. per cycle by Fig. 136 are $0{\cdot}0015 = h$. The frequency is $\frac{pN}{60} = 16{\cdot}66$, so that by eq. (26*a*) the hysteresis loss in the core is $0{\cdot}0015 \times 16{\cdot}66 \times 23{,}800 = 595$ watts.

The volume of iron in the teeth of mean width $0{\cdot}42''$ is $1{\cdot}4 \times 0{\cdot}42 \times 75 \times 8{\cdot}55 \times 16{\cdot}38 = 6170$ c. cm., and the density at the tips of the teeth which will give the highest value is $B_{t1} = \frac{6{,}380{,}000}{371}$ $= 17{,}200$, for which h may be taken as $0{\cdot}0014$. Hence the loss in the teeth may be assumed to be

$$0{\cdot}0014 \times 16{\cdot}66 \times 6170 = 144 \text{ watts}$$

and the total hysteresis loss is $H_w = 739$ watts.

By eq. (141) the eddy-current coefficient for the iron core is

$$21^3\{5 \times 7{\cdot}6^2 \times 11 + 3 \times 12{\cdot}07^2\} \times 10^{-10} = 0{\cdot}00334$$

and for the copper bars is

$$80 \times 6^2 \times 75 \times \frac{1{\cdot}4 \times 0{\cdot}4}{6} \times 11 \times 21^{\frac{3}{4}} \times 0{\cdot}3125^{\frac{1}{4}} \times 2^{\frac{5}{4}} \times 10^{-10} = 0{\cdot}00038$$

whence $F = 0{\cdot}00372$ and $FN^2 = 930$ watts, of which 95 watts are to be assigned to the bars and 835 to the core. It is of interest to compare the latter with the figure of 220 watts which would be obtained from eq. (138) with discs $0{\cdot}025''$ thick and an average $B_a = 15{,}500$ assumed for the total iron volume of 29,970 c. cm.

The total loss in the armature is therefore by eq. (143)

$$W = C_a^2 \,.\, R_a + HN + FN^2 = 2090 + 740 + 930 = 3760 \text{ watts.}$$

The cooling surface reckoned as in Chap. XIX. § 19 is in sq. ins.

$$\begin{aligned} 3{\cdot}14 \times 21 \times (11 + 13{\cdot}2) &= 1600 \\ 3{\cdot}14 \times 15{\cdot}8 \times 13{\cdot}2 &= \underline{655} \\ & 2255 = S_c \end{aligned}$$

and for the peripheral velocity of 2750 ft. per min. the heating coefficient for degrees Fahrenheit is 28·5 by Fig. 390. The rise of temperature is therefore $t^{\circ} = \frac{28{\cdot}5 \times 3760}{2255} = 47{\cdot}5^{\circ}$ F. Our first assumption of 60° F. is therefore higher than necessary, and the loss of volts will be less. The original assumptions may therefore be allowed to stand.

The current to be collected by each brush arm at full load is $\frac{355}{2} = 177{\cdot}5$ amperes, and the area of contact surface of five carbon brushes each $1\frac{1}{2}''$ wide $\times \frac{3}{4}''$ thick will be $1{\cdot}125 \times 5 = 5{\cdot}62$ sq. in. The current-density will therefore be $\frac{177{\cdot}5}{5{\cdot}62} = 31{\cdot}6$ amperes per sq. in. The loss of volts by Fig. 342 with hard carbon brushes is then 1·8 over the two sets of brushes, provided the current is distributed uniformly over their surface, and of watts is 637. To allow for some want of uniformity in the distribution of current over the brush face, the watts may be increased 10 per cent., say to 710.

With a commutator diameter of 13″ the peripheral velocity $v_k' = 1700$ ft. per min. The total area of contact surface of all brushes is $5{\cdot}62 \times 4 = 22{\cdot}48$ sq. ins., and allowing a pressure p of $1\frac{1}{2}$ lb. per sq. in., the total pressure is $P = 1{\cdot}5 \times 22{\cdot}5 = 33{\cdot}8$ lbs. Taking the coefficient of friction μ as 0·4, the loss in watts from mechanical friction is by eq. (146)

$$0{\cdot}4 \times 33{\cdot}8 \times 1{\cdot}7 \times 22{\cdot}6 = 520 \text{ watts}$$

The external cylindrical surface of the commutator is $3{\cdot}14 \times 13 \times 9\frac{1}{4} = 377$ sq. ins., and the area of one face of the 225 lugs up to a height of 3″ from the commutator is $1\frac{1}{4} \times 3 \times 225 = 845$ sq. ins., making a total of 1222 sq. ins. The rise of temperature of the commutator will therefore be by eq. (147), $\frac{100 \times 1230}{1222(1{\cdot}6)} = 63^{\circ}$ F.

The heating question being thus satisfactorily concluded, it remains to consider whether the commutation will be good with fixed position of brushes.

The pitch of the sectors is $\beta = \frac{\pi \times 13}{225} = 0{\cdot}181''$. With brushes $\frac{3}{4}''$ wide and mica strips 0·025″ thick, the maximum number of sections short-circuited at a brush is $\left(\frac{b_1 - m}{\beta}\right)_+ = \frac{0{\cdot}75 - 0{\cdot}025}{0{\cdot}181} =$ almost exactly 4. Hence the short-circuited spool-sides in a pair of interpolar zones are as shown in Fig. 366 in the bottom row under the heading of $\frac{3}{4}$ as a remainder, occupying four slots and three slots respectively. Since the slots are open and filled to the top with the coils, $k_2 \cdot \frac{h_3}{w_3}$ does not appear, and by Fig. 366 k_1' and k_2' are respectively 25·11 and 6·28. Since $\frac{D}{2p \cdot w_3} = \frac{21}{4 \times 0{\cdot}4} = 13{\cdot}1$, and the slots are not very disproportionately filled, from Fig. 362, b_1 for the 4-slot zone may be taken as 3·6 and $b_1 \cdot j_{b1} = 3{\cdot}6 \times 8 = 28{\cdot}8 = k_1'''$; b_2 for the 3-slot zone may from Fig. 358 be taken as 5, so that $b_2 \cdot j_{b2} = 5 \times 8 = 40 = k_2'''$. Thence in eq. (115)

$$\lambda_1 = 25{\cdot}11 \times \frac{1{\cdot}4}{0{\cdot}4} + 28{\cdot}8 = 88 + 28{\cdot}8 = 116{\cdot}8$$

$$\lambda_2 = \ 6{\cdot}28 \times \frac{1{\cdot}4}{0{\cdot}4} + 40 = 22 + 40 = 62$$

and

$$l(\lambda_1 + \lambda_2) = 11 \times 2{\cdot}54 \times 178{\cdot}8 = 5000$$

The periphery of a packet of four coil-sides is approximately $2(0{\cdot}7 + 0{\cdot}6) = 2{\cdot}6''$, and the equivalent diameter is $d_s = \frac{2{\cdot}6}{\pi} = 0{\cdot}827''$. The total length of an end-connection has already been calculated as $l' = 20{\cdot}2''$. By eq. (120)

$$\lambda' = \left(4{\cdot}6 \log \frac{20{\cdot}2}{0{\cdot}827} - 0{\cdot}9\right) \times 4 = 21{\cdot}9$$

and

$$2l'\lambda' = 40{\cdot}4 \times 2{\cdot}54 \times 21{\cdot}9 = 2250$$

Therefore

$$L + \Sigma \, . \, M = (5000 + 2250) \times 10^{-9} = 7250 \times 10^{-9} \text{ henrys}$$

$$J = \frac{355}{4} = 88{\cdot}75, \text{ and } T = \frac{5}{v_k}(b_1 - m) = \frac{5}{1700}(0{\cdot}75 - 0{\cdot}025) = 0{\cdot}00213$$

second, whence

$$\frac{2J(L + \Sigma M)}{T} = 0{\cdot}604 \text{ volt.}$$

For a ratio $\frac{c}{l_g}$ nearly $= 8$, the factor by which the normal l_g must be multiplied to give the effective air-gap at the centre of an interpolar zone is $12{\cdot}3 \times 1{\cdot}2 = 14{\cdot}75$, and $l_{gx} = 0{\cdot}3125 \times 14{\cdot}75 = 4{\cdot}61'' = 11{\cdot}7$ cm. The cross induction is therefore $B_q = \frac{1{\cdot}257 J \cdot \tau}{2p \cdot 2l_{gx}} = \frac{1{\cdot}257 \times 88{\cdot}75 \times 450}{4 \times 2 \times 11{\cdot}7}$ $= 535$, and the volts of one section due to the cross field are

$$\frac{\tau}{N_2} \cdot B_q \cdot l \cdot v \times 10^{-8} = 2 \times 535 \times 11 \times 2{\cdot}54 \times \frac{2750}{60} \times 30{\cdot}48 \times 10^{-8}$$
$$= 0{\cdot}417 \text{ volt}$$

The total difference of E.M.F. between the two edges of a brush is therefore by eq. (124)

$$\Delta E_1 = 4(0{\cdot}417 + 0{\cdot}604) = 4{\cdot}08 \text{ volts}$$

which is within the permissible limit.

While the various items have above been given in full, the result may at once be obtained from eq. (126); thus

$$\Delta E_1 = 0{\cdot}026 \times 4 \times 88{\cdot}75 \times 2 \times 500\left(12{\cdot}57\frac{11 \times 2{\cdot}54 \times 450}{4} \cdot \frac{21}{0{\cdot}3125 \times 14{\cdot}75}\right.$$
$$\left. + 2 \times \frac{13}{0{\cdot}725} \times 7250\right) \times 10^{-9}$$
$$= 1{\cdot}67 + 2{\cdot}41 = 4{\cdot}08 \text{ volts.}$$

The ratio $\dfrac{X_g + X_t}{A_w \text{under pole}} = \dfrac{12070}{7250} = 1{\cdot}66$ is also a safe value, and sparkless running with a fixed brush position will be secured.

In conclusion, the mechanical strength of the binding-wires may be considered. The force acting on the cross-section of the binding-wires due to the centrifugal force of the copper bars in the slots within the length of the core is by eq. (58)

$$f_c = 7{\cdot}5 \times 450 \times 0{\cdot}041 \times 11 \times 19{\cdot}6 \times 500^2 \times 10^{-7} = 745 \text{ lbs.}$$

Taking a non-magnetic steel wire 0·040″ in. diameter with an ultimate breaking strength of 200,000 lbs. per sq. in., and three bands, two of $\frac{3}{4}''$ width in the centre and one of $\frac{3}{8}''$ width at each end, the number of wires will be $\dfrac{3 \times 0{\cdot}75}{0{\cdot}04} = 56$, and their sectional area $56 \times 0{\cdot}00125 = 0{\cdot}07$ sq. in. The apparent factor of safety is then $\dfrac{200{,}000 \times 0{\cdot}07}{745} = 18{\cdot}8$, which is more than ample.

Along the bent ends of the winding the copper section is increased in the ratio $\dfrac{\sqrt{m_t^2 + l^2}}{l} = \dfrac{\sqrt{6{\cdot}95^2 + 4{\cdot}9^2}}{4{\cdot}9} = \dfrac{8{\cdot}5}{4{\cdot}9}$, and since the axial projection beyond either core-end is 6·6″, we have at *each* end $f_c = 745 \times \dfrac{8{\cdot}5}{4{\cdot}9} \times \dfrac{6{\cdot}6}{11} = 775$ lbs., or more than along the core. A single band at each end $1\frac{1}{2}''$ wide, and composed of 33 steel wires 0·045″ diam. will give 0·0525 sq. ins. and may be reinforced by making each of the two bands on the ends of the core $\frac{7}{8}''$ wide, and taking $\frac{1}{2}''$ as assisting in retaining the end-winding. The additional wires will thus add at each end $\dfrac{0{\cdot}5}{0{\cdot}04} \times 0{\cdot}00125 = 0{\cdot}0156$ sq. ins., and the apparent factor of safety is $\dfrac{200{,}000 \times (0{\cdot}0525 + 0{\cdot}0156)}{775} = 17{\cdot}6$. In each case the real factor of safety is less than the apparent by an amount depending on the initial tension under which the bands are put on.

The over-all efficiency at full-load is $\dfrac{80{,}000}{86{,}447} = 92\frac{1}{2}$ per cent., the various losses being as follows:—

	Watts.	Percentage of Output.
$C_a^2R_a$	2090	2·62
Hysteresis	740	0·925
Eddy currents	900	1·12
Electrical loss over brushes	710	0·89
Brush friction	520	0·65
Bearing- and air-friction, say	250	0·31
Shunt-winding	820	1·025
Series-winding	417	0·52
Total losses	6447	8·06
Output	80,000	...
Input	86,447	...

The weight of the armature complete is approximately 16¼ cwts., and of the magnet 27 cwts. without base-plate.

§ 7. **Design of 4-pole dynamo with toothed armature for 165 KW.**—Next may be taken the design of a multipolar dynamo having a normal output of 165 kilowatts, and suitable for direct coupling to a high-speed enclosed engine running at 450 revs. per min. The dynamo is to be used both for lighting and for traction purposes, being shunt-wound in the former case and compound-wound in the latter. It has further to comply with the following conditions; when working as a shunt machine, it must give its full output of 165 kilowatts for any load between the limits of 480 volts, 344 amperes, and 520 volts, 317 amperes; it must also be capable of giving 460 volts and 360 amperes, but in this case and for any smaller load at 460 volts the speed may be reduced by 5 per cent., or say to 425 revs. per min., in order to maintain the stability of the magnetism. On a traction load the machine is to give 500 volts at no-load, and to be over-compounded to give 550 volts with the full-load of 300 amperes, while it must further be capable of giving an overload of 20 per cent., or 198 kilowatts made up of 500 volts, 395 amperes for at least 15 minutes without sparking at the brushes or any undue heating. After a continuous run of six hours with either of the two normal full-loads the temperature of the surface of the armature or field when measured on bare copper must not exceed that of the surrounding air by more than 70° F. The shunt regulating resistance and switch is to be large enough to enable all the above conditions of load to be obtained by alteration of the field strength under any normal conditions of temperature of the engine-room and of the dynamo itself; but when the switch is set to give 500 volts at no-load, it must not be used to assist in obtaining the required compounding effect. Lastly, in parallel with the series winding is to be placed a regulating switch and resistance, by means of which the amount of the over-compounding may be raised in four or five steps while the machine is working. A machine to comply with such varied conditions and to work over so large a range of voltage will introduce several of the nicer problems in the art of dynamo design.

The type of dynamo most suitable for the present case will be a 4-pole machine with a lap-wound barrel-wound armature and as many sets of brushes as there are poles. The maximum current being nearly 400 amperes, four sets of brushes will give 100 amperes per armature path, or 200 amperes to be collected per brush arm, which will lead to a reasonable length of commutator. The watts per rev. per min. at the normal full-load are $\frac{165{,}000}{450} = 366$, or with the maximum volts and maximum current $\frac{189{,}000}{450} = 420$; but since the maximum current has to be commuted sparklessly with the field at its weakest, the machine must be somewhat large for its normal full-load, and its specific torque may be reckoned as, say, 480 watts per rev. per min. From Fig. 394 for a strong field the "size constant" may be taken as 21, whence $D_{''}{}^2 L_{''} = 21 \times 480$. Assuming $\lambda = \frac{L}{D} = 0{\cdot}54$, $D_{''} = 2{\cdot}76 \sqrt[3]{\frac{480}{0{\cdot}54}} = 26\tfrac{1}{2}''$, and $L_{''} = 14{\cdot}3''$.

The leading dimensions of the armature will therefore be taken as 27″ diameter × 14″ length of core between the end-plates.

The maximum flux will correspond to the full traction load, and allowing a loss of $2\tfrac{3}{4}$ per cent. over the armature, brushes, and series winding, or 15 volts on 550, $E_a = 565$ volts. If $\beta = 0{\cdot}7$, or the polar arc be 63° with rounded interpolar edges, and the air-gap be assumed for the present to be $l_g = \tfrac{1}{4}''$, increasing slightly at the edges to reduce the magnetic humming, a preliminary estimate of the air-gap area gives it roughly as $(15 + 2l_g) \times 13{\cdot}5 = 210$ sq. in. or 1352 sq. cm. With $B_g = 10{,}000$, the maximum $Z_a = 13{\cdot}52 \times 10^6$ C.G.S. lines, and by eq. (22), allowing for a small drop in speed to, say, 445 revs. necessitated by the range of the governor,

$$\tau \times \frac{445}{60} \times 13{\cdot}52 \times 10^{-2} = 565 \text{ volts}$$

whence $\tau = 562$.

In order to obtain 8 bars per slot in two layers and 5 equalising rings, $\tau = 560$ will be chosen, distributed among 70 slots with 280 commutator sectors, and Z_a becomes $13{\cdot}6 \times 10^6$. The ampere-wires per in. of circumference are then $a_w = \frac{560}{84{\cdot}8} \times \frac{347}{4} = 572$, which by Fig. 393 is rather low; but as this value holds with the weakest field, and the insulation for 550 volts must be taken into consideration, it will be found suitable. The ampere-wires per slot are $8 \times \frac{346}{4} = 690$, which by comparison with Fig. 375 will also allow sufficient margin for the 550-volt insulation.

By Fig. 274 an average depth of slot will be $1\frac{5}{8}''$, and the pitch at the root of the teeth will be $1{\cdot}06''$. The width of tooth at the root must be such that the maximum uncorrected $B_{t2} = 23{,}000$ with $Z_a = 13{\cdot}6 \times 10^6$, *i.e.* an iron area of at least 590 sq. cm. is required. With three ventilating ducts each $\frac{1}{2}''$ wide, the net iron length of core is $12{\cdot}5 \times 0{\cdot}9 = 11{\cdot}25''$, and there are $12\frac{1}{4}$ teeth under the polar arc, or with 10 per cent. increase to allow for spreading of the field $13{\cdot}4$. Thence $w_{t2} \times 11{\cdot}25'' \times 13{\cdot}4 \times 6{\cdot}45 = 590$, or $w_{t2} = 0{\cdot}605$, say $0{\cdot}61''$. The width of slot $= 1{\cdot}06 - 0{\cdot}61 = 0{\cdot}45''$. Allowing $0{\cdot}075''$ for the double thickness of the insulating lining of the slot, and $0{\cdot}035''$ for the insulation on each of the 4 bars abreast, the total insulation thickness per slot is $0{\cdot}215$, or say $0{\cdot}230$, leaving $\frac{0{\cdot}45 - 0{\cdot}23}{4} = 0{\cdot}055''$ for the width of each copper bar.

With a wooden wedge $0{\cdot}175''$ deep, $0{\cdot}160''$ depth of insulating wrappings, and $0{\cdot}045'' \times 2$ for the insulation of the two layers of deep bars, the depth of bar becomes $\frac{1{\cdot}625 - 0{\cdot}425}{2} = 0{\cdot}6''$.

The area of bar $0{\cdot}6''$ high $\times$ $0{\cdot}055''$ thick $= 0{\cdot}033$ sq. in., and the resistance per 100 yds. at 60° F. $= \frac{0{\cdot}0024}{0{\cdot}033} = 0{\cdot}0727$ ohm. At the engine-end the loops will be bent over without joints.

A sufficient amount of chording will be given by making the rear slot-pitch $y_R' = 16$, and $\frac{y_R'}{2}(w_s + w_{t2}) = 8(0{\cdot}45 + 0{\cdot}61)$, while $w_s + d =$ say $(0{\cdot}45 + 0{\cdot}05)$. Thence by eq. (52)

$$l_e = \frac{8 \times 1{\cdot}06 \times 0{\cdot}5}{\sqrt{1{\cdot}12 - 0{\cdot}25}} + 0{\cdot}5 + 0{\cdot}562 + 0{\cdot}725$$

$$= 4{\cdot}57 + 1{\cdot}787, \text{ say } 6\tfrac{3}{8}''$$

On the circumference at the centre between the two layers, $m_t = 8 \times 1{\cdot}13 = 9{\cdot}05$, and by eq. (53) the copper length from slot to slot is $2\{\sqrt{82 + 21} + 1{\cdot}787\} = 24''$. The length of a half-loop is thus $24'' + 14'' = 1{\cdot}055$ yd., and

R_a hot $= \frac{560}{16} \times 1{\cdot}055 \times 0{\cdot}000727 \times 1{\cdot}18 = 0{\cdot}0316$ ohm with an engine-room temperature of 70° F., and a surface-rise of 70° F. above this.

To collect the current, 5 carbon brushes on each arm, each $1\frac{1}{2}''$ wide $\times$ $\frac{3}{4}''$ thick, will give $5{\cdot}62$ sq. ins., and current-densities of $\frac{347}{2 \times 5{\cdot}62} = 31$ on the normal lighting load, of $\frac{304}{11{\cdot}24} = 27{\cdot}1$ on the traction load, and a maximum of $\frac{400}{11 \cdot 24} = 36$ amperes per sq. in. on the overload,—sufficiently low values to ensure cool and satisfactory collection, if the sparking constants are also reasonable. With a commutator diameter of 18″, and mica strips 0·03″ thick, each copper bar will be 0·172″ thick at the top, and the connecting lugs must be riveted on at the side of each strip. By eq. (123), $\frac{0 \cdot 75 - 0 \cdot 03}{0 \cdot 202} = 3 \cdot 56$, or the number of sections simultaneously short-circuited by a brush is $S_k = 4$. A coil when in the position of greatest inductance has three simultaneously short-circuited bars by its side in each slot, and in each interpolar zone the short-circuited coil-sides are spread over three adjacent slots with unsymmetrical distribution, one outer slot having 4 bars short-circuited, and the remaining two having two each, or 8 in all in each zone. The height of the slot below the wedge is $h_s = 1 \cdot 45''$, and of the wedge is $h_3 = 0 \cdot 175''$, while the mean width of the wedge is $w_3 = 0 \cdot 544''$. The ratio $\frac{D}{2p \times w_3} = \frac{27}{4 \times 0 \cdot 45} = 15$, and the interpolar zones being alike, the value of b in each case by the lower curve of Fig. 358 is 5·6; or since the outer slot contains twice as many short-circuited bars as either of the other two, it will be a little higher, say 6. We thus have

$$\begin{array}{llr|llr}
 & & \lambda_1 & & & \lambda_2 \\
k_1' \cdot \frac{h_s}{w_s} & = 4(2 \cdot 09 + 6 \cdot 28) \times \frac{1 \cdot 45}{0 \cdot 45} & = 108 & k_2' \cdot \frac{h_s}{w_s} & = 4 \times 2 \cdot 09 \times \frac{1 \cdot 45}{0 \cdot 45} & = 27 \\
k'' \cdot \frac{h_3}{w_3} & = 4 \times 12 \cdot 57 \times \frac{0 \cdot 175}{0 \cdot 544} & = 16 \cdot 1 & & & = 16 \cdot 1 \\
k''' & = b \cdot j_b = 6 \times 8 & = \underline{48} & & & = \underline{48} \\
 & & 172 \cdot 1 & & & 91 \cdot 1
\end{array}$$

and $l\,(\lambda_1 + \lambda_2)$ in cm. units $= 14 \times 2 \cdot 54 \times (172 \cdot 1 + 91 \cdot 1) = 9360$.

The periphery of the packet of 4 bars with intervening insulation is calculated as follows:—

Width of copper	$4 \times 0 \cdot 055 = 0 \cdot 22$
„ insulation . . .	$3 \times 0 \cdot 03 = 0 \cdot 09$
Height of bar	$0 \cdot 60$
	$0 \cdot 91$

Total periphery $= 0 \cdot 91 \times 2 = 1 \cdot 82''$ and diameter of equivalent circle $d_s = \frac{1 \cdot 82}{3 \cdot 14} = 0 \cdot 58''$. Thence by eq. (120)

$$\lambda' = (4 \cdot 6 \,.\, \log \frac{24}{0 \cdot 58} - 0 \cdot 9) \times 4 = 26 \cdot 2$$

and

$$2l'\lambda' = 2 \times 24 \times 2 \cdot 54 \times 26 \cdot 2 = 3200$$

Since $\frac{c}{l_g} = \frac{3 \cdot 18}{0 \cdot 25} = 12 \cdot 7$, by Fig. 318, for $\gamma = 140°$, the multiple to give l_{gx}

is $19{\cdot}5 \times 1{\cdot}5 = 29{\cdot}2$, and $l_{gx} = 29{\cdot}2 \times 0{\cdot}25 = 7{\cdot}3''$. By eq. (126) therefore as a shunt machine for $J = \frac{347}{4}$

$$\Delta E_1 = 0{\cdot}026 \times 4 \times 86{\cdot}75 \times 2$$

$$45 \times 4 \left(12{\cdot}57 \frac{14 \times 2{\cdot}54 \times 560 \times 27}{4 \times 7{\cdot}3} + \frac{2 \times 18 \times 12{,}560}{0{\cdot}72}\right) \times 10^{-9}$$

$$= 1{\cdot}86 + 5{\cdot}05$$

$$= 6{\cdot}91 \text{ volts.}$$

Under the normal traction load this is reduced to $6{\cdot}91 \times \frac{304}{346} = 6{\cdot}06$, so that the conditions for a fixed brush position are fairly well attained.

Since $\frac{w_{t1}}{w_s} = \frac{0{\cdot}76}{0{\cdot}45} = 1{\cdot}69$, and $\frac{w_s}{l_g} = \frac{0{\cdot}45}{0{\cdot}25} = 1{\cdot}8$, m from Fig. 273 $= 1{\cdot}11$, and by eq. (67) $X_g = 0{\cdot}8 \times \frac{Z_a}{1352} \times 2 \times 1{\cdot}11 \times 0{\cdot}25 \times 2{\cdot}54 = 833\, Z_a \times 10^{-6}$.

With $Z_a = 13{\cdot}6 \times 10^6$ on the normal traction load, $B_g = 10{,}050$, and $X_g = 11{,}320$. The area of iron in the teeth at the tip, centre, and root being 740, 668, and 595 sq. cm., the inductions are—

$B_{t1} = \frac{13{,}600{,}000}{740} \times 0{\cdot}84 = 15{,}400$, corresponding to 24 ampere-turns per cm. length of path

centre $B_t = \frac{13{,}600{,}000}{668} \times 0{\cdot}95 = 19{,}300$, corresponding to 150 ampere-turns per cm.

and uncorrected $B_{t2} = \frac{13{,}600{,}000}{595} = 22{,}900$.

Since $\frac{w_{t2}}{w_s} = 1{\cdot}355$, the corrected density at the root by Fig. 277 is 21,900, corresponding to 900 ampere-turns per cm. by Fig. 278. Thence by eq. (69)

$$X_t = \frac{24 + 4 \times 150 + 900}{6} \times 2l_t$$

$$= 254 \times 2 \times 1{\cdot}625 \times 2{\cdot}54 = 2100$$

and $X_g + X_t = 13{,}420$.

On the normal lighting load as a shunt machine 12·8 volts are lost over the armature and brushes, and the necessary flux to give an armature E.M.F. of 492·8 volts at 445 revs. per min. is $Z_a = \frac{492{\cdot}8 \times 60 \times 10^8}{560 \times 445} = 11{\cdot}85 \times 10^6$ lines; $B_g = 8750$, and $X_g = 9860$. On carrying out the same calculations as above for the teeth,

$B_{t1} = 13{,}400$ corresponding to 9 ampere-turns per cm.
Centre $B_t = 16{,}800$ " 55 " "
$B_{t2} = 20{,}000$
corrected $= 19{,}750$ } " 176 " "

whence $X_t = 67{\cdot}5 \times 8{\cdot}25 = 557$ ampere-turns, and $X_g + X_t = 10{,}417$.

The ampere-wires per pole in the latter case as a shunt machine are $\frac{J \cdot \tau}{2p} = \frac{87 \times 560}{4} = 12,180$, and under the pole are $12,180 \times \beta = 8520$; the ratio $\frac{X_g + X_t}{A_w \text{ under pole}} = \frac{10417}{8520}$ is therefore only 1·22, and by the rules of Chap. XVIII. § 36 it would be advisable to increase this, so as to lessen the distortion and render a fixed brush-position more secure.

The air-gap will therefore finally be increased to $\frac{9}{32}'' = 0·281$, and $m = 1·1$. The axial length of the pole-face being made $\frac{1}{2}''$ shorter than that of the armature core, $\frac{a}{l_g} = 0·89$, and by Fig. 263, $K_1 = 1·1$. The width of pole-face $L_f = 13·5$, and by Fig. 267, $K_3 = 0·25$ for $\frac{w_d}{l_g} = \frac{0·5}{0·281} = 1·78$. From Fig. 265 for $\frac{c}{l_g} = 11·3$, and a sloping edge to the pole making an angle of about 140° to the core $K_2 = 2·5$. The virtual area of the air-gap is therefore $(13·5 - 0·375 + 0·31)(15 + 0·7) = 211$ sq. in. $= 1360$ sq. cm., as against 1352 in our first approximate calculation. Finally, therefore, $X_g = 925\ Z_a \times 10^{-6}$, and the ratio $\frac{X_g + X_t}{A_w \text{ under pole}}$ is raised to $\frac{11,500}{8520} = 1·35$. On the traction load $X_g + X_t$ correspondingly becomes $12590 + 2100 = 14,690$.

The inside diameter of the armature discs may be made 13″, giving a radial depth below the teeth of $5\frac{3}{8}''$, and an effective area of iron core $5\frac{3}{8} \times 11·25 \times 2 \times 6·45 = 780$ square centimetres, and a maximum $B_a = \frac{13,600,000}{780} = 17,420$. The discs will be carried on a cast-iron hub with 4 arms, and in order to relieve the shaft from the torsional strains due to rapid changes of load the hub has a cylindrical prolongation cast on it by which it is bolted directly to the fly-wheel of the engine with 6 coupling bolts on a large pitch circle.

We are now in a position to calculate the eddy loss in the two cases by eq. (141). On the traction load

$$F = 19,600\,(5 \times 10^2 \times 14 + 3 \times 14·69^2) \times 10^{-10}$$
$$+ 80 \times 5·9^2 \times 70 \times \frac{0·45 \times 1·625}{8} \times 14 \times 11·8 \times 0·73 \times 2·38 \times 10^{-10}$$
$$= 0·01495 + 0·00051 = 0·0155$$

The eddy watts are therefore $FN^2 = 0·0155 \times 445^2 = 3070$.

The mean diameter of the core below the teeth being $18\frac{3}{8}''$, the net volume of iron is $18·375 \times 2·54 \times \pi \times 390 = 57,200$ cubic cm. In the teeth the iron volume is $1·625 \times 0·685 \times 11·25 \times 70 \times 16·38 = 14,380$ cubic cm. The frequency is $\frac{pN}{60} = \frac{2 \times 445}{60} = 14·8$, and the coefficient h, for the given inductions may be taken from Fig. 136 as 0·0014 and 0·0012. The total hysteresis loss is

$$\begin{aligned} 0{\cdot}0014 \times 14{\cdot}8 \times 57{,}200 &= 1184 \\ 0{\cdot}0012 \times 14{\cdot}8 \times 14{,}380 &= \underline{\ \ 254} \\ & \ 1438 \text{ watts} = \mathrm{H}_w \end{aligned}$$

The sum of the armature losses on the traction load is therefore

$$\mathrm{C}_a^2\mathrm{R}_a + \mathrm{H}_w + \mathrm{FN}_2 = 2900 + 1438 + 3070 = 7410$$

On the normal lighting load the copper loss is considerably increased to 3830, but the eddy loss is decreased in nearly the same proportion, F becoming 0·0114. On the other hand, the hysteresis loss in the core is increased, and h may be taken as 0·0015 for both the body of the core and the teeth. The sum is therefore increased to $3830 + 1590 + 2250 = 7670$.

The peripheral speed of the armature being 3150 ft. per min., the heating coefficient from Fig. 390 is 25, and the cooling surface is

$$\begin{aligned} 3{\cdot}14 \times 27 \times (14 + 13) &= 2285 \\ 3{\cdot}14 \times 20{\cdot}8 \times 13 \qquad &= \underline{\ \ 850} \\ & \ 3135 = \mathrm{S}_c \end{aligned}$$

The calculated rise of temperature is therefore on the lighting load,

$$\frac{25 \times 7670}{3135} = 61{\cdot}2^\circ \text{ F}$$

which would again be increased on the maximum current of 360 amperes, so that the design of armature may stand.

The loss of volts over the two sets of brushes may be taken from the upper curve of Fig. 343 for hard carbon, and the following table is reached for the different loads.

	C_a	Loss of Volts over Brushes.	Loss of Volts over Armature.	$\mathrm{C}_a(\mathrm{R}_a + \mathrm{R}_b)$.
Normal lighting loads	348	1·8	11	12·8
	321	1·76	10·15	11·91
Maximum lighting load	364	1·82	11·5	13·32
Normal traction load	304	1·75	9·6	11·35
Overload	400	1·86	12·62	14·48

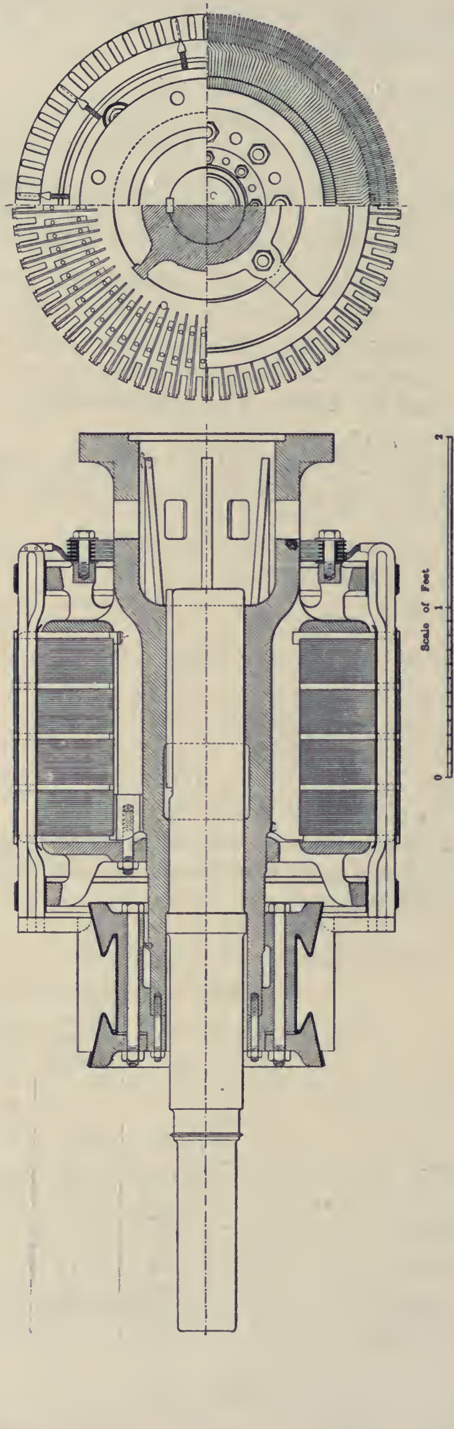

FIG. 396.—Armature of 165-kilowatt dynamo.

The necessary cross-section for the magnet which is to be of cast steel will require careful consideration, owing to the long range of volts over which the dynamo is to work. At the lowest flux the magnetism must not become unstable; while at the upper limit the exciting power must not be unreasonably great, and the proportion of the series to the shunt winding must not be too large. With the maximum flux of $13{\cdot}6 \times 10^6$ lines, $B_a = 17{,}420$, and $f'(B_a) = 75$ from the lower curve of Fig. 131; for $\frac{l_a}{2}$ may be taken the same value as c, or half the interpolar gap measured on the armature core, *i.e.* $3{\cdot}18 \times 2{\cdot}54 = 8{\cdot}06$ cm.; hence $\frac{X_a}{2} = 605$. The ampere-turns over the teeth and air-gap have already been calculated, and $\frac{X_g + X_t}{2} = 7345$. Assuming a brush position with 5° of lead, $\frac{X_b}{2} = \frac{76 \times 560 \times 5}{360} = 590$, and the sum of the four items is $\frac{X_P}{2} = 8540$.

From eq. (73) with deep laminated pole-shoes,

$$\mathcal{P}_l = 12{\cdot}4 \times \frac{14 \times 4}{27} + 12{\cdot}8 + 2{\cdot}25 \times 14 + 0{\cdot}77 \times 27 = 91, \text{ say } 100$$

The leakage lines are therefore $\zeta = 1{\cdot}257 \times 8540 \times 2 \times 100 = 2{\cdot}15 \times 10^6$, making with the useful lines a total $Z_m = (13{\cdot}6 + 2{\cdot}15) \times 10^6 = 15{\cdot}75 \times 10^6$. The densities in the pole and yoke are now required to be high, and may be made 17,300 and 15,900. We are thus led to a sectional area for the pole of 12″ axial length × $11\frac{7}{8}$″ width with corners rounded to 1″ radius, giving 141 sq. in. = 910 sq. cm., and the two dimensions bearing suitable proportions respectively to the axial length of pole-face (= 13·5″) and to the chord of the polar arc (= 14·4″). The yoke will have a single section $a_y = 495$ sq. cm., or 76·7 sq. in., say $4\frac{1}{2}$″ × 17″. To assist the stability of the magnetism and to accommodate the large amount of exciting power required by the varied conditions of service, the magnet-core may have a somewhat greater length than usual, so as to give a bobbin of $9\frac{7}{8}$″ over-all length. A rough sketch then shows for the length of path in the pole and through half of the yoke $\frac{l_m}{2} = 32{\cdot}2$ centimetres and $\frac{l_y}{2} = 57$ centimetres. From the lowest curve of Fig. 130, $f'(B_m) = 70$ and $f'(B_y) = 28$, whence $\frac{X_m}{2} = 70 \times 32{\cdot}2 = 2255$, and $\frac{X_y}{2} = 28 \times 57 = 1595$. Thus $AT_c = 8540 + 2255 + 1595 = 12{,}390$, which can be obtained within the designed length of bobbin.

Other values under different conditions are similarly worked out and tabulated on the following page.

In order to allow for the range of the governor, a normal no-load speed of 455 revs. per min. is assumed. The plotting of a few

Volts	480			500			480			520			460			550		
Revs. per min.	455			455			445			445			425			445		
Load	Shunt, no-load.			Traction, compound-wound no-load.			Shunt, full-load.			Shunt, full-load.			Shunt, maximum current.			Traction, compound, full-load.		
Amperes	0			0			344			317			360			300		
E_a	480			500			492·8			531·9			473·3			565		
Z_a in 10^6 lines		11·3			11·75			11·85			12·8			11·9			13·6	
$B_a = \frac{Z_a}{780}$	14,500			15,100			15,200			16,400			15,250			17,420		
$f'(B_a)$	16			21			23			40			24			75		
$\frac{X_a}{2} = f'(B_a) \times 8·06$			129			170			185			323			193			605
Mean $f'(B_t)$	48			65			67·5			150			69			254		
$\frac{X_t}{2} = f'(B_t) \times 4·13$			198			268			279			620			285			1050
$\frac{X_g}{2} = 462·5 \times Z_a \times 10^{-6}$			5225			5440			5480			5920			5500			6295
Angle of lead, λ							$6\frac{1}{2}°$			$6\frac{1}{2}°$			$6\frac{1}{2}°$			$5°$		
$\frac{X_b}{2} = \frac{J \times 560 \times \lambda}{360°}$			0			0			875			807			919			590
$\frac{X_P}{2}$			5552			5878			6819			7670			6897			8540
$\zeta = 1·257\ X_P \times 100$		1·4			1·475			1·715			1·93			1·73			2·15	
Z_m in 10^6 lines		12·7			13·225			13·565			14·73			13·63			15·75	
$B_m = \frac{Z_m}{910}$	14,000			14,550			14,950			16,200			15,050			17,300		
$f'(B_m)$	12			13·6			15			33			16			70		
$\frac{X_m}{2} = f'(B_m) \times 32·2$			386			438			483			1060			515			2255
$B_y = \frac{Z_m}{990}$	12,810			13,350			13,700			14,900			13,800			15,900		
$f'(B_y)$	7·8			10			11·5			15			12			28		
$\frac{X_y}{2} = f'(B_y) \times 57$			445			570			655			855			684			1595
$\frac{X}{2}$ = AT per pole			6383			6886			7957			9585			8096			12,390

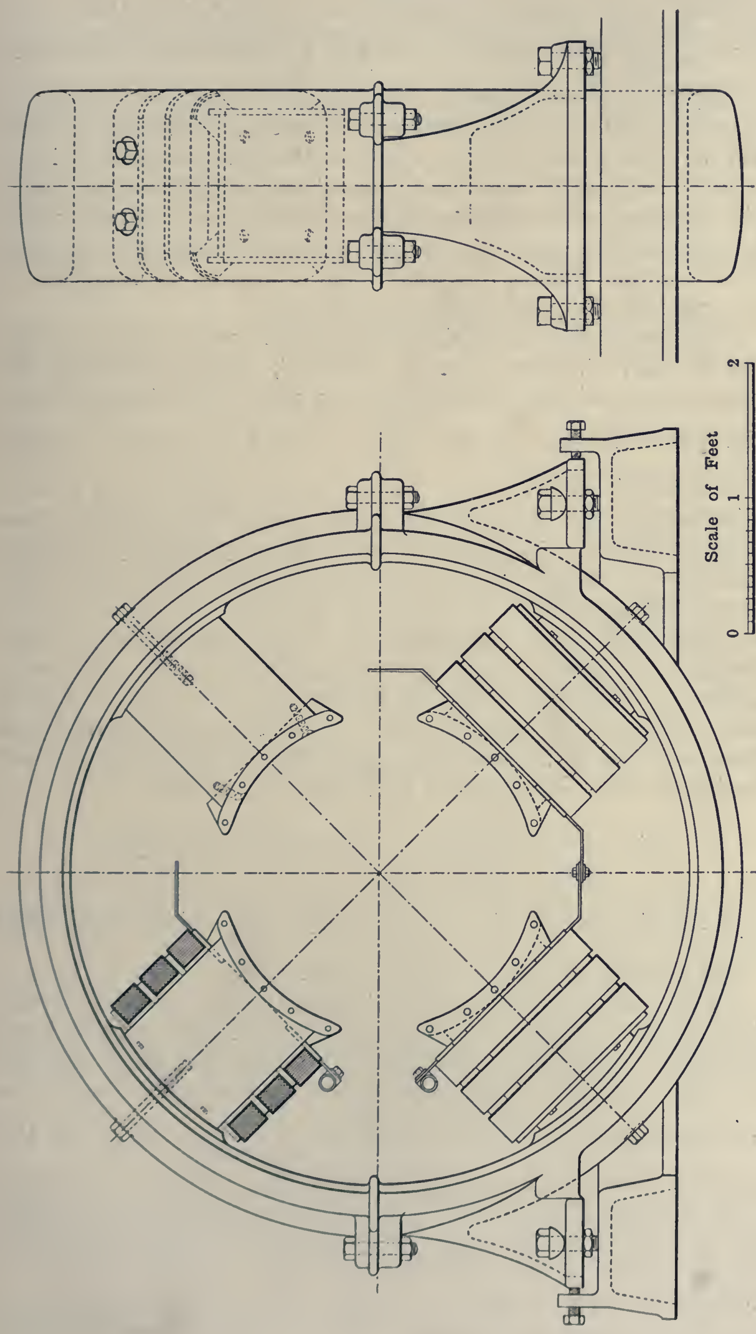

FIG. 397.—Field-magnet of 165-kilowatt dynamo.

additional points on the no-load flux curve shows that a tangent to $Z_a = 11 \cdot 3 \times 10^6$ gives a ratio $\frac{Z}{z_1}$ exceeding 3, so that on the question of stability the machine complies with the limiting condition named in Chap. XVI. § 10. When the volts are lowered to 460 on open circuit with the same no-load speed of 455 revs., the flux would fall still lower to $10 \cdot 8 \times 10^6$, and the ratio approaches closely to the limit, but under this condition we are permitted a lower speed of say 435 revs., which increases the flux again to $11 \cdot 3 \times 10^6$, so that this value may be taken as the smallest working flux, and so far as stability of magnetism is concerned the design of magnet is satisfactory.

The lowest ratio of exciting volts to shunt ampere-turns fixes the size of shunt wire that must be employed, since it determines the lowest value of ω', the resistance per 100 yds. The minimum occurs with 520 volts and $\frac{X}{2} = AT_c = 9585$. The total number of shunt ampere-turns is then $AT_c \times P = 38,340$. With an allowance of $\frac{3}{8}''$ wood strips between coil and magnet-core the length of the innermost turn is $2(10 + 9 \cdot 875) + 2\pi \times 1\frac{3}{8} = 48 \cdot 4''$, and after a preliminary trial and adjustment a suitable depth of winding is found to be $t = 2 \cdot 1''$. The outer periphery is then $2(10 + 9 \cdot 875) + 2\pi \times 3 \cdot 475 = 61 \cdot 55''$, which will be required for the calculation of the cooling surface, and the mean length of a turn is $l_x = 55'' = 1 \cdot 53$ yd., and the mean temperature coefficient in comparison with the standard wire table at 60° F. may be taken as 1·184. In order to retain a little margin of control by the rheostat in case of necessity, an exciting voltage of 510 will be assumed, leaving 10 volts to be absorbed in the rheostat. Thence

$$\omega' = \frac{510 \times 100}{38,340 \times 1 \cdot 53 \times 1 \cdot 184} = 0 \cdot 734 \text{ ohm per 100 yds.,}$$

and the diameter of wire $= \frac{0 \cdot 0554}{\sqrt{0 \cdot 734}} =$ say $0 \cdot 065''$, which when single-cotton-covered has an overall diameter of $0 \cdot 073''$.

With two dividing strips of fibre $\frac{3}{8}''$ thick to form ventilating channels in planes at right angles to the axis of the coil the net winding length of $9 \cdot 875'' - 0 \cdot 75''$ may be divided into two shunt sections $3\frac{1}{4}''$ deep and one series section of $2\frac{5}{8}''$ depth. Each of the former will take 42 turns per layer, and the number of layers $= \frac{2 \cdot 1}{0 \cdot 9 \times 0 \cdot 073} = 32$. The total number of shunt turns on the 4 bobbins is then $32 \times 42 \times 2 \times 4 = 10,752$ with a length of 16,450 yds., and a resistance at 60° F. of 121 ohms, or when hot $R_s = 143$. The shunt current is $C_s = \frac{510}{143} = 3 \cdot 57$, and the watts $= 510 \times 3 \cdot 57 = 1820$.

The cooling surface of the two shunt sections with one end-flange is

$61 \cdot 55 \times 7 + 55 \times 2 \cdot 1 = 546$ sq. in., and $S_c = 4 \times 546 = 2184$. With the given peripheral speed of armature the coefficient for the surface rise of temperature is 72, and the ratio of watts per sq. in. $\frac{1820}{2184} = 0 \cdot 833$, whence the surface-rise under the conditions of greatest heating $= 72 \times 0 \cdot 833 = 60°$ F.

It remains to determine the compounding and the series winding. At no-load with 500 volts at the terminals 6886 ampere-turns per pole are required, and the rheostat must be set to give $\frac{6886 \times 4}{10,752} = 2 \cdot 56$ amperes; on the full traction load this will increase to $2 \cdot 56 \times \frac{550}{500} = 2 \cdot 82$ and 7570 ampere-turns, or a little more if arranged as a short shunt. The necessary series ampere-turns per pole are therefore $12390 - 7570 = 4820$, which will be given by 16 turns per pole of flat copper strip $2\frac{1}{2}''$ wide. Its thickness can be $\frac{2 \cdot 1''}{16} = 0 \cdot 131''$ when insulated, or with interleaved calico strips say $0 \cdot 115''$ bare. All the series coils will be arranged in series, and their junctions will be effected by right-angled strips of thin copper riveted and soldered to the ends of the spiral, an extra half-turn being added to each coil to bring the beginning and finish to opposite sides of the coil. The area of the copper tape is $2\frac{1}{2}'' \times 0 \cdot 115'' = 0 \cdot 288$ sq. in., its resistance per 100 yds. $\omega' = \frac{0 \cdot 0024}{0 \cdot 288} = 0 \cdot 00834$, its total resistance $16\frac{1}{2} \times 4 \times 1 \cdot 53 \times 0 \cdot 0000834 = 0 \cdot 00842$, and when hot $0 \cdot 00842 \times 1 \cdot 184 = 0 \cdot 01$ ohm. The loss of volts is then $3 \cdot 03$ and of watts 917. The cooling surface with one end-flange is $61 \cdot 55 \times 2\frac{5}{8} + 55 \times 2 \cdot 1 = 279$ sq. in., and $\frac{917}{279 \times 4} = 0 \cdot 822$ watts per sq. in., giving practically the same surface rise as for the shunt coils. The heating of the latter under the traction load will of course be less. The total loss of volts over armature, brushes, and series winding being $11 \cdot 35 + 3 \cdot 03 = 14 \cdot 38$ is thus slightly less than the 15 volts originally assumed.

The maximum number of ohms that the shunt rheostat must furnish is given by the condition of 480 volts on open external circuit when the shunt is quite cold, and has its resistance of 121 ohms. The required shunt current is then $\frac{6383 \times 4}{10,752} = 2 \cdot 38$ amperes, and the total resistance of shunt and rheostat must be $\frac{480}{2 \cdot 38} = 202$ ohms. The difference of 81 ohms must be supplied by the rheostat, and in order to give a margin it may be designed for 90 ohms divided between 50 contacts. The series regulator will be further explained in Chap. XXI. § 6.

	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	Full-load.
$C_a{}^2R_a$	51	190	738	1640	2900
Hysteresis	1330	1360	1380	1360	1438
Eddies	2360	2480	2650	2820	3070
$C_a{}^2R_b$	48	101	223	368	530
$C_s{}^2R_s$	950	980	1040	1090	1137
Rheostat	346	356	376	396	413
$C_e{}^2R_m$	16	60	234	519	917
Brush friction	645	645	645	645	645
Bearing and air friction	500	500	500	500	500
Total losses	6246	6672	7786	9338	11,550
Output	19,000	38,400	78,700	121,000	165,000
Input	25,246	45,072	86,486	130,338	176,550
Efficiency	75 %	85·1 %	91·1 %	93 %	93·5 %

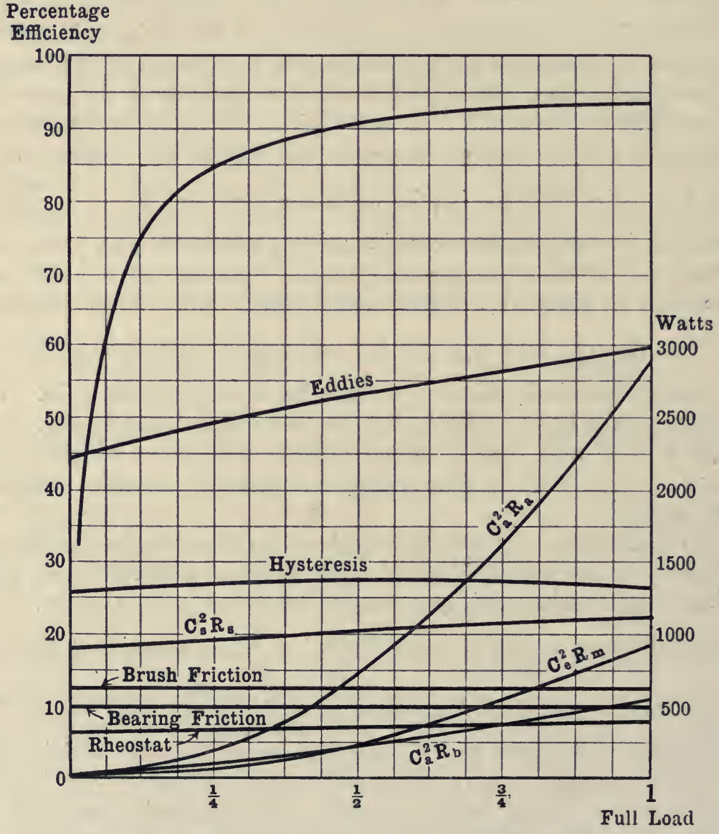

FIG. 398.—Efficiency and loss curves of 165-KW. dynamo.

The peripheral speed of the commutator is $v_k' = 2120$ ft. per min., and the total brush pressure is $P = 1\frac{1}{2} \times 5{\cdot}62 \times 4 = 33{\cdot}7$ lbs. Therefore by eq. 146 the watts lost in mechanical friction of the commutator are

$$0{\cdot}4 \times 33{\cdot}7 \times 2120 \times 0{\cdot}0226 = 645$$

The loss from the friction of the single outer bearing of the dynamo and from windage may be estimated as a constant quantity of 500 watts.

For the traction load as a compound-wound generator the efficiency is calculated as above (Fig. 398). For convenience the calculations are made for currents of full, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ value, so that the results are not strictly those for $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ full-load owing to the proportionate rise of the volts from 500 to 550, but the error is inappreciable.

§ 8. **Design of 500-KW. dynamo with commutating poles.**—The design of a 500-KW. machine with commutating poles is tabulated below in a form which also serves to illustrate a dynamo-designing sheet. The items are entered, generally speaking, in the order which the designer follows, although it will be understood that the process of checking and correction requires a considerable amount of cross-reference.

Kilowatts	500
Volts	480/550
Amperes	1040/910
Revs. per min.	330
Watts per rev. per min.	1510
Number of poles	8
External diam. of armature core, D . .	48″
Gross length of armature core, L . .	12″
$D_{''}^2 L_{''}$	27,600
Size constant, $\frac{D^2L}{\text{watts per rev. per min.}}$ . .	18·25
Frequency	22
Polar arc	30°
Ratio of pole-arc to pole-pitch, β . .	0·68
Style of winding	Simplex lap
Number of armature paths . . .	8
Number of active conductors . . .	960

	Compound-wound Full-load 550 volts	Compound-wound No-load 500 volts	Shunt-wound Full-load 480 volts
Amperes per armature path . . .	115		131
Ampere-wires per inch of circumference . .	735		837
Ampere-wires within pole-pitch . . .	13,800		15,700
Loss of volts over armature . . .	9·7		11
Loss of volts over brushes . . .	1·6		1·7
Loss of volts over commutating pole-winding .	1·1		1·2
Loss of volts over series-winding. . .	2		

	Compound-wound Full-load 550 volts	Compound-wound No-load 500 volts	Shunt-wound Full-load 480 volts
Total loss of volts	14·4		13·9
E_a	564·4	500	493·9
Z_a in megalines	10·7	9·45	9·35
Number of slots	120		
Number of conductors per slot	8		
Arrangement within slot	2 layers,	4 abreast	
Ampere-wires per slot	920		1048
Number of commutator sectors	480		
Number of turns per section	1		
Number of sectors per pole	60		
Average volts per sector	9·15		
Diam. of commutator	36″		
Pitch of sectors	0·235″		
Number of equalising rings	8		
ARMATURE WINDING—			
Depth of slot	1·85″		
Depth of wooden wedge	0·175″		
Net depth below wedge	1·675″		
Width of slot	0·450″		
Depth of bare conductor	0·700″		
Width of bare conductor	0·055″		
Area of bare conductor, sq. in.	0·0385		
Ohms per 100 yds. at 60° F.	0·0625		
Rear-pitch in elements	113		
Rear-pitch in slots	14		
Diameter at bottom of slots	44·3″		
Diameter at centre of slots	46·15″		
Tooth-pitch at bottom of slots	1·16″		
Tooth-pitch at centre of slots	1·208″		
Tooth-pitch at top of slots	1·256″		
w_s+d	0·48″		
Axial projection $l_e=(3{\cdot}7+2)$	$=$ 5·7″		
Length of one end-connector $l'=2\ (9{\cdot}2+2)$	$=$22·4″		
Mean length of half-loop	34·4″		
Resistance of armature at 60° F. in ohms	0·00895		
Resistance of armature $\times 1{\cdot}18=$ at 140° F.	0·01055		
$C_a^2R_a$ in watts	8900		11,500
ARMATURE CORE—			
Number and width of ventilating ducts	Three, ½″		
Insulation between discs	10%		
Net length of iron in core, b	9·45″		
Internal diam. of discs	34″		
Radial depth of iron below slots, a	5·15″		
Double section of core in sq. cm., $2\ ab$	627		
B_a, flux-density in core	17,000	15,050	14,900
Number of teeth under pole, say	11		
Width of tooth at top	0·806″		
Width of tooth at centre	0·758″		

	Compound-wound Full-load 550 volts	Compound-wound No-load 500 volts	Shunt-wound Full-load 480 volts
Width of tooth at bottom	0·71″		
Section of teeth under pole, top, in sq. cm.	540		
Section of teeth under pole, centre, in sq. cm.	508		
Section of teeth under pole, bottom, in sq. cm.	476		
B_{t_1} at top uncorrected	19,800	17,500	17,300
B_t at centre uncorrected	21,000	18,600	18,400
B_{t_2} at bottom uncorrected	22,400	19,800	19,600
Volume of iron below slots in c. cm.	97,800		
Volume of iron in teeth in c. cm.	26,050		
Joules per c. cm. per cycle in core	0·0014		0·0015
Joules per c. cm. per cycle in teeth	0·0008		0·001
Hysteresis loss in core	3000		3250
Hysteresis loss in teeth	460		575
Eddy-current coefficient, F	0·086		0·0626
Eddy-current loss in watts	9350		6800
Total loss in watts	21,710		22,125
Axial over-all length of winding	25·25″		
Circumference of armature	150·7″		
Peripheral surface, sq. in.	3910		
Diameter at bottom of bars at ends	43·5″		
Length of exposed inner surface	11·4″		
Corrected internal surface, sq. in.	1410		
Total cooling surface, S_c	5320		
Watts per sq. in.	4·08		4·16
Peripheral speed in ft. per min.	4150		
$\frac{t \times S_c}{W}$ heating coefficient	17		
Rise of temperature in deg. Fahr.	69.5		70·8
MAGNETIC CIRCUIT—			
Armature core, ampere-turns per cm. length	70	24	22
$\frac{l_a}{2}$ in cm.	8		
$\frac{X_a}{2}$	560	192	176
Armature teeth, $\frac{w_{t_2}}{w_s} = 1 \cdot 575$			
Corrected B_t, at top	16,800	14,900	14,700
Corrected B_t, at centre	20,600	18,500	18,350
Corrected B_t, at bottom	21,600	19,600	19,500
Mean ampere-turns per cm. length	322	102	92
Length of tooth in cm.	4·7		
$\frac{X_t}{2}$	1510	480	432
Air-gap, length $\frac{1}{4}''$ = in cm.	0·635		
Axial length of pole-face	$11\frac{1}{2}''$		
$\frac{a}{l_g} = 1 \quad K_1 = 1 \cdot 25 \quad \frac{w_d}{l_g} = 2 \quad K_3 = 0 \cdot 28$			
$L + K_1 l_g - n_d \cdot K_3 \cdot w_d$	11·4″		
Polar arc at centre of gap	12·85″		

	Compound-wound Full-load 550 volts	Compound-wound No-load 500 volts	Shunt-wound Full-load 480 volts
$\frac{c}{l_g}=12{\cdot}6 \quad K_2=3$			
$A_1+K_2l_g$	$13{\cdot}6''$		
Effective area of air-gap in sq. cm.	1000		
B_g	10,700	9450	9350
$\frac{w_s}{l_g}=1{\cdot}8 \quad \frac{w_{t1}}{w_s}=1{\cdot}79 \quad m=1{\cdot}1$			
$\frac{X_g}{2}=0{\cdot}8\ B_g \cdot ml_g$	5980	5280	5225
Demagnetising turns with comm. poles	0	0	0
$\frac{X_P}{2}$ ampere-turns	8050	5952	5833
Leakage permeance, $\mathcal{P}_l$	128		
Leakage lines, $1{\cdot}257\ X_P \times \mathcal{P}$	$2{\cdot}59$	$1{\cdot}92$	$1{\cdot}88$
Z_m megalines	$13{\cdot}29$	$11{\cdot}37$	$11{\cdot}23$
Magnet Core—	Cast Steel		
Width of magnet core parallel to shaft	$11\frac{1}{2}''$		
Breadth of magnet core across shaft	$10\frac{5}{8}''$		
Area (with 1″ radius corners) in sq. cm.	760		
Pole-Shoes—	Laminated		
Flux-density in magnet-core, B_m	17,450	14,950	
Ampere-turns per cm. length	70	16	
$\frac{l_m}{2}$ in cm.	$30{\cdot}5$		
$\frac{X_m}{2}$	2135	488	
Yoke—	Cast Steel		
Single cross-section in sq. cm.	410		
Flux-density in yoke, B_y	16,200	13,850	
Ampere-turns per cm. length	$34{\cdot}5$	12	
$\frac{l_y}{2}$ in cm.	$35{\cdot}5$		
$\frac{X_y}{2}$	1225	425	
$\frac{X}{2}$, ampere-turns per pole	11,410	6865	
Shunt ampere-turns at 550 volts	7550		
Series ampere-turns	3860		
Magnet Winding—	Sectional ventilated		
Length of bobbin between flanges	$8\frac{3}{4}''$		
Number of sections	3		
Width of 2 wood dividing strips	$\frac{7}{8}''$		
Net winding length	$7\frac{3}{4}''$		
Depth of winding	$1\frac{5}{8}''$		
Length of inside turn	$45{\cdot}55''$		
Length of outside turn	$55{\cdot}75''$		
Length of mean turn	$50{\cdot}65''$		

	Compound-wound Full-load 550 volts	Compound-wound No-load 500 volts	Shunt-wound Full-load 480 volts
SHUNT—			
Number of sections	2		
Length of each section	$2\frac{13}{16}''$		
Resistance of wire per 100 yds. at 60° F. ohms	0·52		
Diameter of wire bare	0·077		
Diameter of wire insulated, s.c.c.	0·085		
Numbers of layers	20		
Number of turns per layer in each section	32		
Total turns per bobbin	1280		
Total turns on machine	10,240		
Total length in yds.	14,400		
Ohms when cold	75		
Ohms when hot, × 1·184	88·7		
Amperes	5.9	5·36	
Volts at terminals of shunt	524	476	
Margin of volts in rheostat	26	24	
Watts in shunt	3090		
Watts in rheostat	156		
SERIES—			
Length of section	$2\frac{1}{4}''$		
Arrangement	two parallels of 4		
Amperes per bobbin	458		
Turns per bobbin	$8\frac{1}{2}$		
Size of conductor	$1\frac{5}{8}'' \times 0{\cdot}190$		
Style of winding	on edge, bare		
Ohms per 100 yds. at 60° F.	0·00775		
Total length in yds.	95·5		
Ohms at 60° F.	0·00185		
Ohms when hot, × 1·18	0·00218		
Loss of volts	2		
Loss of watts	1830		
Total excitation watts	4920		
Cooling surface of bobbin in sq. in.	600		
Cooling surface of all bobbins	4800		
Watts per sq. in.	1·02		
$\frac{l \times S_c}{W}$	62		
Rise of surface in deg. Fahr.	63·5		
Weight of shunt wire in lbs.	775		
Weight of series wire in lbs.	345		
Total weight	1120		
COMMUTATING POLES—			
Number	4		
Material	wrought-iron forgings		
Pole-shoe width along shaft	$11\frac{1}{2}''$		
Pole-shoe breadth across shaft	$2\frac{1}{4}''$		
Pole-core width	$9\frac{3}{4}''$		
Pole-core breadth	$1\frac{3}{4}''$		

	Compound-wound Full-load 550 volts	Compound-wound No-load 500 volts	Shunt-wound Full-load 480 volts
Area with ½″ radius corners, sq. cm.	108·5		
Air-gap, l_{gr}	0·150″		
Effective width of air-gap	11·2″		
Effective breadth of air-gap $= 2\frac{1}{4} + 4{\cdot}5\, l_{gr}$	$=2{\cdot}925''$		
Area of air-gap in sq. cm.	212		
$\frac{w_s}{l_{gr}}=3 \quad m=1{\cdot}16$			
Flux-density, B_r	4710		5345
Ampere-turns over air-gap	1665		1880
$\frac{1}{2} \cdot \frac{J\tau}{2p}$ neutralising turns	6900		7850
Total useful flux in megalines	1·00		1·135
Leakage to adjacent pole of opposite sign	0·73		0·69
Total flux	1·73		1·825
Density in pole-core	15,900		16,800
Ampere-turns per cm. length	20		40
Length of comm. pole in cm.	30·5		
Ampere-turns over core	610		1220
Total ampere-turns per pole	9175		10,950
Arrangement of bobbins	two parallels of 2		
Amperes per bobbin	457		522
Number of turns per pole	21		
Length of coil	7½″		
Depth of winding	1½″		
Length of turn on inside	22·9		
Length of turn on outside	32·3″		
Mean length of turn	27·6″		
Size of copper conductor	1⅜″ × 0·275″		
Style of winding	on edge, bare		
Area of copper in sq. in.	0·378		
Resistance per 100 yds. at 60° F.	0·00635		
Total length in yds.	64·5		
Ohms at 60° F.	0·00102		
Ohms hot × 1·17	0·0012		
Loss of volts	1·1		1.25
Loss of watts	1010		1305
Cooling surface per bobbin, sq. in.	250		
Cooling surface of 4 bobbins	1000		
Watts per sq. in.	1·01		1·3
Weight of wire in lbs.	310		
COMMUTATION—			
Number of brush arms	8		
Number and size of brushes per arm	six 1½″ × ¾″		
Total contact surface of brushes, sq. in.	54		
Normal current-density, amperes per sq. in.	34		38·7
Volts lost over brushes	1·6		1·75
Watts lost over brushes	1465		1830
Peripheral speed of comm. in ft. per min.	3110		
Brush friction watts	2280		2280

	Compound-wound Full-load 550 volts	Compound-wound No-load 500 volts	Shunt-wound Full-load 480 volts
Thickness of mica	0·032		
Number of sections simultaneously short-circuited	3		
$k'_1 . \frac{h_s}{w_s}$	93·6		
$k''_1 . \frac{h_3}{w_3}$	12·2		
k'''_1	42·6		
$k'_2 . \frac{h_s}{w_s}$	23·3		
$k''_2 . \frac{h_3}{w_3}$	12·2		
k''''	165		
	348·9		
Core permeance, $349 \times 12 \times 2{\cdot}54$. =	10,700		
$\lambda' = 19{\cdot}5$ $2\,l'\lambda'$ . . . . =	2220		
$L \times \Sigma M$	$12{,}920 \times 10^{-9}$		
T in secs.	0·001155		
$\frac{2\,J\,.\,(L \times \Sigma M)}{T}$ volts . . .	2·58		2·92
B_k	4110		4660
$l_{gz} = 22{\cdot}8 \times 0{\cdot}635 = 14{\cdot}5$ cm.			
B_q	600		685
B_r	4710		5345

§ 9. **Turbo-dynamos.**—The design of the continuous-current turbo-dynamo, driven at very high speeds by a steam-turbine, calls for so much detailed calculation that space does not permit of more than a short summary of the chief points that require especial attention. The primary consideration must be the mechanical construction of the armature to withstand in perfect safety the centrifugal force from its various parts. As the upper limit which the peripheral velocity rarely exceeds may be fixed at 15,000 ft. per min., and any speed above 7000 or 8000 ft. per min. may be considered as demanding great care in the design and workmanship, every pound revolving at a mean speed of 11,400 ft. per min. at 1 ft. radius exerts a centrifugal force of half a ton, or 1120 times its own weight. To give a solidly constructed rotor the armature coils are usually formed of stiff copper strip, heavily served with tape, and keyed within the slots by wedges which are preferably of fibre as giving greater mechanical strength than wood; and the end-connections firmly bedded on turned metal rings must be held down by end-shields of bronze, turned inside and outside* (cp. Chap. XIII. § 31). With

* In the case of a turbo-generator where nickel-steel end-rings were employed, these rings, in order to avoid a large eddy-current loss, were shielded from the leakage

overhung slots and two conductors abreast the overhang should be entirely on one side of the slot for ease of inserting the loops.

The peripheral speed of the commutator surface may be as high as 7000 to 8000 ft. per min., so that here again considerations of mechanical strength call for special constructions with shrunk-on steel rings as described in Chap. XIII. § 29. The necessity for the most perfect running balance that can be practically attained is evident, especially if carbon brushes which may be thrown out of contact with the commutator by the slightest tendency to vibration are employed. The armature core, wound and unwound, and also the commutator, must all be separately balanced, both statically and dynamically, by running them in bearings which permit of restricted lateral movement.* Even when a balance is attained at the outset, the difficulty of maintaining it is no slight one when it is remembered that in the composition of the coils insulating materials such as paper and mica, etc., have to be introduced, all of which are likely to become compressed or to shift slightly under the stress of prolonged working, and after repeated heating and cooling. The coils are therefore best driven home within the slots during the process of manufacture by powerful pressure firmly and evenly applied. A low temperature rise, and still more an evenly distributed temperature rise, is desirable so as to avoid unequal expansion. Yet from the nature of the case the external cooling surface of the turbo-dynamo bears a smaller ratio to the watts to be dissipated than in the corresponding slow-speed dynamo, and therefore assisted or even forced ventilation comes into question. It may thus become advantageous to totally enclose the magnet frame by cast-iron end-plates or housing with well-defined air inlet and outlet, so that only the commutator projects into the open; instances of such a construction will be found in Messrs. Brown Boveri & Co.'s turbo-dynamos, described in Chap. XXVII. § 3. This tendency is reinforced by the desirability of securing noiseless running, to assist which, in addition to enclosing the machine, a smooth-surface rotor and a careful leading of the air through properly designed channels without abrupt changes in their course or cross-section must be aimed at.

Owing to the high inductive voltage that results from the peripheral speed even when there is only one turn per section of the armature winding, the use of commutating poles is general, accompanied in critical cases by a compensating winding.† The latter, by neutralising the distortion of the main flux, enables a higher value to be given to the average voltage per sector even when the air-gap is small, without

from the poles by iron rings surrounding the armature (*Trans. American I.E.E.*, 1907, vol. xxvi. p. 1751.

* For references, *vide* Chap. XIII. § 36.

† See Chap. XVIII. § 41.

danger of "flashing-over" from sector to sector round the whole commutator under sudden variation of load (cp. Chap. XVIII. § 42). A maximum of 40 volts* per sector gives a practical limit in this direction, and this must allow for the rise of volts under the trailing pole-edge due to armature distortion, so that, assuming the increase to amount to 33 per cent., the mean volts per sector are reduced to a maximum of 30. The great desirability of lower values not exceeding 20 volts has led to the devices of subdividing each loop into two or more sections by auxiliary commutator connections as mentioned in Chap. XVIII. § 39.

It is evident that the necessity for the careful choice of materials and for the most skilful construction, together with the other precautions alluded to above, must to a great extent discount the advantages of the turbo-dynamo in the matter of its cost and floor-space due to its comparatively small number of watts per rev. per min. for a given output. The continuous-current machine, unlike the alternator, is not intrinsically a high-speed machine. Much will no doubt be done in the future to standardise and cheapen the construction of turbo-dynamos, so that it is not possible to lay down strictly the possibilities in this direction, but it may without hesitation be said that it is easy for the speed most suitable for the steam turbine to exceed the speed which is really the most economical for the dynamo of given output. Messrs. Hobart and Ellis, who have investigated the subject at length,† have given as the most economical speeds for 250, 500, 1000 KW. outputs, 1500, 1000, and 750 revs. per min., while at the present time the corresponding steam turbines, according to their type and other conditions, are designed for speeds exceeding the above by 50 to 100 per cent.

An open-type continuous-current dynamo of small size for 125 kw. at 125 or 250 volts, as made by the General Electric Company of Schenectady, N.Y., for direct coupling to a horizontal Curtis steam-turbine running at 2400 revs. per min., is illustrated in Fig. 399. It has four main poles and also commutating poles; the rigid circular frame carrying the brush-gear with hand-wheel for adjusting its position is well shown in the illustration, and in some machines this is again repeated with a commutator between the armature and turbine.

The turbo-dynamo of Messrs. Siemens Bros. Dynamo Works Ltd., as shown in Fig. 400 for 500 KW. 250 volts, 1500 revs. per min., is noteworthy for the special patented construction of subdivided and ventilated commutator.‡ The latter is divided into two halves,

* Dr. Pohl, "The Development of Continuous-Current Turbo-Generators," *Journ. Inst. Electr. Eng.*, vol. xl. p. 239, and the discussion thereon.

† *High-Speed Dynamo Electric Machinery* (New York, John Wiley & Sons, 1908).

‡ Cp. Brit. Patent No. 19,891, 1908.

united by copper radial blades which serve not only as electrical connections between the sectors, but also as fans. As shown in

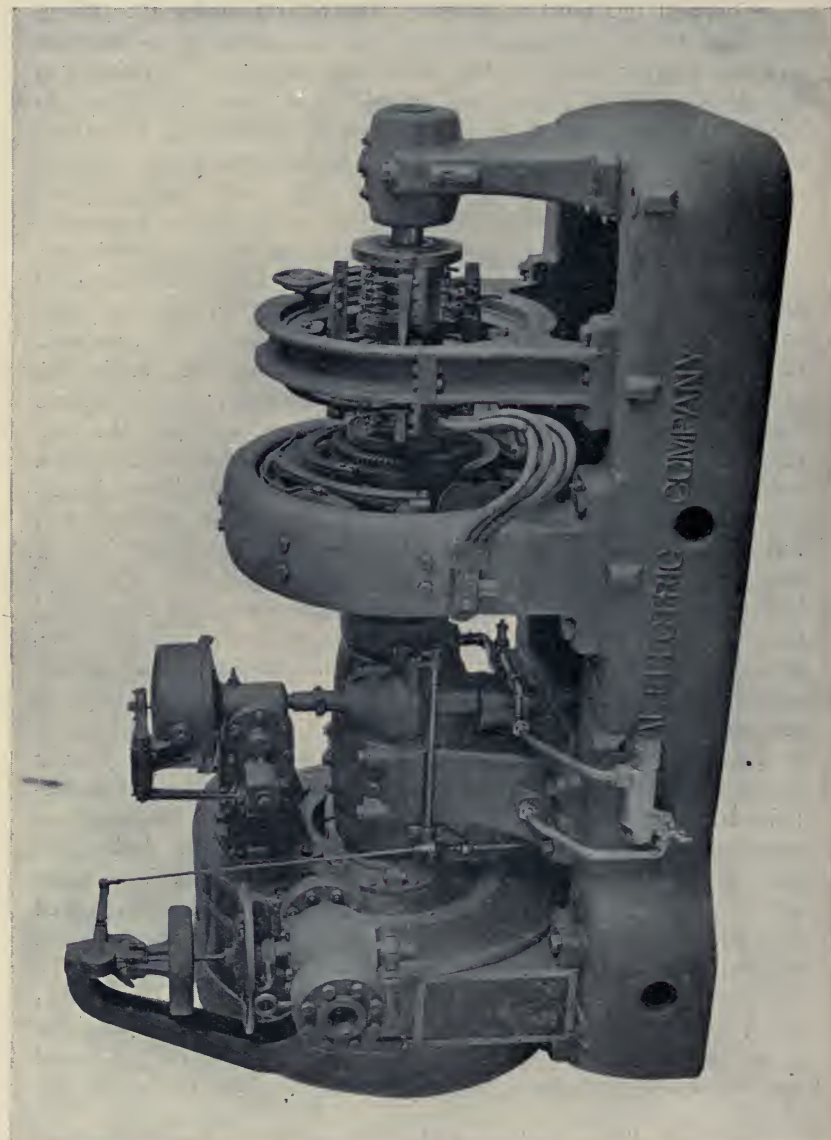

Fig. 399.—125-KW. turbo-dynamo of General Electric Company, U.S.A.

Fig. 401, the commutator bars are provided along their entire length with grooves, which form tunnels through which air is drawn from either end by the action of the radial blades. The connections to the armature winding are specially designed to give a certain amount of flexibility ; they are thus better able to withstand the vibrations that may

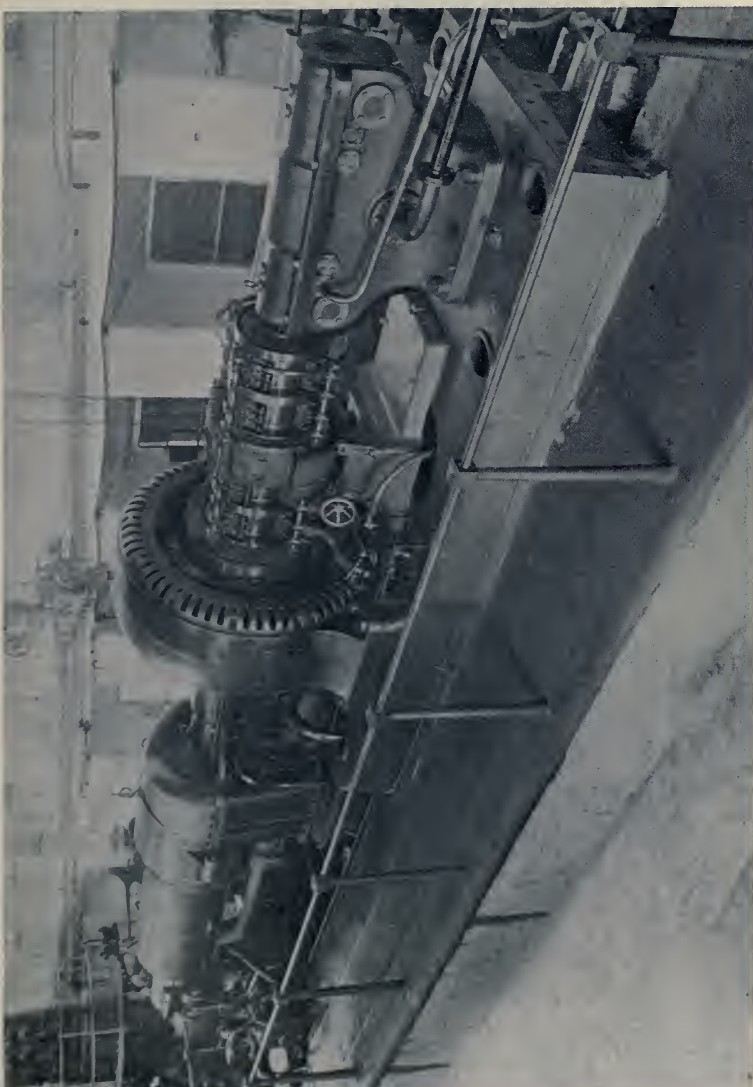

FIG. 400.—500-KW. Siemens turbo-dynamo.

occur on running up to speed or on slowing down, without hardening and subsequent fracture. The sides of the shrink rings on the commutator are shielded by wooden plates, so as to prevent dust

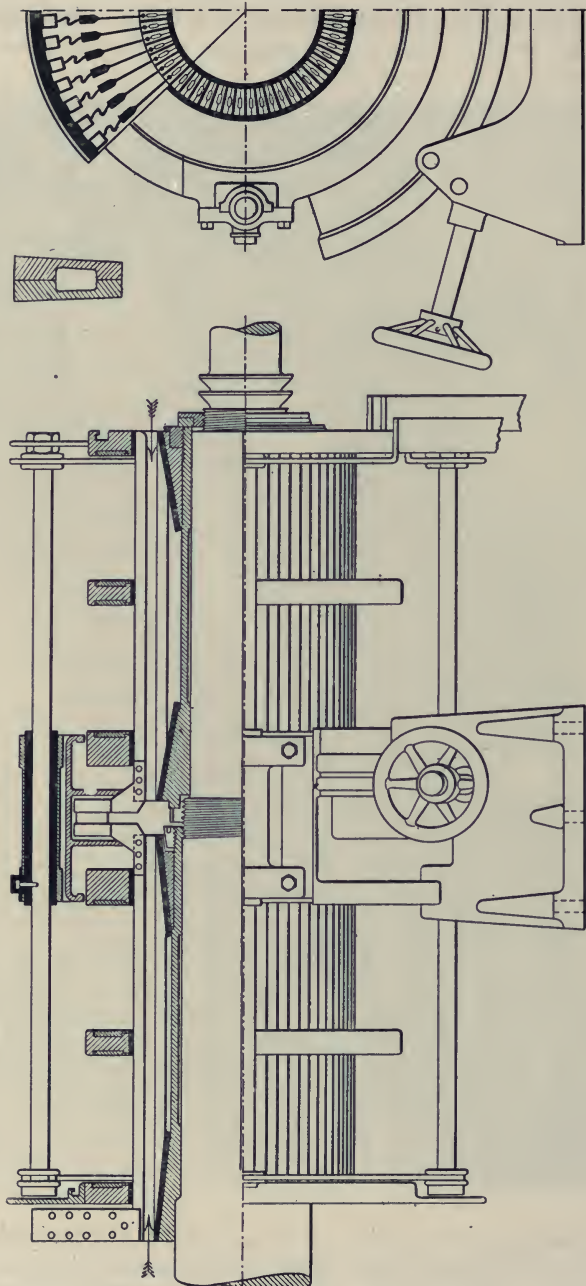

FIG. 401.—Siemens ventilated commutator for turbo-dynamo.

bridging over the mica seatings or a spark from the brushes flashing to the shrink ring and thence being transmitted to other sectors.

The British Westinghouse Company have introduced a type of commutator for turbo-generators, in which a number of upstanding rings are turned on the surface, against the radial sides of which the brushes press. By this construction it is claimed that carbon brushes can be successfully used even at the highest speeds without chattering or vibration, and without any liability of flashing over to the shrink rings ordinarily employed, while at the same time an excellent cooling effect is obtained.*

Much difference of opinion and practice exists on the question of brushes for turbo-generators; a high conductivity is necessary in order to shorten the length of commutator, but on the other hand the conditions are particularly difficult. A specially soft graphitic brush with low friction coefficient is made by Le Carbone for very high-speed commutators, and Morganite brushes have also been largely used with success, as also Endruweit brushes (cp. Chap. XVIII. § 42) with carbon leading brushes.

Owing to the lesser volume of copper and iron in proportion to the magnitude of their losses which turbo-generators have as compared with steam-engine driven dynamos, the former do not permit of such large temporary increases of output above their normal, and their overload capacity is not greatly higher than their continuous rating. The increased cost of special alloyed sheet steels which have a low hysteresis loss, and the use of very thin laminations to eliminate eddy-currents, may become warranted in turbo-generators in view of the importance of reducing the constant core loss to the lowest possible figure.

§ 10. **Motor-current method of measuring losses.**—The two principal methods of measuring the losses which occur in continuous-current dynamos may here be introduced as bearing directly on their design. The first † consists in running the machine as a motor without load at various speeds, and noting the current through its armature, the E.M.F. applied to it, and the speed, the excitation being kept constant at the desired value throughout the test.

The power developed in the armature of a continuous-current motor is $E_a \cdot C_a = (E_e - C_aR_a - C_aR_b)\,C_a$, where E_e = the E.M.F. impressed on the motor armature, C_a = the current through it, and R_a, R_b are the resistances of the armature and of the two sets of brushes. Since the motor is now assumed to be running light, C_a is small, and with a fairly large armature of low resistance C_aR_a is practically negligible

* See W. Hoult, "Direct-Current Turbo-Generators," *Journ. Inst. Electr. Eng.*, vol. xl. p. 625, where an armature to give 250 kw. at 500 volts and 3000 revs. per min. is illustrated; and H. T. C. Beyer, *Electr. World*, vol. l. p. 967, where details are given of a 375-kw. 240-volt, 2500 revs. per min. machine.

† *Electrician*, vol. xxvi. pp. 699, 700, and vol. xxvii. p. 162.

as compared with E_e; the loss of volts over the resistance of carbon brushes should, however, be taken into account even at no-load. Subject to this deduction, the back E.M.F. of the motor is closely equal to the E.M.F. impressed upon it, and with a fixed excitation the speed is easily varied throughout a wide range by increasing or decreasing the applied voltage. Since the field Z_a during the test is kept constant, the back E.M.F. of the armature $E_a = \frac{2}{q} . Z_a . \tau . \frac{pN}{60} \times 10^{-8}$ volts is proportional to the speed, and when plotted in relation thereto gives an inclined straight line passing through the origin (Fig. 403).

Again, since Z_a is constant, the total torque rotating the armature is proportional to the current C_a. The latter may therefore be mentally split up into three portions, the first supplying the torque to overcome the resisting force from friction of the bearings and windage, the second corresponding to the torque from hysteresis, and the third to that from eddy-currents. For the moment let it be supposed that the first item has been deducted from C_a, and that the remainder when plotted in relation to speed gives the line AB (Fig. 402). Now, the losses from hysteresis and eddy-currents being proportional respectively to the speed and to the square of the speed (Chap. XIX. §§ 16 and 18), or $W_H + W_E = HN + FN^2$ (where H and F are two constants for the machine with a given excitation), the torque from hysteresis is $\propto H$ and is a constant, while the torque from eddies is $\propto FN$ and is proportional to the speed. The required division of the current AB is at once obtained as shown in Fig. 402; for it must be made up of a constant portion of height OA and a portion rising in an inclined straight line OD from the origin in proportion to the speed. If we experimentally determine a number of points on the line AB, by producing it backwards to cut the vertical at A the constant H can be determined, and from the difference, say PQ − QG, the constant F can be determined. The rate of loss in watts from both hysteresis and eddies at any particular speed N is then obtained by multiplying together the corresponding ordinates of the current and voltage $= QP \times E_a$; the watts absorbed by hysteresis are $W_H = QG \times E_a$, and by eddies are $W_E = GP \times E_a$. Thence $H = \frac{W_H}{N}$

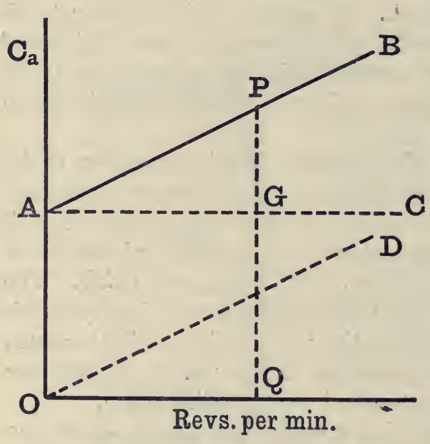

FIG. 402.—Separation of eddy-current from hysteresis and friction losses.

and $F = \frac{W_E}{N^2}$. Or if the iron losses at two speeds N_1 and N_2, the former high and the latter low, are respectively w_1 and w_2, the two coefficients* are

$$H = \frac{N_1^2 \,.\, w_2 - N_2^2 \,.\, w_1}{N_1^2 \,.\, N_2 - N_2^2 \,.\, N_1}$$

$$F = \frac{N_2 \,.\, w_1 - N_1 \,.\, w_2}{N_1^2 \,.\, N_2 - N_2^2 \,.\, N_1}$$

So far the loss from friction of the bearings and from windage has been supposed to be previously known and eliminated. For a constant intensity of pressure per square inch of bearing surface and for a constant temperature the law that the coefficient of friction, and therefore the torque, varies as the square root of the velocity of the shaft, holds as approximately true for all ordinary peripheral speeds ranging between 150 and 500 feet per minute (Chap. XIII. § 12). Hence in ordinary running the friction torque and the component of the no-load motor current which is proportional thereto, when plotted in relation to speed, should be slightly bowed or convex to the horizontal axis.

On the other hand, the torque from the air resistance is app:oximately proportional to the square of the speed, especially when there are ventilating air-ducts with numerous blades interspersed along the armature core; and since the aim of the designer is to produce the maximum of cooling action, by disposing the armature winding to act as an effective fan, the proportion of the air-friction to that of the bearings may be quite appreciable. The loss from bearing friction and windage is thus $w_f = w_{(b+a)} = f_b \,.\, N^{1 \cdot 5} + f_a N^3$, and the effect is to raise the friction current with increasing speeds more nearly to a constant quantity. There is thus some justification for regarding the loss from friction and windage as more or less proportional to the speed, or as equal to the speed multiplied by a coefficient analogous to H for hysteresis. On this assumption, for $W_H = HN$ may be substituted $W_{(H+f)} = (H+f)N$, and the total current is again an inclined straight-line as in Fig. 402, but divisible into the two portions, viz. a constant amount proportional to the torque from hysteresis, friction, and windage, and a portion increasing with the speed, so that

$$H + f = \frac{W_{(H+f)}}{N}, \text{ and F as before} = \frac{W_F}{N^2}$$

Experiment shows that in most cases the total armature current is not far from a straight line, although often slightly bowed. At high speeds the torque from air-friction increasing faster than the speed may render the current-curve even concave.†

* For certain cautions as to the application of these values in designing armatures, *vide* Chap. XIX. § 13.

† Cp. the experimentally obtained figures, *Electrician*, vol. lii. p. 831.

Fig. 403 shows the observed results of a test on a small dynamo, the values of the E.M.F. being given by the inclined straight line passing through the origin, and the corresponding values of the armature current by the inclined straight line which, when produced backwards, gives the initial or starting value of 2·85 amperes. The true curve is probably shown by the dotted line to which the straight line is a fair approximation. At 600 revs. per min. with an E.M.F. of 85 volts the total current taken is 6·85 amperes; since the current required to balance the torque from hysteresis and friction is 2·85

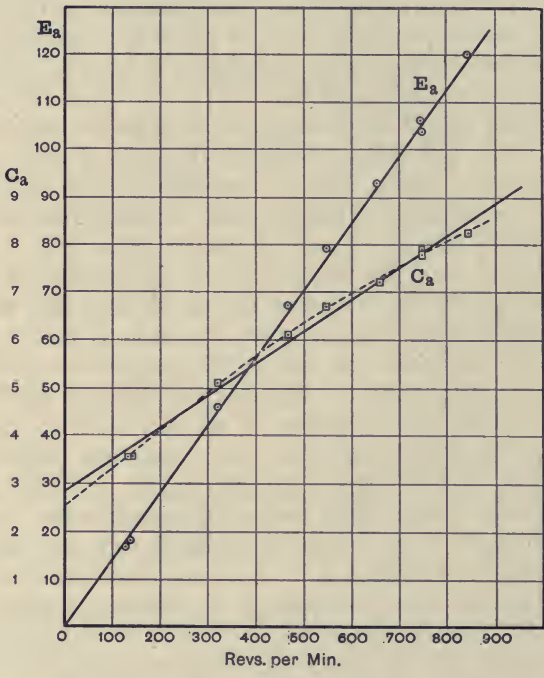

FIG. 403.—Motor-current test of dynamo with armature 12″ diam. × 7″ long.

amperes, the friction, windage, and hysteresis loss is $85 \times 2·85 = 242$ watts, and the difference of 4 amperes multiplied by 85 volts gives the loss by eddy-currents in the armature, namely, 340 watts.

Fig. 404 gives the corresponding values of E_a and C_a for a larger 4-pole machine with toothed armature 21″ diam. × 11″ long under three different degrees of excitation; the ampere-turns per pair of poles were respectively 9700, 14,500, and 19,350, and the air-gap densities 6200, 7140, and 7600. At the normal speed of 400 revs. per min., and the intermediate excitation, the losses by friction, windage, and hysteresis and by eddies are respectively 545 volts × 0·945 amperes = 515 watts, and $545 \times 1{\cdot}035 = 565$ watts, making a total of 1080 watts in all.

Owing to the theoretical objection that the loss from the mechanical friction of the bearings and by windage is not strictly proportional to the speed, the *motor-current test* yields in accuracy to the retardation

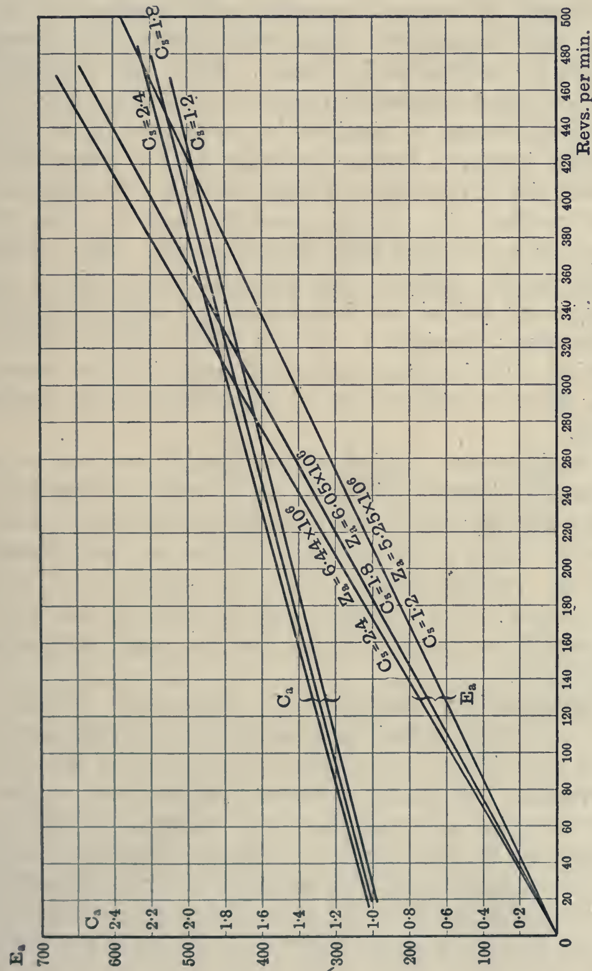

FIG. 404.—Motor-current test of armature 21" × 11".

method to be described in the next section; yet it is simpler and on the whole yields much useful information, so long as the current readings do not diverge greatly from an inclined straight line. It is essential with carbon brushes that their position should be adjusted to give the minimum loss, since by an incorrect setting a considerable additional loss can result from unequal current-density over the contact surfaces of the brushes. There remains, too, the objection that it is difficult in practice to keep the bearings in a steady normal state under the changes of speed which are a necessary part of the test. Since the friction loss is inversely proportional to the temperature of the oil, the machine must be run for a sufficient time to allow the bearings to reach a constant temperature.* Usually three or four hours are required to attain this state, without which all motor-current tests are open to considerable inaccuracy. Even then a change from a high to a low speed temporarily reduces the friction loss to a value below that which would be obtained in steady running, or *vice versa* for the reverse change, so that some minutes must be allowed to elapse before reading the current after a change of speed. It is best to take two sets of readings, the one with ascending and the other with descending speeds, and to note their exact sequence. Slight changes in the pressure on the shaft from the magnetic pull causing a slightly different alignment and minute deformation are sufficient to alter the small friction loss appreciably.† At very low speeds the friction may increase owing to the lesser amount of oil swept into the bearing by the rotating shaft, or owing to the oil rings failing to act steadily and satisfactorily.

If two machines are coupled together rigidly, one can be run as a motor and its losses calibrated. The change of current in its armature can then be used to determine the several losses in the other machine when excited or not excited, and in this way an unwound core can be tested. In such cases the accurate alignment of the shafts within the bearing is very important, and the frictional loss should be specially checked and compared with that of a single machine under normal conditions.

§ 11. **Retardation method of measuring losses.**—The second, or as it may be called the *retardation method*, first applied by M. Routin,‡ necessitates a knowledge of the moment of inertia of the rotating armature, but possesses several incidental advantages. No assumption is made as to the law which the friction obeys, and its actual amount can be very accurately measured; since the changes of speed are automatically made in a definite sequence and with perfect regularity in each repetition of the test. The armature is first run up to speed, and the driving power is cut off; it is then allowed to come

* Cp. Finzi, *E.T.Z.*, 1903, p. 917. † *Electrician*, vol. xliv. p. 323.

‡ *L'Eclairage Electrique*, vol. ix. p. 169. A full description of the method and its various modifications has been given by Dr. Alfred Hay, *Electr. Review*, vol. xlvii. p. 287, from which the following account has largely been drawn and to which the reader is referred for a more detailed discussion. Cp. also Chas. F. Smith, "The Experimental Determination of the Losses in Motors," *Journ. Inst. Electr. Eng.*, vol. xxxix. p. 437.

to rest, while at small intervals of time the speed is measured either directly or indirectly. If the speeds are converted into angular velocities $\left(\omega = \frac{2\pi N}{60}\right)$ in radians per sec. and plotted in relation to time in seconds, at any given moment let ω be the angular velocity, and let a tangent be drawn to the curve at the point corresponding to the moment in question. The tangent then measures the time-rate of change of the angular velocity, or $\frac{d\omega}{dt}$. Let $I = Mk^2$ be the moment of inertia of the revolving part in C.G.S. units, *i.e.* in grammes of mass $\times$(centimetres)^2. At any point of time as the rotor comes to rest the rate of change of its angular momentum is equal to the retarding torque acting on it in C.G.S. units; or

$$T = -I \frac{d\omega}{dt} \text{ dyne-centimetres,}$$

and the rate at which the stored kinetic energy is expended in overcoming the retarding torque is

$$T\omega = -I\omega \frac{d\omega}{dt} \text{ ergs per second,}$$

or

$$= -I\omega \frac{d\omega}{dt} \times 10^{-7} \text{ watts} \quad . \quad . \quad . \quad . \quad (155)$$

Since a kilogramme-(metre)^2 is 10^{-7} C.G.S. units, the moment of inertia I is best expressed in kilogramme-(metres)^2, when the factor 10^{-7} cancels out, and the expression reduces to

$$\text{watts} = -I \, . \, \omega \frac{d\omega}{dt}$$

Since the velocity is usually measured in revs. per min., and plotted in relation to seconds of time, this may also conveniently be expressed as

$$\text{watts} = -I \, . \, \frac{2\pi N}{60} \, . \, \frac{2\pi}{60} \, . \, \frac{dN}{dt} = -I \, \frac{4\pi^2}{3600} N \, . \, \frac{dN}{dt}$$

$$= -0{\cdot}0109 \, I \, . \, N \, . \, \frac{dN}{dt}. \quad . \quad . \quad . \quad . \quad (156)$$

where I is in kg-(metres)^2. But the tangent to the curve must then be read off in terms of the scales to which the curve is plotted.

The watts thus obtained are at any moment equal to the rate at which energy is dissipated in friction of bearings and brushes, windage, hysteresis, and eddy-currents. Thus if the retardation curves be determined and plotted, first with the field unexcited and without brushes, then with brushes down, and lastly with the field excited to various strengths, a complete analysis of the various losses may be made. After the experimental curves have been taken, the derived curves of $\frac{d\omega}{dt}$, or of $\frac{dN}{dt}$, should be plotted in relation to ω or N, the tangents at a number of points being taken so as to check the correctness with which they have been deduced. The torque is then proportional to

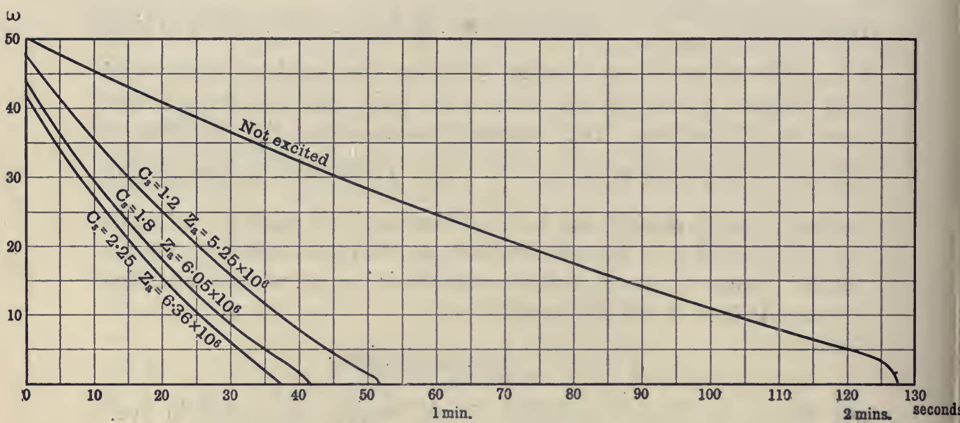

FIG. 405.—Retardation curve of 21″ × 11″ armature.

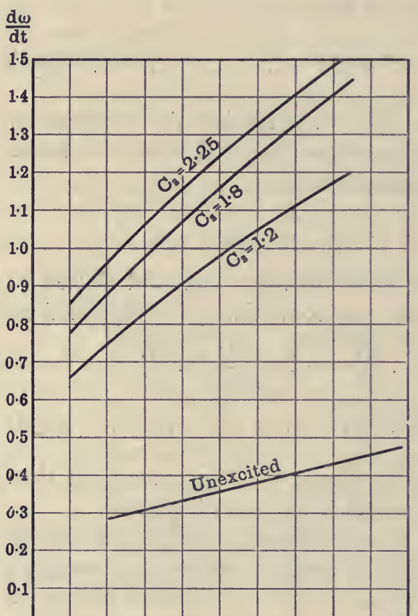

FIG. 406.—Derived curves of rate of change of angular velocity.

the $\frac{dN}{dt}$ curves, and the watt curves are obtained by multiplying $\frac{dN}{dt}$ by N and by the constant 0·0109 I.

Fig. 405 gives the retardation curves of the same dynamo as that for which the motor-current readings are given in Fig. 404; thence are obtained the derived curves of Fig. 406, and finally the total and the friction *plus* windage losses of Fig. 407. By deduction of the friction and windage loss from the total the curves of Fig. 408, for the losses by hysteresis and eddy-currents, are reached. The moment of inertia of the armature was $I_a = 17{\cdot}875$ kg-(metres)^2.

In order to determine the bearing friction and windage, the retardation curve when the field is not excited is best taken by the following method. The machine is coupled by belt to a small motor, and by its means run up to or rather above its full speed; the revolutions are taken on a speed-counter, and the voltage due simply to residual

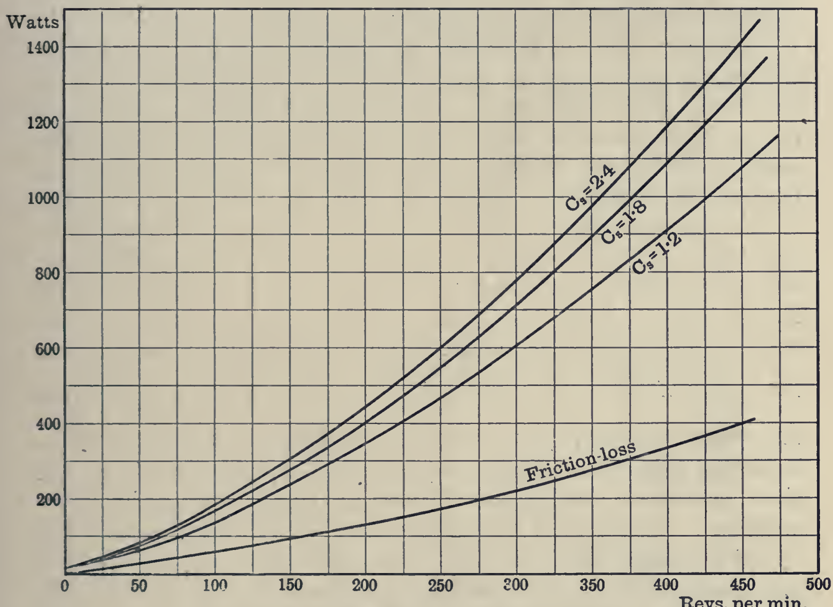

FIG. 407.—Losses in 21″ × 11″ armature.

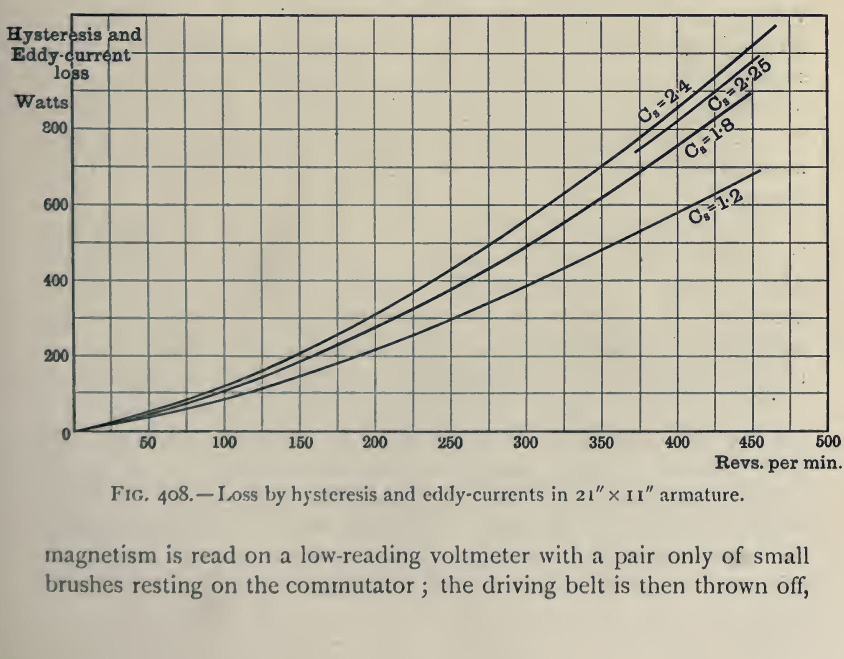

FIG. 408.—Loss by hysteresis and eddy-currents in 21″ × 11″ armature.

magnetism is read on a low-reading voltmeter with a pair only of small brushes resting on the commutator; the driving belt is then thrown off, and at intervals of, say, three seconds the voltage is read as it gradually dies away. The speed of the armature as it comes to rest is then simply proportional to its voltage, and by means of the residual magnetism can be measured indirectly, and more accurately than by a tachometer, which in itself adds an indeterminate amount of friction and inertia comparable in small machines with that which is to be measured. Next, the field is excited, and the machine is run at its full speed as a motor. The armature circuit is broken while the voltmeter leads are left attached to the brushes; the same readings are taken of the voltage as it dies away, and a second curve is obtained. If the time of coming to rest is too short to allow of accurate readings being taken at successive intervals of a few seconds, the process must be continually repeated, a stop-watch being started each time that the driving power is cut off, and again stopped when the voltmeter needle passes a prearranged point. The difference between the two values of $\frac{dN}{dt}$ on the two curves for the same speed N of the armature, when multiplied by $0{\cdot}0109IN$, measures the power absorbed by the hysteresis and eddy-currents in the excited field, and the latter may be given any desired value in order to test the effect of the flux in a given armature. The subsequent separation of the hysteresis loss from that by eddy-currents must be made on the approximate assumption that the former loss is proportional to the speed, and the latter to the square of the speed. Or graphically, by the principles of § 10, if the watts due to hysteresis and eddies are divided by the corresponding voltage, and the current so derived is plotted with the voltage as abscissæ, an inclined straight line is obtained, of which the intersection with the axis of ordinates measures the current required to overcome the torque due to hysteresis, while the line drawn parallel to the straight line of total current but passing through zero measures the current required to overcome the torque due to eddy-currents.

If a heavy fly-wheel be attached to the armature, the great advantage is gained that the time of slowing-down is extended, and especially when the field is excited and the retarding torque is considerable the measurements can be much more accurately taken. Further, since the moment of inertia of a fly-wheel with heavy rim can, owing to its symmetrical shape, be easily and accurately calculated as I_w, an approximate calculation of the moment of inertia of the armature alone I_a will lead to but little error, since the total value $(I_w + I_a)$ has now to be substituted for I in equation (156). Care must, however, be taken that the bearing friction is not seriously altered by the addition of too heavy a fly-wheel, causing deflection of the shaft. This again may be checked by taking two no-load retardation curves, with and without the fly-wheel; taking the calculated value of I_a, the friction loss deduced from the two curves should coincide, or *vice versa*, the value of the moment of inertia obtained for the armature alone from the formula,

$$I_a = I_w \cdot \frac{d\omega_1/dt}{d\omega_2/dt - d\omega_1/dt} \quad . \quad . \quad . \quad . \quad (157)$$

(where $\frac{d\omega_1}{dt}$ and $\frac{d\omega_2}{dt}$ are respectively the rates of change with and without fly-wheel for the same value of ω) should agree with the calculated I_a. Thus in Fig. 409 a fly-wheel of known moment of inertia $I_w = 1 \cdot 8$ kilogramme-(metres)^2 was substituted for the usual pulley of the same dynamo for

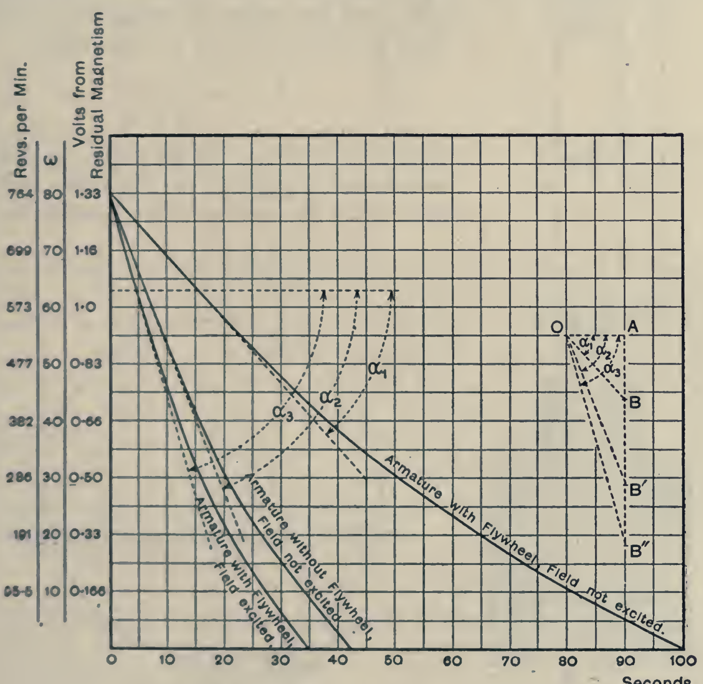

FIG. 409.—Retardation test of 12″ × 7″ dynamo.

which the curves of Fig. 403 were found. The upper retardation curve was then obtained with the field unexcited, the speed being measured by means of the volts from residual magnetism as given in the third scale of ordinates. At 600 revs. per min. or $\omega = 62 \cdot 8$, $d\omega_1/dt = -1 \cdot 1$, angular velocities and seconds of time being plotted to the same scale and the angle α_1 being $47 \cdot 75°$; at the same speed and under the same conditions but without the fly-wheel $d\omega_2/dt$ was $-2 \cdot 62$, the angle α_2 being $69 \cdot 1°$.

Hence

$$I_a = 1 \cdot 8 \times \frac{-1 \cdot 1}{-2 \cdot 62 + 1 \cdot 1} = 1 \cdot 3$$

and the total moment of inertia of armature and fly-wheel = 3·1 kilogramme-(metres)^2. In any such method since the denominator is the difference between two quantities, accurate readings are necessary to avoid considerable error.

The moment of inertia of an armature of moderate size can be conveniently determined by direct measurement of the periodic time of a complete oscillation when it is hung vertically by a bifilar suspension. If the two parallel wires, each of length l, are at equal distance of a from the vertical axis (Fig. 410) the radius of gyration k is found from the formula for the periodic time *

$$T_p = 2\pi \sqrt{\frac{l}{g} \cdot \frac{k^2}{a^2}}$$

whence if M_1 and m are the masses of the armature and clip and of the clip alone, and T_{p1} and T_{p2} are the corresponding periodic times.

$$I_a = \frac{a^2 \cdot g}{4\pi^2 l}(M \cdot T_{p1}{}^2 - m \cdot T_{p2}{}^2).$$

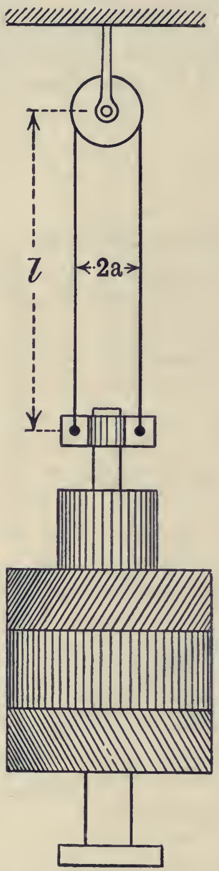

FIG. 410.

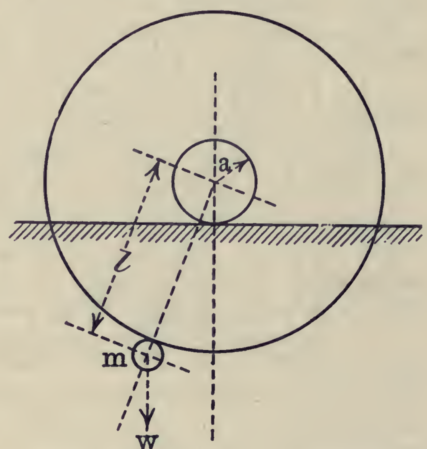

FIG. 411.

To give I_a in kilogrammes-(metres)^2, a and l must be expressed in metres, $g = 9·81$ and M and m are in kilogrammes of mass; or if a' and l' are in feet, and W and w are the weights in lbs.,

$$I_a = 0·0344 \frac{(a')^2}{l'} (W \cdot T_{p1}{}^2 - w \cdot T_{p2}{}^2) \text{ kilogramme-(metres)}^2 \ldots (158)$$

The length of l should be great as compared with $2a$.

* Cp. *Journ. Inst. Electr. Eng.*, vol. xxxi. p .664, "The Breaking of Shafts in Direct-coupled Units," by Messrs. Frith and Lamb.

In the case of a large armature, if its shaft of radius a is supported on knife-edges and it is set swinging with a small weight attached at radius l from the axis of the shaft (Fig. 411), the radius of gyration of the small weight itself being negligible as compared with l and k, then for small oscillations the square of the radius of gyration of the armature alone is

$$k^2 = \frac{m}{M}\left\{g.l.\frac{T_p^2}{4\pi^2} - (l-a)^2\right\} - a^2$$

where m is the mass of the attached weight, and M is that of the armature, and T_p is the time in seconds of a complete swing to and fro. Neglecting a^2 as small in comparison with k^2, we then have

$$I_a = Mk^2 = m\left\{g.l.\frac{T_p^2}{4\pi^2} - (l-a)^2\right\}$$

and if w is the weight in lbs. of the small attached mass, and l' and a' are in feet

$$= 0{\cdot}0344\ wl'\ \left\{T_p^2 - 1{\cdot}22\frac{(l'-a')^2}{l'}\right\}\ \text{kilogramme-(metres)}^2 \quad . \quad . \quad . \quad (159)$$

With still larger armatures having considerable moment of inertia, a retarding torque may be applied by means of a mechanical brake formed by a band or cord passed over the shaft or pulley, with one end carrying a weight and the other end anchored to a spring balance. Thus the second intermediate curve of Fig. 409 might equally well have been obtained by an additional mechanical torque T_o; if the difference between the reading of the spring balance and the weight had been 4·16 lbs. = 1·885 kilogrammes weight at a speed of 600 revs. per min. and the radius of the fly-wheel or pulley to which the brake was applied was 10 in. = 0·254 metre.

$$T_o = -1{\cdot}885 \times 9{\cdot}81 \times 0{\cdot}254 = -4{\cdot}7 \text{ metre-hectokilodynes.}$$

The moment of inertia of the armature and fly-wheel is then

$$I_o = \frac{T_o}{d\omega_2/dt - d\omega_1/dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (160)$$

$$= \frac{-4{\cdot}7}{-2{\cdot}62 + 1{\cdot}1} = 3{\cdot}1 \text{ kg.-(metres)}^2.$$

When fully excited and with the fly-wheel fitted in order to prolong the time of coming to rest, the lowest curve of Fig. 409 was obtained, whence at 600 revs. $d\omega_3/dt = -3{\cdot}27$, the angle α_3 being 73°. The watts expended in overcoming all losses are at 600 revs. $3{\cdot}1 \times 62{\cdot}8 \times 3{\cdot}27 = 636$, while the rate of loss by friction and windage is $3{\cdot}1 \times 62{\cdot}8 \times 1{\cdot}1 = 214$, the difference of 422 watts being the loss by eddy-currents and hysteresis, which may be compared with the figure of 340 obtained for the eddy-loss alone from Fig. 403. Graphically, as shown at the side of Fig. 409, if the watts absorbed by friction are w_f, the watts absorbed by eddies and hysteresis are $w_f \times \frac{BB''}{AB}$, or if with the fly-wheel in position a brake had been applied to give the intermediate curve and the watts absorbed by the brake were $w_1 = T_o\omega$, the watts absorbed by eddies and hysteresis are $w_1 \times \frac{BB''}{BB'}$. The curves, such as Figs. 403 and 404, may also be themselves used to determine the moment of inertia of the armature; if at any speed and excitation the watts taken to drive the armature as a motor are w, and the rate of change of the revolutions at the same speed and excitation is $\frac{dN}{dt}$ from the retardation curve, $\frac{w}{N\frac{dN}{dt}} = C$ should give in all cases consistent values of the constant $C = 0{\cdot}0109I$. Indeed the two methods of the preceding and the present sections when combined afford a useful check upon the readings taken by the motor-current method, since for all speeds the determination of I from the retardation curve must give the same result if the readings are correct.*

If the armature has a very large moment of inertia, the additional torque required to calibrate the readings of the retardation curve may also be applied directly by electrical means; a circuit of suitable resistance is closed on the terminals of the armature when it is in process of coming to rest, and the additional watts thus added are calculated from the volts and amperes given off at the different speeds.†

For retardation as for motor-current tests it is essential that the machine should be run beforehand for a sufficient number of hours to allow of the bearings reaching a steady temperature, since upon this depends so largely the friction loss.

§ 12. **The friction loss.**—Experiment shows that, as the speed increases from rest, the coefficient of friction at first falls rapidly, is then nearly constant at its minimum value within a small range of speed, and lastly rises, giving a loss after the 1·5th-power law. Correspondingly each retardation curve when very accurately taken shows towards its end an inversion point and sharp bend (cp. Fig. 405). The exact location of this point at which a kind of seizure takes place is chiefly dependent upon the speed at which the coefficient of friction reaches its minimum, but it also depends upon the proportion of the other retarding causes, whether hysteresis or eddies, at this speed; it occurs, in fact, at that speed for which the combined retarding torque from all causes is a minimum. Thus it is reached at a relatively higher speed when the friction of bearings and of the air is alone acting; but when the field is excited and the braking action of eddy-currents is added, since their torque is proportional to the speed, the minimum combined torque occurs at lower speeds. At very slow speeds, the coefficient of

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* Cp. Professor Peukert, "Measurement of Losses in Dynamos," *E.T.Z.*, vol. xxii. p. 393.

† See Dr. Sumpner, "The Testing of Motor Losses," *Journ. Inst. Electr. Eng.*, vol. xxxi. p. 632.

friction is also affected by the rapidity of the speed-changes. There is a certain time-lag of the oil-film in point of thickness both when the machine is slowing down and also when it is being run up ; but in the former case the coefficient of friction is smaller than would be the case if a steady state was reached, while in the latter case the opposite holds. The coefficient of friction at the end of the retardation curve therefore depends upon the time during which the process lasts, and its steady value cannot be thence obtained for very slow speeds.*

A convenient method of determining the friction losses has been given by C. Kinzbrunner (*E.T.Z.*, 1903, p. 451), which consists in passing a current through the armature at a low voltage with the brushes shifted away from the symmetrical position so that the forward ampere-turns of the armature within twice the angle of lead excite sufficient field to run the armature ; no other excitation is present. If the applied voltage be from 10 to 30 per cent. of the machine's full voltage, and can be closely regulated, a single setting of the brushes will enable the machine to be run at any speed up to or even above its full normal speed. This—the most favourable position for the brushes—can be found by running the machine up to the highest desired speed, and adjusting the brushes until the current is a minimum ; it is usually near to the pole-edge. The current and voltage taken by the armature at different speeds are then measured ; the current and watt consumption are plotted in relation to the speed, and the intersection of the watt curve with the vertical axis taken in connection with the starting current enables the resistance of the armature and brushes to be calculated, and the loss over them for other currents to be calculated. The subtraction of this loss from the total watts gives a curve of the loss over the friction of the bearings, of the air, and of the carbon brushes. The latter can next be separately determined by employing one or two brushes per arm. The actual field is so small that the losses from hysteresis and eddies during the experiment are entirely negligible.

Given the values of the friction watts w_{f1} and w_{f2} at two widely different speeds, N_1 and N_2, the former high and the latter low, on the assumption above mentioned that the loss from friction of the bearings increases as $N^{1.5}$ and that by windage as N^3, the two coefficients pertaining respectively to the bearings and air may be obtained in the manner suggested by Dr. Finzi † as

$$f_b = \frac{N_1^3 . w_{f2} - N_2^3 . w_{f1}}{N_1^3 . N_2^{1.5} - N_2^3 . N_1^{1.5}}$$

$$f_a = \frac{N_2^{1.5} . w_{f1} - N_1^{1.5} . w_{f2}}{N_1^3 . N_2^{1.5} - N_2^3 . N_1^{1.5}}$$

A rough-and-ready empirical formula which gives reasonable values for the total watts from the friction of two bearings and from air resistance is

$$\text{watts} = 0{\cdot}0005\ W_a N$$

where W_a = weight of armature in lbs. and N = revs. per min.

§ 13. Efficiency test by Hopkinson's method.

—The *efficiency* of a dynamo, or the ratio—

$$\frac{\text{power supplied from the terminals of the dynamo to the external circuit}}{\text{power given to the shaft of dynamo by the prime mover (engine or belt, etc.)}}$$

is often calculated as in §§ 6 and 7 by adding the various losses in field, armature, and friction to the output, and dividing the output by the sum of output + losses. But completely reliable results are only to be obtained

* *E.T.Z.*, 1903, p. 916 (Finzi). † *Electr. Eng.*, vol. xxxii. p. 318.

by direct measurement of the efficiency.* In cases where the dynamo is coupled directly to the engine, the power indicated in the engine cylinders, less the portion of this power which is wasted in the engine itself, gives the brake horsepower supplied to the dynamo; the results, however, of such a method of calculation cannot be regarded as very accurate, owing to the difficulty of ascertaining the exact value of the waste in the engine. Another method is to transmit the power from the prime mover to the dynamo through a transmission dynamometer which registers the power passing through it; unfortunately, such a dynamometer when of large size is both costly and difficult to manage, and, further, does not admit of very accurate readings being obtained from it. A much more exact method is that due to Dr. Hopkinson,† by which two similar machines are so coupled together that the one acts as a motor driving the other as a dynamo. The output from the latter, being returned to the armature of the motor, supplies the greater part of the power required to drive it; and it is thus only necessary to supply to the motor from some external source the amount of power expended in the losses within the two machines. As this is but a small fraction of the total power developed, it is more easily measured on a transmission dynamometer, and even a large error in its determination, since it only affects the comparatively small item of the waste power, produces but slight error in the result. The only objection to the method is that it requires two machines of exactly similar size and output. These are driven at their normal speed and approximately at their normal voltage; the field of one, M, is, however, slightly weakened by a rheostat in its magnet circuit, so that its internal E.M.F. (E_2) is less than the terminal E.M.F. (E_1) of the other machine D; hence D sends a current through M as a motor, and by means of the rheostat the amount of this current is regulated until it corresponds to the normal armature current of either machine. Let $W =$ the total mechanical power in watts supplied from an external source to the motor armature, and $C =$ the dynamo armature current which is also passed through the motor armature, the fields of both being separately excited. Neglecting the loss over the connecting leads, which can be made as small as desired, if we deduct from W the losses in the armature resistances of D and M, the remainder, $W - C^2(R_{aD} + R_{aM})$ is the total loss by eddy-currents, hysteresis, and friction in the two armatures. The field of the one is stronger than that of the other, but if the E.M.F. of D is made slightly greater, and the E.M.F. of M slightly less than the normal voltage of either machine, the error will be very small when this loss is

* Cp. especially Dr. C. V. Drysdale, *Engineering*, vol. lxxx. p. 679, where a method of testing giving very accurate results is described.

† A description of this method is to be found in Dr. Hopkinson's paper on "Dynamo Electric Machinery," *Phil. Trans.*, 1886, reprinted among his *Original Papers on Dynamo Machinery* (Whittaker & Co.), p. 112 ff.

assumed to be equally divided between the two so long as the machines are of such size, say over 30KW., that the efficiency of each is not less than about 90 per cent. Let

$$\frac{W - C^2(R_{aD} + R_{aM})}{2} = \frac{L}{2};$$

then the commercial efficiency of the dynamo is

$$\eta = \frac{E_1 C}{E_1 C + C^2 R_{aD} + \frac{L}{2} + E_{f1} C_{f1}} \quad . \quad . \quad . \quad . \quad . \quad (161)$$

$E_{f1}C_{f1}$ being the watts expended in magnetising the field of the dynamo. Where it is not necessary to obtain such great accuracy, part of the armature current of the dynamo D may also be used to excite the field-magnets of both machines; in this case, assuming the machines to be shunt-wound, if C_1 = the sum of the currents C_{aM} and c_{sM} supplied respectively to the armature and field of the motor, the efficiency of the dynamo is

$$\frac{E_1 C_1}{E_1 C_1 + C_{aD}{}^2 R_{aD} + \frac{L}{2} + E_1 c_{sD}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (162)$$

where $\qquad L = W - C_{aD}{}^2 R_{aD} - C_{aM}{}^2 R_{aM} - E_1(c_{sD} + c_{sM}).$

It is convenient in carrying out these tests to couple the two shafts of motor and dynamo rigidly together, in one line; but it should be remarked that when this is done in the case of machines intended for belt-driving, the loss by friction in the bearings may not reach its normal amount, since the pull of the belt corresponds to the transmission of but a small fraction of the normal power. On the other hand, when the motor and dynamo are coupled together by belt, an extraneous loss of power in bending the belt is introduced.

A further improvement of the above method consists in the use of a third dynamo as the external source whence the waste of energy in the system is supplied. This auxiliary dynamo need be of but small size, and may be coupled either in series or preferably in parallel with the two machines which are to be tested. When the method is thus modified, all the measurements can be made electrically by one voltmeter and one ammeter, and further, as regards the efficiency of the two armatures, great accuracy in the calibration of the instruments is not of such vital importance.

Fig. 412 shows the series arrangement, in which the auxiliary dynamo A adds volts to the terminal E.M.F. of the dynamo, and must be capable of carrying the full current of the machines to be tested. The voltage of the auxiliary machine, being approximately $= \frac{\text{total losses in the two machines}}{\text{armature current}}$, will be from 20 to 40 per cent. of the voltage of either dynamo or motor according to their efficiency. The field of the dynamo D must be weakened by the rheostat *r*, and the two-way switch enables the observer to read in quick succession either the combined voltage on the motor (E_m) when the arm is placed on contact *a*, or the terminal

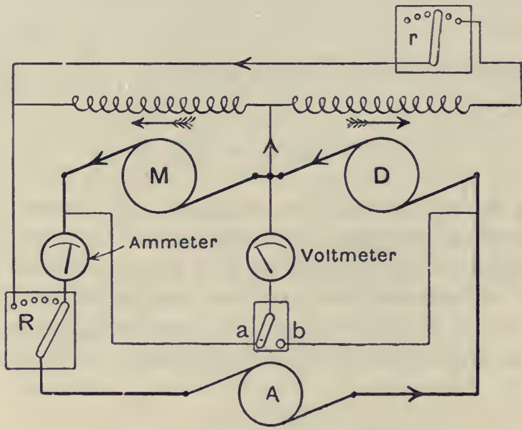


FIG. 412.—Series electrical arrangement of Hopkinson efficiency test.

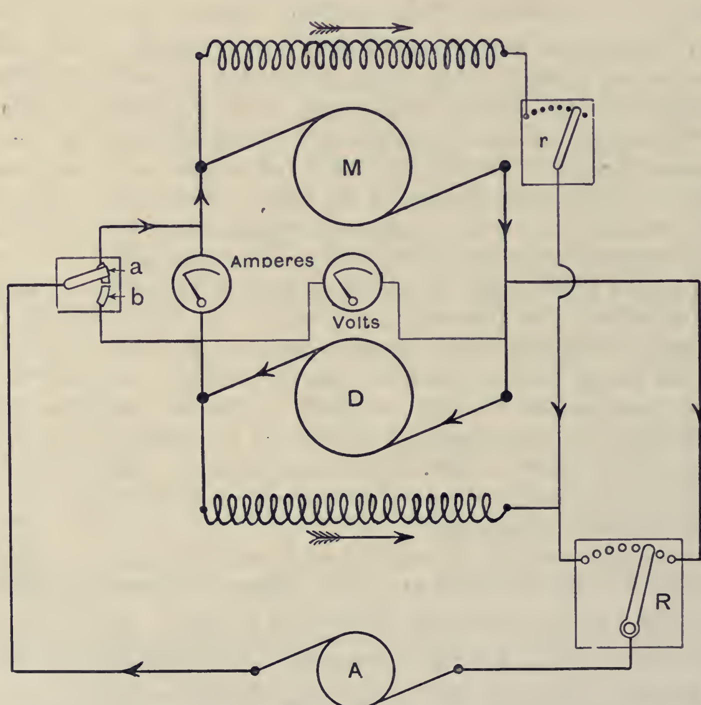


FIG. 413.—Parallel electrical arrangement of Hopkinson efficiency test.

voltage of the dynamo (E_1) when it is placed on *b*. If the dynamo and motor are both separately excited from a fourth dynamo, and C be the amperes passing through the system as read on the ammeter, the power supplied to M is CE_m, while the power obtained from D is CE_1. The voltage of the auxiliary dynamo is $E_m - E_1$, and the power added by it is $C(E_m - E_1)$. The combined efficiency of the two armatures of M and D is thus $\frac{CE_1}{CE_m} = \frac{E_1}{E_m}$, and the efficiency of each separately is very approximately $\eta = \sqrt{\frac{E_1}{E_m}}$. A small percentage error in the voltmeter produces but little error in the result, since the efficiency is only proportional to the square root of the voltage ratio. The fourth dynamo employed for separate excitation of the fields may also be dispensed with by the arrangement shown in the diagram, but in this case the system must be started by means of the switch and resistance marked R. When the arm of the switch is placed on the contact farthest to the left at starting, resistance is inserted in series with the armatures, and sufficient fall of potential is obtained to pass a small shunt current through the two fields in order to supply an initial excitation and start the motor. As the speed rises, the arm is brought over to the right until all the resistance is cut out of the armature circuit, while its presence in the shunt circuits will produce little effect. In calculating the efficiency, allowance must then be made for the fact that the dynamo armature is carrying not only its own shunt current but also that of the motor; hence, as in expression (162), if $C_1 = C + c_{sM}$, the combined efficiency of dynamo and motor is $\frac{E_1 C_1}{CE_m + c_{sM}{}^2 r_{sM}}$, and the square root of this gives approximately the efficiency of either machine; or, separately, the efficiency of the dynamo

$$= \frac{E_1 C_1}{\text{output of motor}} = \frac{E_1 C_1}{CE_m - C^2 R_{aM} - \frac{L}{2}}$$

and the efficiency of the motor

$$= \frac{CE_m - C^2 R_{aM} - \frac{L}{2}}{CE_m + c_{sM}{}^2 r_{sM}}$$

where L = the watts added by the auxiliary machine minus the electrical losses over the armatures and fields of the dynamo and motor.

The parallel arrangement is shown in Fig. 413, and it will be seen that the auxiliary dynamo adds amperes and must be capable of giving the full E.M.F. of the machines to be tested. It is now the field of the motor M which must be weakened by means of the rheostat *r*, and readings are taken of the amperes in quick succession for the two positions of the arm of the two-way switch. When this is placed on the contact *a*, the dynamo current is read, exclusive of its own shunt current, and when multiplied by the volts on the voltmeter gives the nett output of the dynamo. When the two-way switch is placed on contact *b*, the ammeter reads the motor current inclusive of its own shunt current, or its total input. The combined efficiency of the two machines is then $\frac{C_d}{C_m}$, and the efficiency of each machine is very closely $\eta = \sqrt{\frac{C_d}{C_m}}$, the voltage being maintained constant during the short time necessary to take the two readings of amperes. The system is started by means of the switch and resistance R, with the arm thrown over to the left, and the same switch will also enable the voltage to be regulated to the right amount if that of the auxiliary dynamo is slightly higher than that of the machines to be tested. As before, a small percentage error in the ammeter produces an almost negligible error in the result.

In both cases care must be taken to ensure the connections to the fields being such as to cause the one machine to act as a motor and the other as a dynamo, and any change of the rheostats must be made gradually, so that its effect on the system may not be masked by the inertia of the revolving armatures.*

§ 14. **Efficiencies of continuous-current dynamos.**—In Fig. 414 are given the efficiency curves of two typical dynamos for various proportions of their full loads. The exact shape of the curve depends upon the relative proportions of the constant and variable losses. Thus in a 1000-kilowatt traction generator, the two are nearly equal at full load, each being about 3 per cent. of the output, and giving efficiencies—

at full load	of	94·5	per cent.
,, $\frac{3}{4}$	,,	94·1	,,
,, $\frac{1}{2}$	,,	93	,,
,, $\frac{1}{4}$	,,	87·7	,,

Fig. 415 shows the efficiency at full-load that may be obtained under ordinary commercial conditions with machines of different outputs.† From this curve it will be seen that the rise in efficiency is but little after an output of some 75 kilowatts is reached. Any such curves

* For a full description of these two methods, see a paper by G. Kapp on "The Determination of the Efficiency of Dynamos," *Electr. Eng.*, January 22 and 29, 1892. Cp. E. Wilson, *Electr. Eng.*, vol. xxxii. p. 432 ; and for the extension of the Hopkinson principle to the testing of a single continuous-current multipolar machine, E. Kolben, *Elektrotech. u. Maschinenbau*, vol. xxvi. pp. 25–27, and *Electr. Eng.*, vol. xli. pp. 341, 380 and 415, and *Electrician*, vol. lx. p. 884.

† Cp. R. Goldschmidt, *Journ. Inst. Electr. Eng.*, vol. xl. p. 455 ; and for detailed curves of efficiency and losses in a Siemens 1500 kw. generator, see *Electr. Eng.*, vol. xlii. p. 15.

are, however, greatly affected by conditions of the design, and in especial by the speed, a low-speed machine being in general less efficient for the same output than one which runs at a high speed.

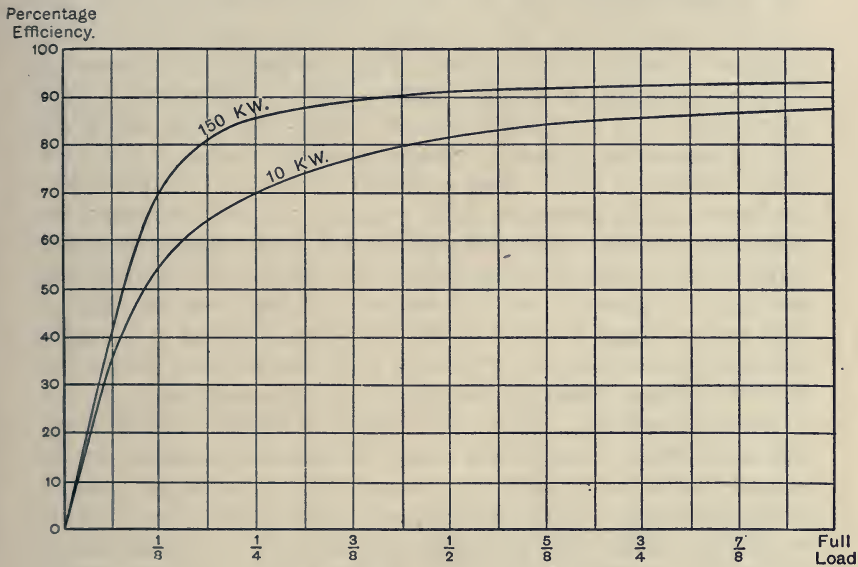


FIG. 414.—Efficiency at various loads.

For a given carcase the losses in excitation and over the armature resistance remain practically the same whatever the speed. The loss from hysteresis, friction, and windage bears a nearly constant ratio

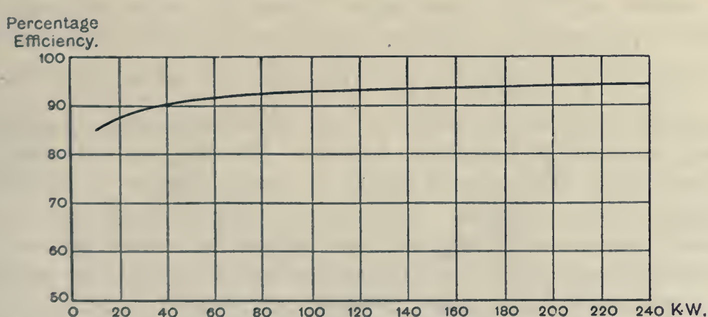


FIG. 415.—Efficiencies at full load.

to the output, and the only loss that increases faster than the speed is that from eddy-currents in the armature or pole-pieces. The output itself rises nearly proportionally with the speed, so that, on the whole, within ordinary limits the higher speed is favourable to the efficiency. There is, however, for a given armature with a given winding a certain speed at which the efficiency reaches a maximum, the assumption being made that the increase of the output with increasing speed is simply due to an increase in the volts; this speed is reached when the loss by eddy-currents is equal to the constant losses in the copper resistance of the armature and field-windings.* Such a fact is, however, of little assistance in the process of designing a machine in the first instance, since it presupposes a fixed copper loss. For a constant terminal voltage and speed, and given no-load losses, the maximum efficiency is reached when the variable losses proportional to the square of the armature current (*i.e.* practically the losses over the resistance of the armature and series windings) are equal to the no-load losses, but whether such a value for the armature current can be reached depends upon the limits set by heating and sparking. In plating dynamos, owing to the large proportion of the total voltage which is lost over the contact-resistance of the brushes and other connections, and also owing to the considerable friction loss from the numerous brushes, the efficiency is necessarily low; *e.g.*, in a 10-kilowatt machine giving 4 volts and 2500 amperes, the efficiency may fall between 60 and 70 per cent. Very small dynamos are unable to excite themselves owing to the comparatively large air-gap required for mechanical reasons, and when separately excited if the excitation is equal to the output, the efficiency is zero; hence, in very small sizes, as in models, ohmmeter generators, etc., a permanent magnet is employed.

§ 15. **Weights of continuous-current dynamos.**—Owing to the great diversity of the types and designs of dynamos, no expression even approximately accurate can be given for the weight of a machine in terms of its output per rev. per min.; yet for purposes of rough calculation and with a given type of dynamo the weight may often be taken as proportional to the two-thirds power of the watts per rev. per min., or $W = c\left(\frac{w}{N}\right)^{\frac{2}{3}}$ lbs., and c has then such values as 250–200 in the case of multipolar machines with slotted armatures, the weights being exclusive of that of the bedplate. The weight of the armature bears a higher proportion to that of the entire machine in multipolar machines than in bipolar; in the former it rises gradually from 20 per cent. in small to 42 in large machines, and averages about 35 per cent. Correspondingly its cost as compared with that of the whole machine is comparatively high, namely, about 45 per cent.

The above approximate formula for the weight may in the case of large machines with $D_{''}^2 L_{''} > 15{,}000$ be also expressed as $27\,(D_{''}^2 L_{''})^{\frac{2}{3}}$, since $\frac{w}{N}$ will then be $\simeq \frac{D^2 L}{21}$. But in the case of smaller machines the weight is proportional rather to the three-fourths power of D^2L, and may be given as $W = 8{\cdot}8\,(D_{''}^2 L_{''})^{\frac{3}{4}}$ lbs.

* A. G. Hansard, *Electrician*, vol. xxxviii. p. 401.

CHAPTER XXI

THE WORKING AND MANAGEMENT OF CONTINUOUS-CURRENT DYNAMOS

§ 1. **The interconnection of dynamos.** — In many cases it is required to connect two or more dynamos to the same pair of mains in order to increase the amount of electrical energy that can be supplied to their common circuit. If the dynamos are coupled together in series, although the maximum current that may be passed through them is no more than the current permissible in the smaller of the two, yet the available voltage is increased. On the other hand, if two dynamos of the same voltage are coupled in parallel, the total amount of current can be increased, although the terminal voltage remains unchanged. Any such coupling together of dynamos must necessarily be so arranged that the working of one dynamo does not interfere with the proper working of the other or others; and as this requires certain precautions in their interconnections, the more usual cases which occur in practice will here be shortly considered.

§ 2. **The coupling of series-wound dynamos in series.**— The simplest case is the coupling of two series-wound continuous-current machines in series. To effect this, it is only necessary to connect the + terminal of one machine with the − terminal of the other, the external circuit being then connected to the remaining terminals of the pair. Such an arrangement is not unusual in cases of transmission of power over long distances, where it is necessary to work with high pressures in order to combine economy in the first cost of the copper leads with a high efficiency of transmission. It has been already mentioned that the delicate nature of the commutator hardly permits of more than 4000 volts being generated in any one continuous-current dynamo; but by the use of two or more similar dynamos of the closed-coil class coupled in series, and each giving, say, 3000 volts, a combined E.M.F. of 30,000 or more volts is obtained on the external circuit.

The employment of such a system for the transmission of power over long distances by high-tension direct current * has been brought into prominence by M. Thury. In the St. Maurice to Lausanne

* Cp. Highfield, *Journ. Inst. Electr. Eng.*, vol. xxxviii. p. 471; and *Electr. World and Engineer*, vol. xlviii. p. 755.

transmission six turbines are employed, each driving two generators, and each of the latter giving 2250 volts and 150 amperes at 300 revs. per. min.; being coupled in series, the total line pressure is 27,000 volts. Between Moutiers and Lyons, four generating sets, each consisting of a turbine driving four dynamos in series, give a total line pressure of 57,600 volts, the output of each dynamo being 75 amperes, 3600 volts at 300 revs. per min. The turbine is connected to the dynamos by an insulating coupling, and the base of each machine is separately insulated from earth, the foundation block itself being also of special insulating material.

§ 3. **The coupling of shunt-wound dynamos in parallel.**— The coupling of two or more dynamos in parallel is even more frequent. In all large installations and central electric works, as the load increases, more dynamos have to be brought into use, without interruption to the supply from the machines already running, and in such cases, in order to obtain the greatest economy, each dynamo should be worked as long and as closely as possible at its maximum output: this is best attained if they are all capable of being worked in parallel, an additional machine being switched on to the same mains as soon as the load exceeds the combined output of those already at work. To connect two shunt-wound dynamos in parallel it is simply necessary to join their positive terminals to form a common + and their negative to form a common − terminal. If, as is usually the case, one machine is already running and excited, care must be taken that it is not joined in parallel with a second machine while the latter is at rest or is unexcited. If this were done, the armature of the second machine would form a short-circuit to the first, and would present no E.M.F. opposing an excessive rush of current through its low resistance. Hence the second machine, B, must be run up to its normal speed, and before it is thrown into parallel it must either be allowed to excite itself to approximately the same voltage as that of machine A, or B's shunt circuit must be closed on the first machine so as to excite B's field before its armature circuit is closed: the machines may then be safely thrown into parallel, and each will supply a certain part of the load, the exact proportion in which the total current is divided between them depending on their respective internal E.M.F.'s and armature resistances. The condition which determines this division is that, after deducting the volts lost by the passage of the current over the armature resistance of either machine from its internal E.M.F., the remainder or the terminal voltage must be alike in both machines. Thus, if two similar machines, similarly excited and run at the same speed, be coupled in parallel, each will take half of the total current. If the speed of one be now lowered or its field weakened, its current will gradually pass over to the other machine; when its internal E.M.F. falls to equality with the terminal voltage of the second machine, it supplies no current at all, the whole of the load being thrown on to one machine. To take a numerica example, suppose that each machine runs normally at 1000 revolutions, and is then excited with 13,000 ampere-turns giving $Z_a = 4{,}300{,}000$, and $E_a = 103$ volts; further, that the loss of volts over the armature at the full load of 100 amperes reduces the terminal voltage E_b to 100 volts. If the speed of machine B falls, a larger portion of the current passes over to machine A. The increased loss over the armature resistance of A and the increased reaction of the armature current on its field combine to reduce both its internal and terminal voltages. When it takes the whole of the current, let $Z = 4{,}100{,}000$ be the number of lines that are produced by the 12,000 ampere-turns due to the terminal E.M.F. of 92 volts; in other words, $C_e = 200$ amperes, and $E_b = 92$ volts, give a point on its characteristic curve for a constant speed of 1000 revolutions. This same terminal voltage will, however, in the case of machine B, which is carrying no current, give, say, $Z_a = 4{,}250{,}000$, and this flux will give an internal E.M.F. of 92 volts when B is running at 900 revolutions. Thus the speed of B may be reduced by 10 per cent. before the whole of its load passes on to machine A, provided that the speed of the latter is kept strictly constant. On a further reduction in the speed or strength of field of B, its internal E.M.F. falls below the terminal voltage of A, and the latter then drives a reverse current through B's armature: the internal E.M.F. of B *plus* the volts lost over its armature resistance due to the reverse current are then equal to the terminal voltage of A. The effect of the reverse current through B's armature is to assist in turning it as a motor with the same direction of rotation as before, without mechanically damaging the brushes, and thereby it tends to keep up B's speed. The electrical interaction of the two machines with drooping characteristics thus exerts a considerable inherent influence, tending to equalise their speeds and loads; it is therefore easy to work shunt-wound dynamos in parallel, and the share of the current which each machine takes is easily regulated by altering its speed or its field excitation. In fact, without interfering with the speed, the rheostat in the shunt circuit affords a ready means of loading or unloading the continuous-current machine to any desired extent. In practice, when, as is most often the case, the machines are driven by separate prime movers, the equalising tendency is also assisted by the mechanical action of the prime movers themselves. Thus, when the load of one machine increases, the speed of its engine or other prime mover falls and tends to check the increase, while, for a similar reason, when a machine fails to maintain its fair share of the load, the speed of its prime mover rises, and tends to keep up its E.M.F.

The use of shunt-wound dynamos in parallel is common in central stations for electric lighting: as soon as the load becomes too great for a single machine, a second is run up to speed, excited to an equal voltage and switched into parallel with the first, the same process being repeated with other dynamos as often as required. When so connected, they all supply current to a common pair of "omnibus bars," whence the feeders are run to the network of mains. As a precautionary measure, a magnetic switch is frequently inserted between each dynamo and the omnibus bars: this automatically flies off and breaks contact if the armature current falls below a certain minimum; and hence, if for any reason a dynamo begins to slow down, it is cut out of circuit before a reverse current passes through it, while its load is taken up and divided among the other dynamos at work, without interruption to the general supply of current.

§ 4. **The coupling of series-wound dynamos in parallel.**—When series-wound dynamos are connected together in parallel after the same fashion by joining their like terminals, we are met by the difficulty that if, for any reason, the E.M.F. produced by one machine, B, falls considerably below that of the other machine, A, then the current through B is reversed; and this, since the machine is series-wound, reverses its polarity. The direction of B's E.M.F. is consequently reversed, with the result that both dynamos simply act in series round their own internal resistances, and in a short time would be damaged by the excessive current. This difficulty is, however, at once overcome by the addition of an *equalising* wire connecting the ends of the series coils adjoining the brushes (Fig. 416).

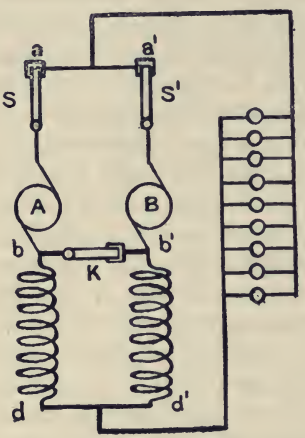


FIG. 416.—Two series-wound dynamos coupled in parallel.

It is especially important that this equalising lead, *b b'*, should be of such large area that its resistance is practically negligible as compared with the resistance of either of the series windings. By it the extremities of the two field-windings are joined in parallel, and the current supplied to the external circuit flows in the same direction through both and magnetises each equally. Should the internal E.M.F. of machine B fall below the voltage at the brushes of A, a reverse current passes through B driving it as a motor, but as this motor current flows through the lead *b b'*, and does not pass in a reversed direction through the series winding of B, it does not reverse B's polarity. If the fall of volts over *b b'* is considerable, a certain proportion of the motor current may pass through the alternative path of *b d d' b'*, but this is prevented by making *b b'* of sufficiently low resistance, as mentioned above. If it be required to switch B into parallel with A while the latter is running, it is necessary, as in the case of shunt machines, to excite B to approximately the same voltage: this is effected by closing the switch K while B is running, and a few seconds later switch S′ may be closed.

§ 5. **The coupling of compound-wound dynamos in parallel.**—Series-wound dynamos are, however, but seldom required to work in parallel, and the above remarks are merely introduced owing to their applying equally well to the series windings of compound machines when worked in parallel. The arrangement usually adopted for these is shown in Fig. 417, from which it will be seen that when the switches are closed, exactly as in the case of series machines and for the same reason, there are three junction-points, namely, the leads from one set of brushes *a a′* to the switchboard, the leads from what may be called the "outer" ends *d d′* of the series coils, and also the equalising terminals, which are themselves the junctions of the "inner" ends *b b′* of the series coils to the other set of brushes. Again, the equalising wire *b b′* must be of negligible resistance as compared with the series windings, since any reverse current through the series-coils of a compound-wound dynamo will partially demagnetise it and finally overpower the shunt. In order to throw the one machine, say B, into parallel with the other, A, when the latter is running, B must be run up to approximately the same voltage as A, the excitation being provided by the shunt alone; switch K is then closed, and finally switch S′, while, if it be required to withdraw machine B from parallel connection, its speed should be slightly reduced until it is supplying little or no current, after which switch S′ is opened, and lastly switch K. The opening or closing of the two connections in due order may be conveniently effected by a composite switch, by which the contact at K is made first on closing the switch, and broken last on opening it.

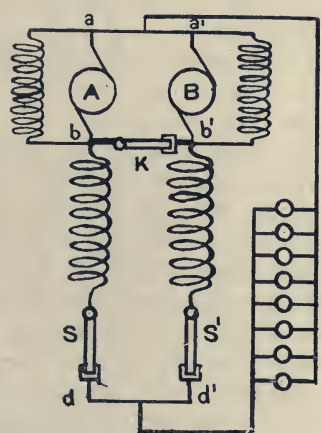


Fig. 417.—Two compound-wound dynamos coupled in parallel.

If the dynamo ammeters are placed in the leads to the switchboard bus-bars on the same side *d d′* as the series coils, the sum of the currents gives the total amperes supplied to the external circuit, but the separate readings of the instruments in no way indicate how the load is divided between the several armatures, and one or more might be running as motors with current supplied by the other machines. It is, therefore, very desirable to place the ammeter of each dynamo in the lead to the bus-bar which is on the side *a a′* opposite to the series coils, when it will measure the current actually passing through the machine to which it is connected. Further, it is advantageous to employ moving-coil ammeters with a zero position and a short scale on one side, so that the readings of the needle to one or the other side of the zero render the state of affairs evident at a glance. In every case, either the hand-wheel governing the speed of the engine or the shunt-regulating switch must be employed to secure equal divisions of the load.

§ 6. **Regulation of load between compound-wound dynamos in parallel.**—In the case of over-compounded dynamos, it is not easy to secure an equal division of the load between several machines owing to small differences in their characteristic curves, and even if at full load the division is proportionate to their respective capacities, it may not retain this strict proportionality throughout a wide range. The first requirement for running compound-wound machines of different make or of different output in parallel is that the drop of volts over the series windings and the leads connecting the outer positive and negative terminals to the bus-bars on the switchboard should be the same for all machines when carrying their proper share of the total current delivered to the board; in other words, the resistances of the series coils and connecting leads should vary inversely as the normal full-load current for each machine. If the junction-points or the bus-bars were immediately upon the machines, as in Fig. 417, and therefore added practically no resistance, it would suffice that the external characteristics of the several machines at their terminals should be alike to secure proper division of the load. But such is not the case in practice, since the switchboard is usually at some distance and also is at different distances from the machines; the connecting leads must therefore have appreciable resistances, and these may differ even when the machines are of equal size. It is then necessary to insert in the connecting leads small resistances to carry the full-load current, and of such amounts that the drop of volts over both series-winding and connecting leads again becomes in each case equal. The external characteristics at the bus-bars on the switchboard are thus once again made similar, and although over-compounded each machine will take its due share of the total load, so long as it is running at the correct speed. But, whatever precautions are taken, satisfactory parallel working must in the end depend upon the mechanical governor as definitely fixing the speed for each amount of steam supply to the prime mover. Thus suppose that two dynamos of equal size are running in parallel with the load equally divided between them, and that for some reason the speed of one falls slightly; its load decreases, while that of the other machine rises, and this change is accompanied by a similar change in their voltages, so that the load tends to pass completely over to the machine which has maintained its speed, and this tendency is only held in check by the mechanical governor; no inherent electrical effect comes into action until a motor current is actually supplied from the machine of higher voltage to assist in keeping up the speed of the machine whose voltage has fallen. Perfect division of the total current between the several series coils, and also at the same time practically zero resistance between the "inner" ends of the series coils, can be realised if the junction-point between the "inner" ends is transferred to the switchboard and is effected by the equaliser bus-bar (marked =) as

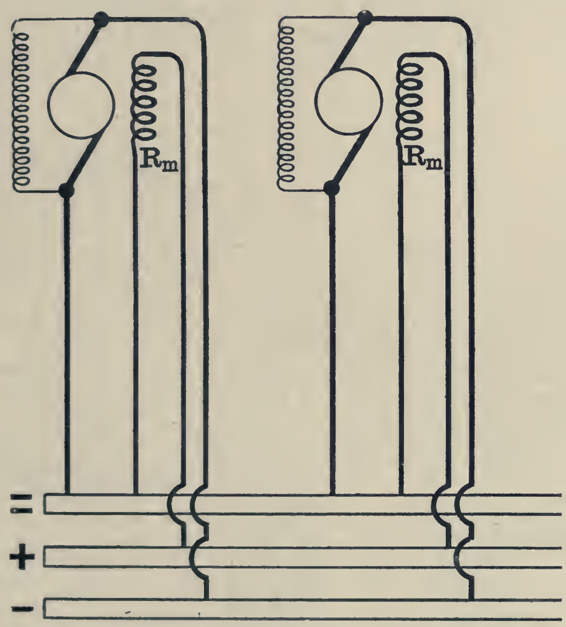


Fig. 418.—Compound-wound dynamos in parallel.

shown in Fig. 418. The resistance of the connecting leads is then adjusted so that the drop of volts at full load is in each case equal and the machines are on complete equality. But such an arrangement requires four leads to the switchboard from each dynamo, and a triple-break switch for each dynamo.

The amount of the compounding action of a machine is frequently regulated by a *diverting switch and resistance*, which shunts a portion of the current away from the series turns through a bye-pass. When this is the case, any alteration of the diverting switch of one machine also affects the current through the series turns of the other machines. This may be corrected by the employment of a *compensated series* *regulator*, of which the principle is shown in Fig. 419 in connection with a dynamo which is used as a shunt machine on a lighting load and as a compound-wound machine on a traction load. In the position of the switch shown in the diagram, the current passes immediately through the arm from *a* to *b*, and there is no compounding action at all. But when the arm is moved in a clockwise

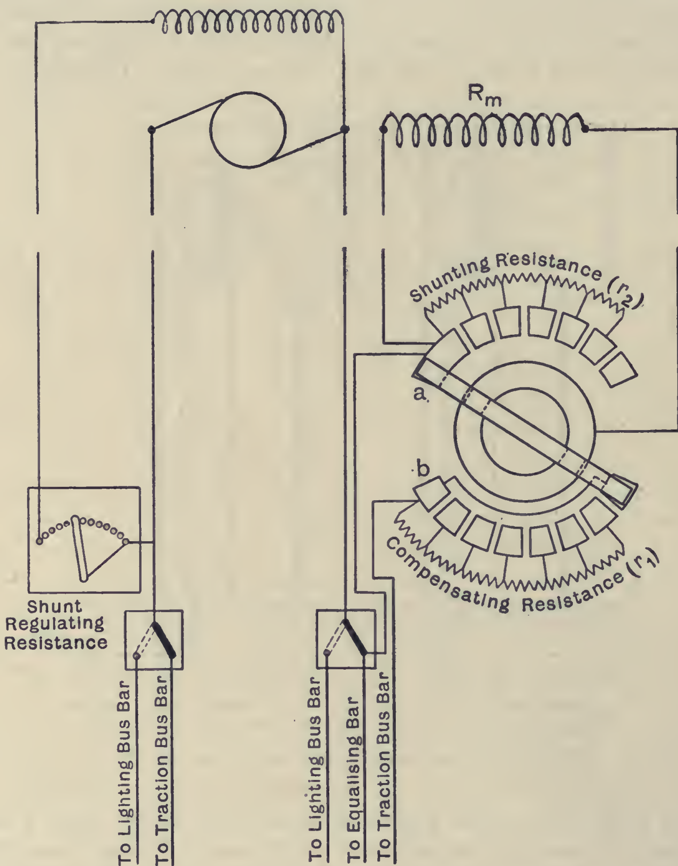


FIG. 419.—Series regulator for compound-wound dynamos in parallel.

direction, a shunting resistance r_2 is inserted in parallel with the series turns R_m, and as the resistance r_2 increases step by step, more and more of the total current C passes through the series turns. At the same time, with a movement of the switch from its starting position a compensating resistance r_1 is inserted in series with the parallel branches of the series turns and shunting resistance, and this resistance

r_1 is gradually cut out until with the arm on contact b all the current passes through the series turns, and the maximum compounding action is obtained. On any contact (except the starting position) $r_2 = \frac{C_m}{C - C_m} \cdot R_m$, and $r_1 = \frac{C - C_m}{C} \cdot R_m$, where R_m is the resistance of the series turns and of the leads therefrom to the regulating switch. By this means the resistance between the points a and b is maintained perfectly constant at a value $= R_m$, and an increase of C_m from 0 to C is obtained by six equal steps without in any way affecting the other machines which are in parallel.

§ 7. **Three-wire dynamos.**—For the purpose of using a single dynamo on a three-wire system without any auxiliary balancer, it is necessary to make connection for the third wire with a point nearly midway in potential between the positive and negative brushes. If a third set of brushes were applied at points on the commutator intermediate between the positive and negative brushes, their position would, strictly speaking, require adjustment according to the amount of load on the machine and also according to the amount of out-of-balance current, but, apart from this, such destructive sparking would ensue as to render the method practically inadmissible. An armature wound with two independent windings, each for half the main voltage and the two connected in series, meets the case better, as the third wire can be connected to the common junction uniting the positive brushes of one armature with the negative brushes of the other; but it involves two commutators, one at each end of the armature, and two sets of brush-gear, so that its cost is great, and it does not admit of the voltage of one side being regulated independently of the other save by external means.

A simpler method by which, without moving machinery and with a single armature, a central point may be found for attachment of the third wire, is due to von Dolivo Dobrowolsky, and is not infrequently employed in small generating stations, as in factories, where some proportion of the motors or other plant is better suited by a low voltage, such as the half of the main voltage between the outers. It consists in the addition to the armature of two or more slip-rings each connected to the armature winding at a point or at points corresponding to an equivalent number of electrical degrees according as it is wave or lap, just as in a rotary converter, and in the connection of these slip-rings to a choking coil. The latter has a laminated closed magnetic circuit and is wound for as many phases as there are slip-rings. The central point of the windings of the choking coil, which is also called the "static balancer," is then practically midway in potential between the positive and negative brushes, and furnishes a point to which the third wire may be attached.

In its simplest form two slip-rings are employed, and the choking coil becomes "single-phase," although, strictly speaking, one-half represents a phase 180° displaced from the other half, and the third wire is attached to the middle of the winding of the coil. The two slip-rings are joined to any two points 180° apart on a wave-wound armature, or on a lap-wound multipolar each is connected to p points separated by a distance corresponding to a pair of poles, and these two sets of p points are mutually displaced by a distance corresponding to the pole-pitch. When the armature is at work, an alternating E.M.F. is set up between the slip-rings, and so long as the two sides of the network are exactly balanced or on open external circuit, this simply causes a lagging alternating current of frequency $\frac{pN}{60}$, which magnetises the choking coil. As the windings of the coil should have a very low resistance, the value of the magnetising current is practically fixed by the reactance $2\pi f L$, and the loss in watts is thereby kept small. The effective voltage applied to the coil is dependent upon the ratio of the pole-arc to the pole-pitch, but on the assumption of a sine-law distribution of the magnetic flux is with two slip-rings $\frac{1}{\sqrt{2}} = 0{\cdot}707$ of the E.M.F. at the brushes on the commutator, and in general with increased numbers of slip-rings bears the same ratios to the direct-current voltage as in a rotary converter.

The action of the balancer is as follows. With given unequal resistances on the two sides of the network, the potential of the third wire before the balancer is added is not midway between the potentials of the outers, but the side of higher resistance receives more than half of the total potential difference. When the balancer is added, there arises an out-of-balance current or difference in the respective currents on the two sides, and the amount of this difference depends simply upon the ohmic resistance of the choking coil and the resistance of the armature. Taking the case of a two-phase choking coil and a pair of slip-rings, at the particular moment when the potential difference impressed between the slip-rings is exactly equal to and balances the counter E.M.F. induced in the two limbs of the choking coil so that on a balanced network the instantaneous value of the alternating current would be zero, then on an unbalanced network at this moment the total out-of-balance current in the third wire is naturally divided equally between the two limbs of the coil, and these two halves flow through the two limbs either both outwards from or both inwards to the centre junction-point of the coil according to whether it is the positive or the negative side of the network which is the most loaded. Let a current through one limb of the choking coil be reckoned as + when outwards away from the centre junction-point, and as − when inwards towards it, so that at the above starting-point of time the currents in the two limbs are equal and of the same sign. If the current that would flow right through the choking coil from end to end on a balanced network for a given potential difference between the slip-rings is now defined as the natural alternating current, then on the unbalanced network at any moment or position of the armature the current in either limb of a two-phase coil is equal to the algebraic sum of the instantaneous value of the natural alternating current and half of the out-of-balance current. In other words, the actual instantaneous values of the currents in the limbs of the choking coil may always be reached by supposing the equal division of the out-of-balance current in the two limbs to continue permanently and the natural alternating current to be superposed upon this equal division.

At the starting-point described above or after half a period when again the out-of-balance current is actually divided equally between the two limbs of the choking coil, it is evident that the potential of the centre is either above or below the midway value by the amount necessary to pass half the out-of-balance current over the ohmic resistance of one limb or phase. By this condition with a given resistance of balancer the potential of the third wire and also the total out-of-balance current are both fixed; there is only one value of the centre potential giving a difference between the currents on the two sides of the network equal to twice one of the currents flowing outwards from or inwards to the centre, and this same condition also holds for all other times.

The above is, however, only strictly true when the resistance of the armature is neglected. Owing to the varying amount of this which is included in the circuit of the out-of-balance components, the potential at the centre does not, strictly speaking, remain absolutely constant. Prof. A. Sengel * has shown that if R_a is the resistance of the armature from brush to brush, ρ the resistance of one limb of a choking coil which has n limbs or phases, and i_o the total current in the third wire, the *average* amount by which the potential of the third wire is above or below the midway value is

$$i_o\left\{\frac{\rho}{n}+R_a\left(\frac{1}{12}+\frac{1}{3n^2}\right)\right\}$$

The average *difference* between the terminal volts on the two sides is, of course, twice this amount.

As the ohms in each limb of the balancer are progressively lowered, the potential of the middle wire is raised or lowered more and more nearly to a value midway between the outers, and at the same time the out-of-balance current approaches its ideal maximum when each side is fed with half of the voltage between the outers.

All that has been said above is equally true whether the balancer be a simple resistance or an inductive choking coil, but so long as it is non-inductive, the natural alternating current must always exceed half the out-of-balance current, and if it is to have much effect in improving the potential of the third wire in virtue of its low resistance, a large amount of energy must be wasted as the slip-ring tappings draw up to and pass the brush positions, so that the arrangement is very inefficient. This is obviated by making the balancer inductive, so as to check the growth of the currents towards the epoch of maximum applied voltage by means of the counter induced E.M.F. The two phases are wound on a common core or preferably are intermixed just as the primary and secondary coils of a transformer. Equal currents both flowing away from or both towards the centre junction-point will neutralise one another and have no magnetising action. But the algebraic difference of the instantaneous currents in the two phases will reproduce exactly the natural alternating current which magnetises the core, gives an alternating flux, and provides the requisite counter E.M.F. Half the out-of-balance current can now greatly exceed the natural alternating maximum, and the latter will show itself simply as an undulation superposed on the continuous current, the direction of the actual current in a limb in this case being never reversed.

With an increase in the number of slip-rings, the advantage is gained that the out-of-balance current is more uniformly distributed over the armature winding, and local heating is minimised. But three phases are not to be recommended, owing to the necessity of additional windings on the transformer in order to neutralise the magnetising effect from the out-of-balance continuous current equally divided between the three limbs.

* *E.T.Z.*, May 17, 1900.

§ 8. **Foundations and erection of dynamos.**—It remains to add a few remarks on the fixing and working of dynamos in general. In the first place, good foundations are as much an essential for successful working as a good dynamo, inasmuch as any vibration is most detrimental to the life of the armature, commutator, and brushes, and if transmitted to the surrounding floor and walls may have serious consequences. The ground should be excavated until a sound and solid bottom is reached, on which may be built up a foundation of concrete, or of concrete and brickwork, sufficiently massive to absorb and damp the vibration of a fast-running dynamo. With small dynamos, Lewis bolts having a tapering shank of rectangular section with jagged edges are used to hold down the bed-plate or slide-rails: the square holes required for these bolts are cut into the concrete, their position being marked off usually from a wooden template of the slide-rails, and when these latter are ready to be laid down, the bolts are dropped in and fixed by lead or sulphur cement run in round them. With larger dynamos, long holding-down bolts are used, passing right through to the bottom of the concrete and terminating at their lower ends in large square plates. The holes for such bolts are formed in the concrete by inserting long tapering wooden boxes, of square section and made collapsible for the purpose of withdrawing, through which pass the bolts; when the bed-plate is in position, thin cement is run in round the bolts until the holes are full, and is then allowed to set without disturbance. In all cases, the upper surface of the foundation should be carefully levelled with a straight-edge and spirit-level, or a smooth slab of York stone may be used as a seating on the top of the concrete.

When driven by belt, it is essential that the dynamo should be set square with the driving engine or shafting, in order that the belt may run fairly on the centres of the two pulleys. This is ascertained by taking a length of string and stretching it tightly from the far edge of the driving pulley or fly-wheel to the dynamo pulley; when the string just touches the near edge of the driving pulley, its transverse distance from a centre line drawn round the dynamo pulley should be the same both on the side nearer to the driving pulley and on the side farther from it.

The belt itself should be soft and flexible, and joined into an endless band either by a cemented and riveted joint, or by belt fasteners with a butt joint; a lapped or laced joint causes a jerk each time that it passes on to the dynamo pulley, which gives rise to vibration and fluctuation of the E.M.F., together with sparking. Wherever possible, the direction of rotation should be such that the under side of the belt is the tight or driving side.

Perfect steadiness of driving is of great importance, especially for direct incandescent lighting; a very slight fluctuation in the speed of the dynamo, even though it be not great enough to cause sparking, is immediately discernible as a pulsation in the light of the lamps, owing to the slight change of E.M.F. to which it gives rise. On this account many types of gas or oil engines are inadmissible for direct lighting owing to their great fluctuation of speed during each cycle, even when fitted with heavy fly-wheels. When such prime movers are used, the dynamo is usually worked in conjunction with an accumulator battery, and even then it is advisable for the dynamo to be itself fitted with a heavy disc fly-wheel. In such cases, the secondary cells should never be placed in the same room with the dynamo, since the cotton or other fibrous insulation of the dynamo wires is rapidly attacked and eaten away by the acid spray given off during the process of charging.

After the machine has been fixed in position, the armature should be turned round by hand to see that it revolves freely and that nothing is loose; it should then be run for some time with the brushes raised from the commutator, in order to test the alignment of the bearings and of the machine generally. The adjustment of the brushes when lowered on to the commutator requires careful attention: usually lines are cut on the collar or sectors of the commutator next to the outer bearing (these lines corresponding to the angular pitch of the poles), and to them the brushes must be set, the tips of each set of brushes carried by one arm of the rocker being in line with one setting mark.

All screw contacts should be firmly screwed up, and any dirt or lacquer (if such there be) on the points of binding-screws should be cleaned off. Want of contact of the brushes on the commutator or elsewhere may cause a failure of the dynamo to excite. The electrical connections should be carefully examined and verified; in especial, the connections to the field-magnet coils must be such as to agree with the "hand" of the armature winding. On starting, it should be borne in mind that a shunt-wound dynamo will not excite on a very low resistance, or a series-wound dynamo on a very high resistance. If, therefore, a shunt-wound dynamo fails to excite even when the main switch is open, a short-circuit in the leads is a possible cause. Or if a series-wound dynamo will not excite with its normal external circuit closed by the main switch, there may be a disconnection either within or outside the machine. Any such difficulty will usually be solved by testing with an ordinary linesman's detector; or, in default of a solution by this means, trial may be made with the connection of the brush leads to the field winding transposed, in case it is through misconnection that the field will not excite.

If a machine has been standing for long out of action, and its windings are consequently very damp, it should be run at a low voltage for some little time until it becomes thoroughly dry.

§ 9. **Care of machine in working.**—To bed a new set of carbon brushes, a long strip of emery cloth should be strained taut against the curved surface of the commutator, and then be drawn repeatedly in the direction of rotation under the face of the brushes, which are meanwhile pressed down on to it by their pressure springs. After any such operation the tips of the brushes must be cleaned from any adherent copper dust.

For the filling of the lubricators, copper oil-cans should invariably be used, since iron cans are liable to be drawn to the magnet, and thereby, perhaps, cause damage by catching in the armature. All oil-pipes and waste oil-chambers require occasional attention to see that they are not clogged. If from any neglect in this respect, or from original defective construction, lubricating oil creeps from the bearings on to the surface of the commutator, it becomes carbonised by any sparking at the brush tips, and forms a thin conducting film which bridges across the strips of insulation between the sectors of the commutator, the result being a loss of E.M.F., due to the local leakage which ensues between neighbouring coils; while if oil further makes its way on to the armature winding, it has a deteriorating effect on the protecting varnish, and causes adherent deposits of dirt and copper dust. Every dynamo should therefore be kept clean, and in especial its commutator requires scrupulous care on this point. A small air-compressor, by means of which the dust collected by the armature connections and winding may be blown out at intervals, is a valuable accessory to any large installation. The terminals and insulating washers of the brush gear occasionally require to be wiped to remove any dust which may have lodged on them. With carbon brushes paraffin wax may be sparingly used as a lubricant and to prevent chattering. If carbon brushes show signs of copper dust working into their bearing surface, any such coppering must be carefully scraped away. The dark burnished lustre which may be seen on the commutator of a good non-sparking dynamo is the sure evidence of a careful attendant. Occasionally, before stopping, fine emery cloth may be applied if the surface shows signs of wearing into grooves and becoming uneven. If a flat begins to develop on one or more sectors (Chapter XVIII. § 44) it may often be ground out by applying a hard stone with curved face to the commutator when running. But if the disease has gone too far for such remedy, the armature must be put in a lathe, and the commutator surface turned up true; the tool should be sharp and fine-pointed, and the feed should be light, so as not to drag the copper over the insulating strips of mica: after turning, it should be lightly filed with a smooth file, and finally examined to see that no particles of copper are embedded in the mica, and bridging adjacent sectors. Still better is it to grind the commutator true in place by means of a small motor-driven emery wheel.

CHAPTER XXII

ARMATURE WINDING OF ALTERNATORS

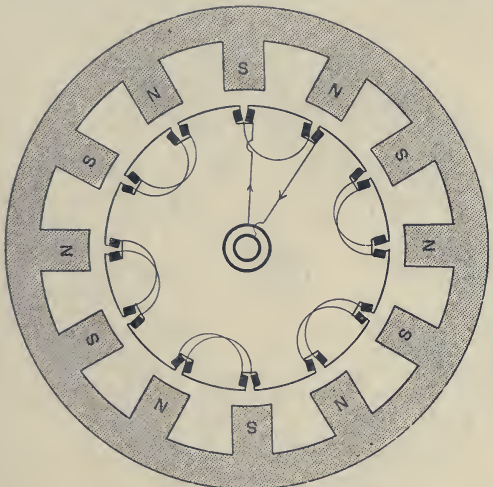
§ 1. **Suitability of the alternator for high pressures.**—While the continuous-current constant-potential dynamo is limited to pressures not exceeding 4000 volts * owing to the difficulty of commutation at the brushes, and partly also of insulating the wires of its rotating armature, the alternator is by contrast well adapted on every score for the production of high voltages. Its armature coils may be stationary and heavily insulated; there is no commutator, with its liability to trouble from sparking; lastly, the alternating-current transformer with its stationary windings affords a simple, economical, and thoroughly reliable means for transforming electrical energy from a high to a low pressure or *vice versa* just as the circumstances demand. In the early days of electric lighting, single-phase alternators were largely employed in central stations for the smaller cities and suburbs which could not be served by the low-tension continuous-current system; at pressures of 1000 or 2000 volts step-down transformers dotted over large scattered areas could be fed without an excessive amount of copper in the transmitting lines. As the demand for lighting increased and also became more concentrated, the spread of the three-wire system and the raising of the pressure at the consumer's terminals to 200 volts enabled the continuous-current system to reach a wider area of dense supply with reasonable economy of copper in the mains; at the same time, for want of a satisfactory single-phase motor, energy could not be supplied for motive purposes on the alternating-current system. On both counts, therefore, the latter found less favour. With the introduction, however, of the induction motor and polyphase alternator, the last-named objection was removed, and the alternating current once again recommended itself from the ease with which the energy could be transmitted at high pressures up to as much as 40,000 volts, and from the flexibility with which it could be adapted to meet every kind of demand. The generating station could now be placed on some convenient and economical site, perhaps near to a waterfall or to a

* A special dynamo with stationary armature giving 25,000 volts was built in 1902 to the design of M. Thury for testing purposes in connection with the St. Maurice-Lausanne transmission.

coal-field, at a considerable distance from the area to be supplied; the alternator could itself be built directly for pressures of 5000, or in large sizes for as much as 10,000 to 15,000 or even 20,000 volts, while the energy could be supplied at high pressure and transformed down at a substation, or again converted into direct-current form for traction purposes by means of the motor-generator or rotary converter. The latter serves as the connecting link between the alternating and continuous systems, and is largely employed for traction or railroad work, for which the continuous-current series-wound motor is especially suited from its great starting torque and accelerating power.

§ 2. **Drum armatures more common than ring or disc armatures.**—Owing to the absence of the commutator, the winding of the alternator armature is somewhat simpler than that of the analogous continuous-current machine. Of the four kinds of windings, the ring and discoidal-ring methods are seldom now used. The advantage of the drum winding for alternators is the same as for continuous-current dynamos, viz., that with a properly proportioned core it requires less wire for a given number of active wires, and the drum loops have less inductance than the ring; in virtue, therefore, of its lower resistance and inductance, it gives a better regulation for constant potential, and a smaller machine for equal temperature rise. Disc winding is precisely analogous in its general principles to drum winding, and the diagrams (Figs. 420–426) of the ensuing sections may be taken as applying equally to either drum or disc armatures. But for practical reasons connected with the ease of manufacture, the drum has now superseded the disc method.

§ 3. **Single-phase drum armature winding.**—In the drum armature of a single-phase alternator, there are usually as many coils as there are poles; each of the coils then has its outside end connected to the outside end of the adjacent coil on one side, and its inside end connected to the inside end of the adjacent coil on the other side. In Fig. 420 the radial portions of each coil in the left-hand side of the diagram mark the active wires which lie on the surface or in the slots of the armature, and the rounded ends are assumed to lie outside the influence of the field. If the winding be connected in series right round the armature, one group, consisting of the adjacent halves of the first and last coils, is subjected to a strain on the insulation of contiguous wires amounting very nearly to the total voltage of the machine. The inside ends of two neighbouring coils are best made the terminals of the winding, but in the disc machine if the E.M.F. is very high, a dummy coil is sometimes interposed so as to more effectually separate the ends of the armature. Or the winding may be divided into two halves in parallel; the coils which are in series are then wound in the same direction, and this direction is reversed at the points where the two halves are united in parallel. When the number of pairs of coils is even, the junctions are made by connecting together contiguous inside ends of one pair of coils and the distant inside ends of the opposite coils. With an uneven number of pairs of coils, contiguous inside and contiguous outside ends are joined together.



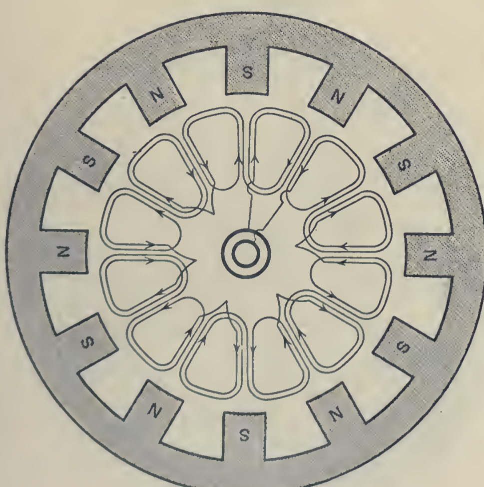


Fig. 420.—Single-phase drum or disc armature with divided coils.

The maximum difference of potential between any neighbouring wires is then equal to that generated by a pair of coils, and so in a machine giving, *e.g.*, 2000 volts of effective E.M.F. with 12 poles does not amount to more than about 950 volts at the moment of maximum induced E.M.F.

The method of parallel division is, however, accompanied by the disadvantage that there are then twice as many active wires, each of half the area, and consequently there is more space wasted in insulation, while, further, it is difficult to secure exact equality of the E.M.F. generated at each instant in the two parallels, from which there may

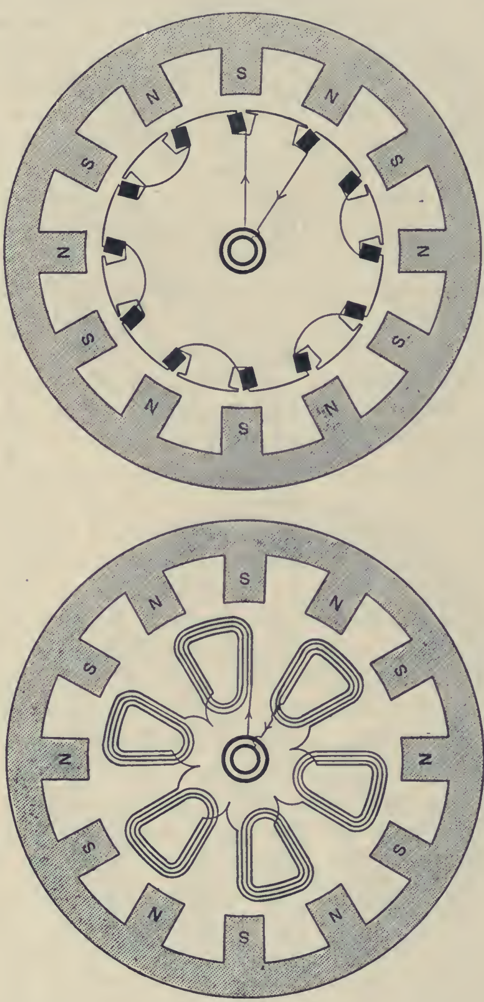


Fig. 421.—Single-phase drum or disc armature with undivided coils.

result a loss of efficiency and greater heating of the armature. On the above score a single large coil per pair of poles corresponding to Fig. 81 has the advantage; it will be seen from Fig. 421 that with this arrangement, which in the toothed armature gives only half the teeth wound, the beginning and the end of the winding when connected in one series are kept well apart, and there is little difficulty in insulating the coils at the point of maximum strain. The method is therefore well suited for high voltages, but has the disadvantages already mentioned in Chapter VIII. § 5, namely, that the space at the ends of the core is

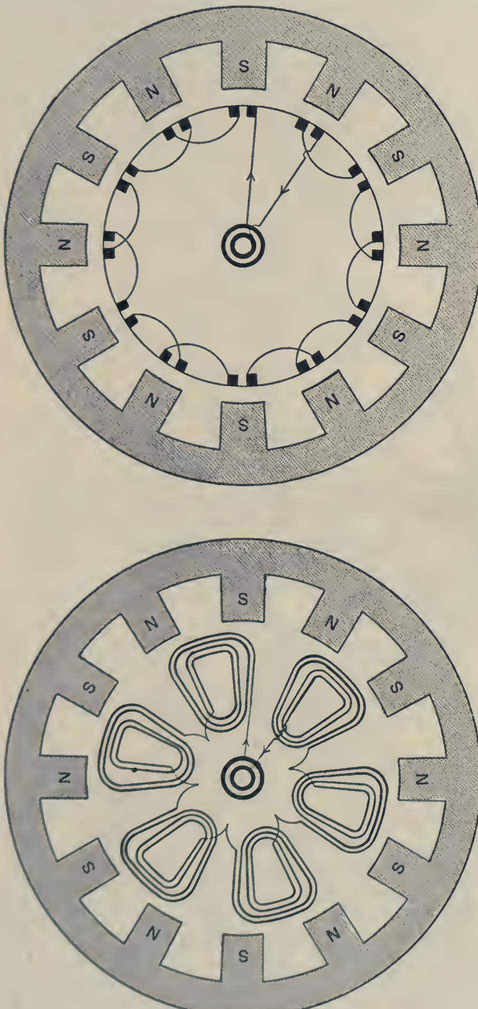


FIG. 422.—Single-phase drum armature with two slots per pole.

not so well utilised, less cooling effect is obtained, and the inductance of the winding is greater.

While the windings on the left-hand side of Figs. 420 and 421 are distributed, the corresponding windings of the toothed armatures on the right-hand side are completely concentrated within a single slot per pole. It is evident that whether the armature be smooth or toothed, there are numberless intermediate cases of grouped windings, the side of each coil being divided into two or more groups, and in the toothed armature the separate divisions being placed in as many separate slots or tunnels. Although not so often used for single-phase machines, such arrangements are of importance in connection with polyphase machines. In the single-phase case the slots might be uniformly distributed over the periphery, although with some loss of E.M.F. relatively to the number of wires, but usually they would be grouped more or less together. Under these circumstances the periphery of the armature would be made with a number of uniformly distributed slots in order to give a more constant reluctance to the magnetic circuit, and some of the slots would be unwound. Fig. 422 shows the

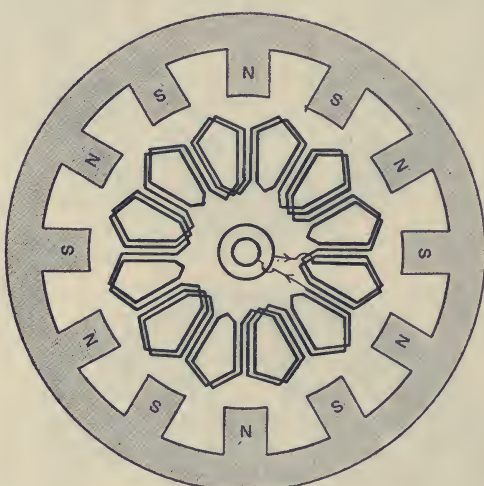


FIG. 423.—Lap-wound single-phase armature with divided coils.

form of the windings of Fig. 421 when the side of each coil is divided into two groups situated closely together.

In many cases step-up transformers are employed where transmission over long distances necessitates the use of very high voltages. The generating machines can then be built for low voltages with few large conductors, and little loss of space in insulation. Thus, instead of the wire coils of Figs. 420–422, which may consist of a great number of turns, bar-winding with few active wires per group becomes suitable.

Each of the coils of Fig. 420 can then be lap-wound, the electrical effect as shown in Fig. 423 being precisely the same as in Fig. 420, save that the terminals of the winding are now formed by the inside and outside ends of a pair of adjacent coils. The bars are bent or are joined by V-shaped end-connectors very similar to those employed in continuous-current dynamos, and the several groups are connected in series in the same progression as before. By a further rearrangement we pass to wave-winding analogous to that employed in the multipolar wave-wound continuous-current machine. At one point where the

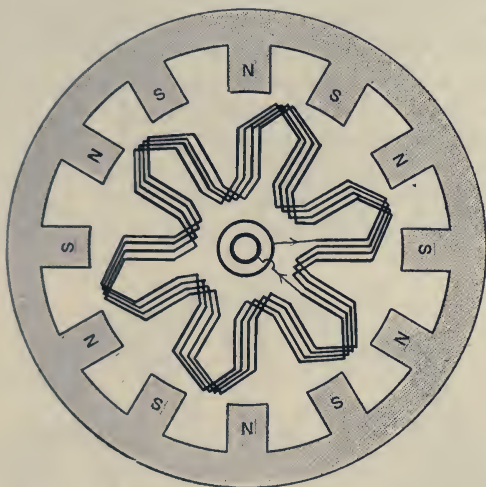


FIG. 424.—Wave-wound single-phase armature.

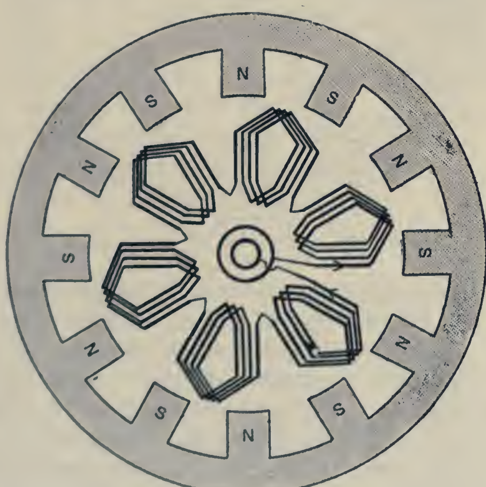


FIG. 425.—Lap-wound single-phase armature with undivided coils.

winding has completed a tour of the armature and starts off again on another tour, the end-connectors are shorter than elsewhere (Fig. 424). The larger coils of Fig. 421 may also be lap-wound, and are then connected in series by a wave method of progression round the armature (Fig. 425). The end-connectors are of course longer than in the case of the smaller divided coils of Fig. 424, and have three different lengths. Finally, with only one active bar per pole, the two diagrams of Figs. 424 and 425 become identical, and the winding is a simple undulatory zigzag (Fig. 426).

§ 4. **Interlinked quarter-phase and four-phase systems.**—If the several phases of a polyphase alternator are kept entirely distinct and independent, each set of coils has its two collector rings or terminals, and requires two transmitting lines to its external circuit. It has, however, already been stated (Chapter VIII. § 9) that it is possible to interlink the several phases, the advantage of so doing being that the number of transmitting lines to the external circuits and the weight of copper in them may be reduced for a given loss in the

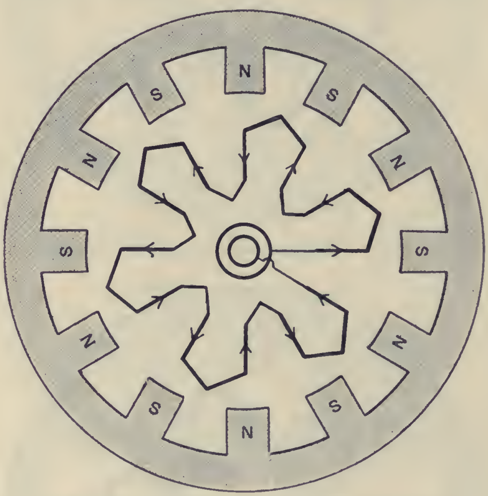


FIG. 426.—Single-bar drum winding.

transmission. Before passing to the winding of polyphase alternators, the question of interlinking in general may first be considered. Obviously, if such interlinking is not to interfere with the proper flow of the currents, the algebraic value of the instantaneous currents reckoned as positive or negative according as they flow away from or towards any junction must be zero when in their original state before the one or more junctions are effected. To investigate the methods of interlinking, each circuit of the armature may be diagrammatically represented by a coil having a certain angular displacement relatively to the other circuits or coils. Thus, Fig. 427 shows two pairs of coils in quadrature with one another and giving E.M.F.'s which differ in phase by a quarter of a period. If the two wires b and c are replaced by the single wire bc (Fig. 428), this third wire will serve as a common path for the currents of the two phases either away from or towards the armature, and we have a *quarter-phase interlinked system.* For all calculations as to effective values, the radii vectores representing the R.M.S. values as derived from the equivalent sine waves (pp. 104–5) may be compounded together even if the actual periodic curves are not sinusoidal. Thus if the circuits, both external and internal, are alike in every respect, in resistance, inductance, and capacity, so that the curves

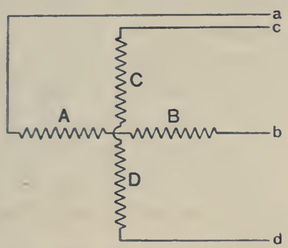


FIG. 427.—Two E.M.F.'s in quadrature.

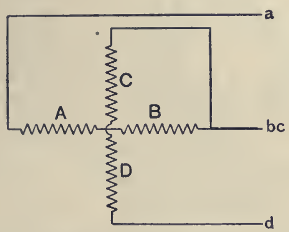


FIG. 428.—Quarter-phase interlinked system with three wires.

of their instantaneous E.M.F. and current and therefore their equivalent sine waves are identical, the effective value of the current in the common wire *bc* is the vector sum of the effective values of the currents through the two pairs of coils; *i.e.*, it is $\sqrt{2}$ or 1·414 times the current in either of the two outer wires, and the effective value of the voltage between the outer pair of wires *a* and *d* is $\sqrt{2}$ or 1·414 times the effective voltage between either of the outer wires and the common wire *bc*. In the above method the two sets of coils are independent, and are only interlinked at one of the external terminals or collecting rings. Although copper is thereby saved, the method is seldom used.

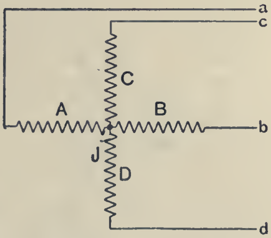


FIG. 429.—Star-connected four-phase system.

A second method unites the four coils at a common junction J, and four terminals and transmitting lines are again required (Fig. 429). Such an interlinked arrangement is known as a *star-connected four-phase system*; although often called a two-phase system, it is best regarded as composed of four phases given respectively by the coils A, B, C, and D, just as a single-phase alternator can be regarded as composed of two phases, E and –E, the one being as it were the counterpart or return of the other. If e be the effective voltage of one phase or coil A, the voltage between the two line wires *a*, *b* or *c*, *d* is $2e$, and that between any other pairs is $\sqrt{2}e$.

The four coils may also be connected up into a continuous helix just as in a continuous-current closed-coil machine, and four equidistant points are then connected to the collecting rings or external terminals (Fig. 430). When so interlinked, the arrangement forms a *mesh-connected four-phase system* (also called "ring-connected"). Again, since four lines are necessary, no copper is saved by its use in generators, but the mesh connection is employed in rotary converters, since in them it is necessary that the winding of the armature should form a closed circuit. If e be the effective voltage of one phase A, the voltage between any pair of adjacent lines in our diagram is e and between m and o or n and p is $\sqrt{2}e$. The current in any line is the vector sum of the currents in the two phases connected to it, and its effective value is $\sqrt{2}i$, where i is the effective value of the current in one phase.

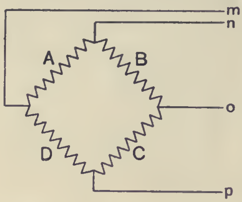


FIG. 430.—Mesh-connected four-phase system.

§ 5. **Interlinked three-phase systems.**—With three phases 120° apart, the same methods of star and mesh connections may be followed out. The three coils of Fig. 431 can be united at their centre, and three external leads are alone required. At any instant one wire will carry away from the centre the currents which are flowing towards the centre in the other two wires, or *vice versa*. Such a star method of interlinking is also known as the *Y-connected three-phase system*. If e be the effective voltage of each phase, or

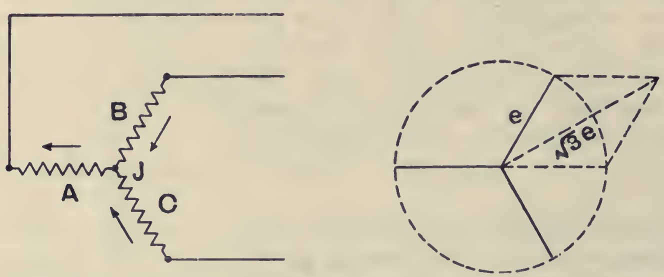


FIG. 431.—Y-connected three-phase system.

the voltage between any one of the three collecting rings and the common central junction J, the volts between any pair of transmitting lines will be $E = \sqrt{3}e$, the effective current I in each of the three lines will be equal, and the total output in watts will be $W = 3eI = \frac{3EI}{\sqrt{3}} = \sqrt{3} \cdot EI$, or 1·732 times the product of the effective voltage between the lines and the current in any single line.

On the other hand, if the three coils are closed upon themselves, and three transmitting lines are connected to the three junctions of the coils, we obtain the mesh or ring connection which in the three-phase case is also called the Δ or *delta-connected winding* (Fig. 432). The voltage E between any pair of wires is that of one phase, and the current in a line wire is the vector sum of the currents in two adjacent phases; *i.e.*, if i be the current in one phase, the current in a line wire beyond a collecting ring is $I = \sqrt{3} \cdot i$. The total watts in the external system are

$$W = 3Ei = \frac{3EI}{\sqrt{3}} = \sqrt{3} \cdot EI,$$

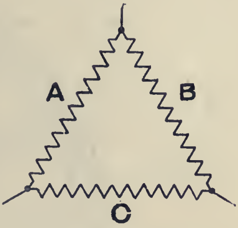


FIG. 432.—Δ-connected three-phase system.

which gives exactly the same numerical value as before. Hence any three-phase winding can be changed over from the Y to the Δ connection, and will then give 1·732 times as much current but only $\frac{1}{1{\cdot}732}$ of the voltage, so that the output is unaffected although its two factors are altered.

For transmission purposes the three-phase system is now almost always adopted in practice in preference to the quarter-phase with common return or the four-phase with four wires. Further, except in rotary converters and some motors, the star or Y connection is much more common than the mesh or Δ connection, since in the latter any ripples superposed on the main sinusoidal E.M.F. curve may cause internal parasitic currents round the closed triangle (cp. § 10).

§ 6. **Polyphase armature windings.**—The conversion of the preceding winding diagrams into forms suitable for quarter- or four-phase generators is easily effected by superposition upon the original set of coils of a second set exactly similar but displaced through an angle corresponding to half the pole-pitch; while for three-phase machines, two new sets of coils must be added, displaced respectively through one-third and two-thirds of the pitch. Thus Fig. 433 shows the quarter-phase equivalent of Fig. 420 with as many coils in each phase as there are poles; the adjacent sides of the coils of one phase may be wound in the same slot, or may be divided between two slots (as in Fig. 433), or between a still larger number. Fig. 434 shows the same arrangement in connection with an external stationary armature. In one set of coils (marked B) the end-connections after clearing the tunnels are bent up at right angles and lie against the sides of the armature; in the other set of coils (marked A) the end-connections after clearing the B set can then run practically straight across from tunnel to tunnel, care being taken that the length of the two sets of coils is alike. Fig. 435 is the quarter-phase equivalent of Fig. 421, with one coil per pair of poles and per phase, and with the winding concentrated in one slot per pole and per phase. Or, as in Fig. 422, there may be one coil per pair of poles, but with the winding grouped in two slots per pole and per phase. In all such cases where there is in effect only one large coil per pair of poles, the advantage is gained that the stationary ring of the armature can be constructed in two halves to enable the internal magnet to be removed; since at certain points

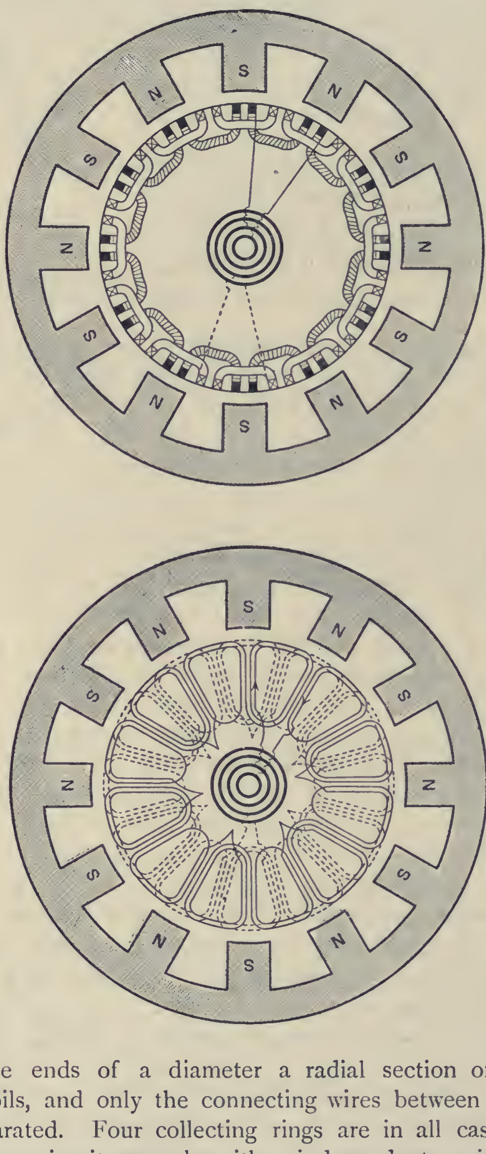


Fig. 433.—Quarter-phase winding with divided coils for rotating armature.

at opposite ends of a diameter a radial section of the armature cuts no coils, and only the connecting wires between the coils need to be separated. Four collecting rings are in all cases shown, and the armature circuits may be either independent or interlinked as a star at their central points. The three-phase equivalent of Fig. 420

with as many coils as there are poles is shown in Fig. 436. With a stationary armature (Fig. 437) it will be seen that the third or C set of coils would be most conveniently bent inwards in the opposite manner to the B set, but that they would then project in front of the magnet and, since the armature cannot be split, would prevent its withdrawal sideways; they must therefore in practice be bent up outwards. When, however, we pass to the three-phase equivalent of Fig. 421 with one large coil per pair of poles, two cases are possible. Thus if each of the two additional windings has a relative displacement of two-thirds of the pitch, although the armature cannot be split, only two kinds of coils are required, and the series of coils of any one phase consists alternately of straight and bent coils (Fig. 438); the appearance

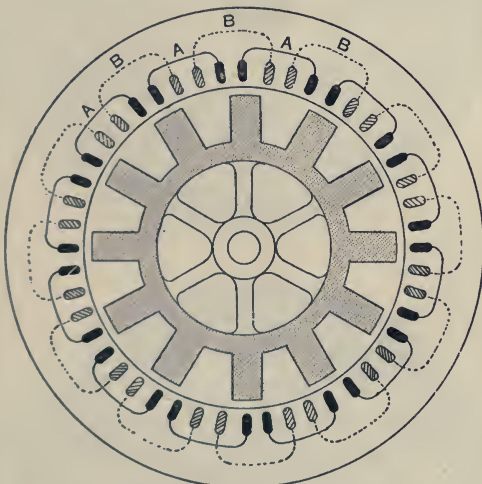


FIG. 434.—Quarter-phase winding with divided coils for stationary armature.

of such a three-phase armature is therefore exactly the same as that of the quarter-phase armature of Fig. 434, except in the number of sides of coils embraced within the pole-pitch. But if the three windings are relatively displaced through one-third of the pitch (Figs. 439 and 440), although a section of the armature cuts through three sets of coils at certain points and therefore three kinds are required, one of these can be bent inwards, since the armature can be split at certain points where no coils are cut. Fig. 439 is shown star- or Y-connected, as indicated by the common junction J, while Fig. 440 is shown delta-connected, as indicated by the two armature wires led to each of the three collecting rings. In Y-connected windings after completing one phase it is immaterial whether the start of the second and third phases is made from a common junction of three points lying close beside each other on the armature or from a common ring joining points approximately 120° apart, but in order to reduce the difference of potential between adjacent coils to a minimum the latter is preferable;

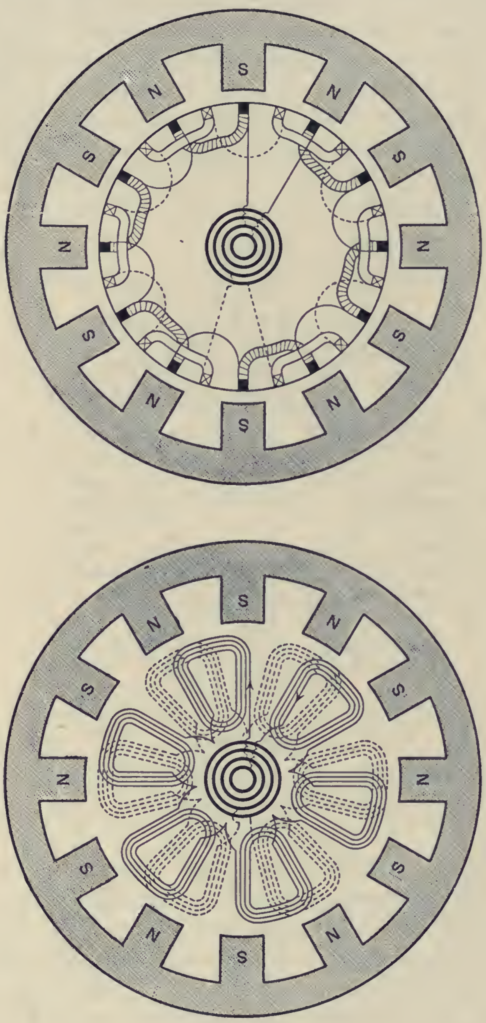


Fig. 435.—Quarter-phase winding with undivided coils.

and for the same reason each ring of the Δ-connection should unite two points approximately 120° apart.* Bar windings are shown in Figs. 441 and 442, the former being the lap-wound armature of Fig. 425,

* If the number of poles is itself divisible by three, the junction points cannot be exactly 120° apart.

and the latter the wave-wound armature of Fig. 424, both adapted to three phases, and both Y-connected.

All the above forms have been derived from the single-phase methods by duplication or triplication, and some are more convenient than others

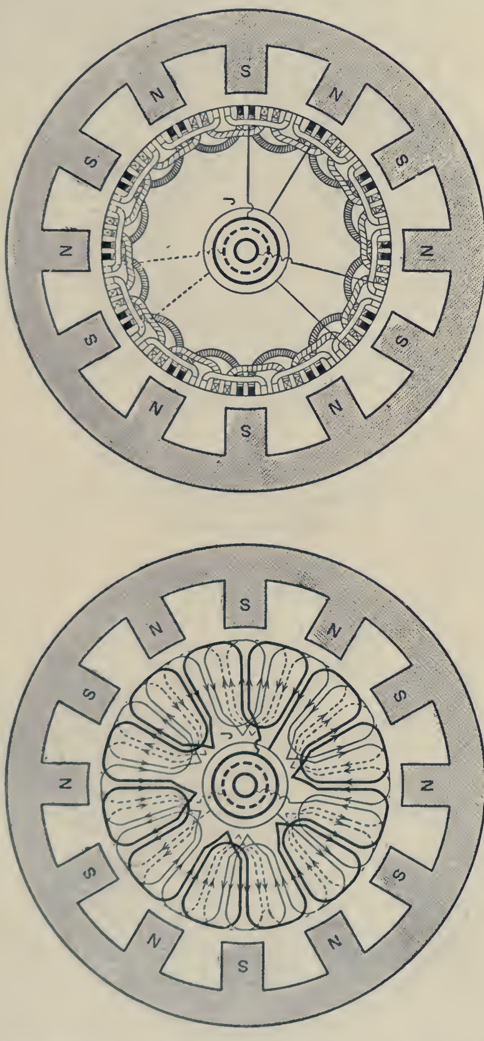


Fig. 436.—Three-phase winding with divided coils for rotating armature.

according to the nature of the output which the generator is to give. There are, however, in addition other windings which find advantageous application in polyphase work, although not suitable for single-phase machines. In general it may be said that while the forms with one arge coil in each phase per pair of poles lend themselves readily to quarter- and three-phase work, the forms with as many coils as there are poles, although utilising the end-space well and having a good cooling effect and small inductance, yet involve a considerable amount of over-

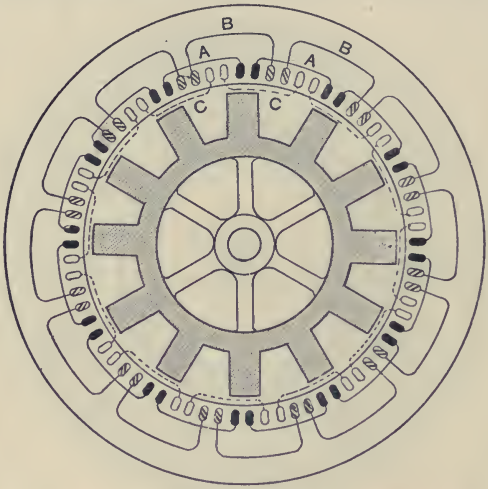


FIG. 437.—Three-phase winding with divided coils for stationary armature.

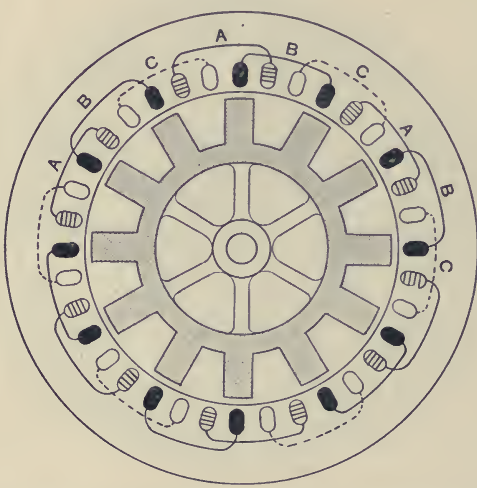


FIG. 438.—Three-phase winding for stationary armature with large coils of two kinds.

lapping of the coils. This disadvantage may be lessened by the employment of *shortened* coils. The coils of the quarter-phase machine are divided so that in each phase there are as many as there are poles;

their span is then narrowed until their over-all width is only equal to half the pole-pitch (Fig. 443), and there is no overlapping. In the three-phase machine there are two stages of the shortening process. If the coils are divided, a symmetrical winding is obtained by narrowing

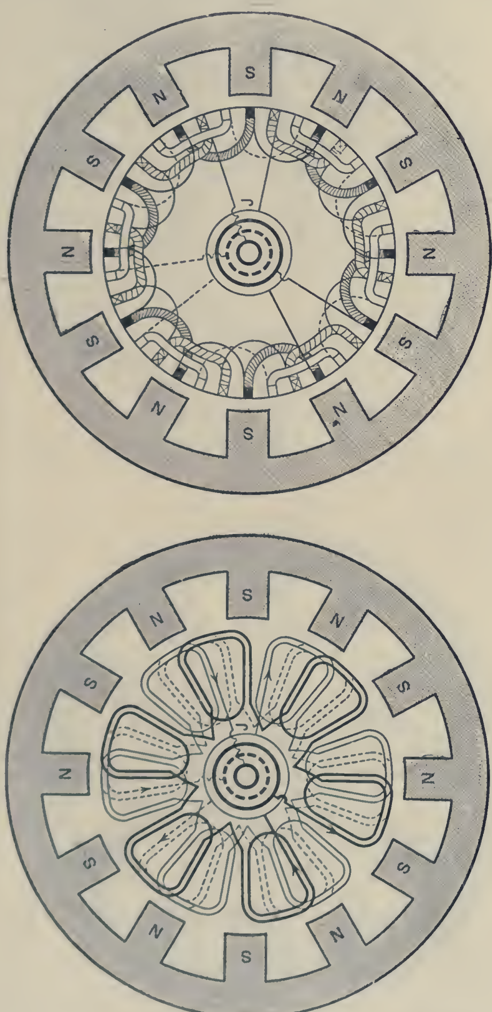


Fig. 439.—Three-phase winding with undivided coils of three kinds.

the coils until their *mean* width is equal to half the pole-pitch, as in Fig. 444 (ii.). Above (Fig. 444, i.) are shown four slots per pole and per phase with long coils divided, and below the coils are narrowed until their mean width is equal to half the pole-pitch. A further shortening with undivided coils until their over-all width is no more than two-thirds the pole-pitch, as shown in Figs. 444 (iii.) and 445, gives entirely non-overlapping coils with a three-phase machine, but is not to be recommended. Either the pole-faces must be correspondingly narrowed until their width is about equal to the widths of the coils,

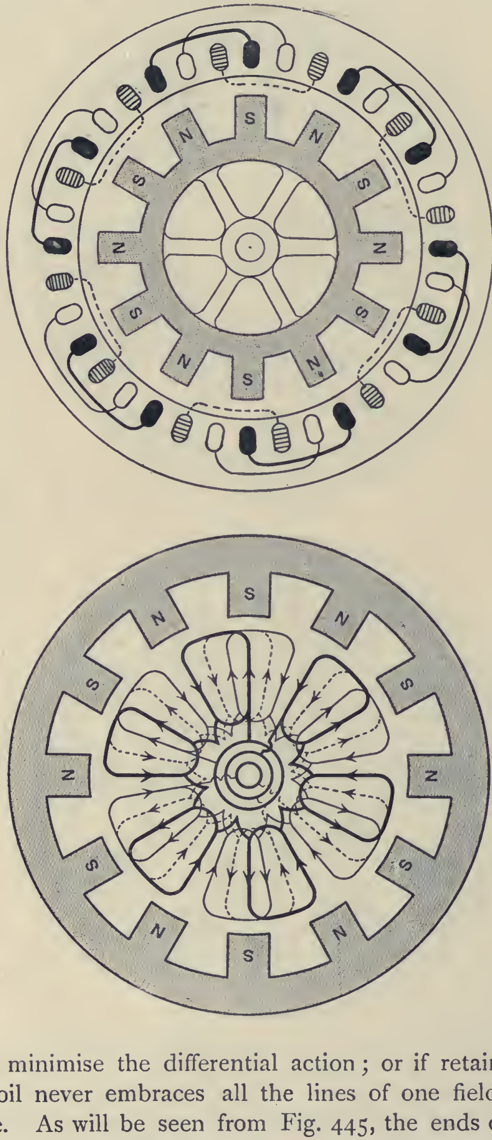


FIG. 440.—Three-phase winding for stationary armature with three kinds of coils.

in order to minimise the differential action; or if retained of normal width, a coil never embraces all the lines of one field, and K has a low value. As will be seen from Fig. 445, the ends of the coils are still well utilised for cooling, yet the reduction of the effective pole-area is so far disadvantageous that entirely non-overlapping coils are seldom employed, any gain from the reduction in the mean length of an armature turn being of small importance. On the other hand, with

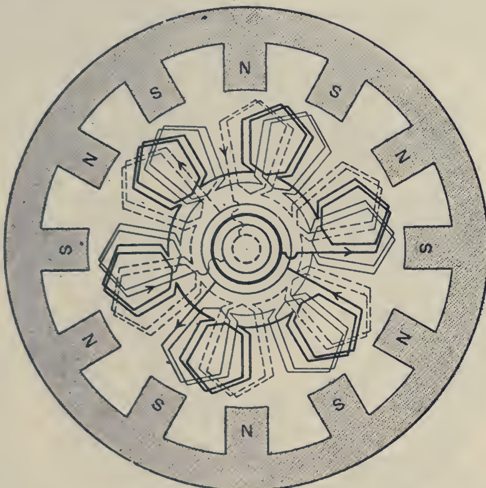


FIG. 441.—Three-phase lap-wound bar armature.

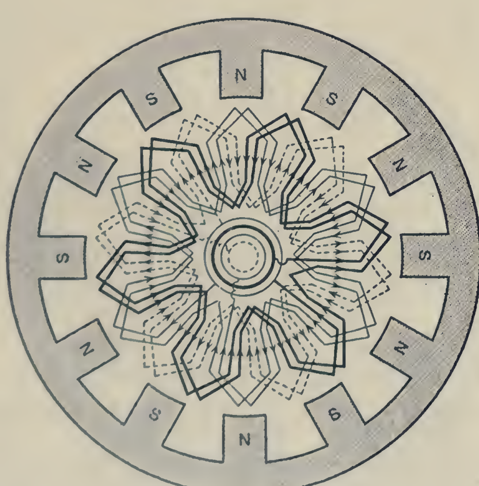


FIG. 442.—Three-phase wave-wound bar armature.

very high speed alternators having few poles and coils of very wide span, a certain amount of chording has much in its favour.*

There still remain to be considered certain forms special to the

* J. Bache-Wiig, *Trans. Amer. I.E.E.*, vol. xxvii. May 1908.

ring- or delta-connected polyphase machine, in which the winding must necessarily form a closed circuit. Any re-entrant drum or ring winding such as those employed in continuous-current dynamos can then be used; in the four-phase machine four approximately equidistant points

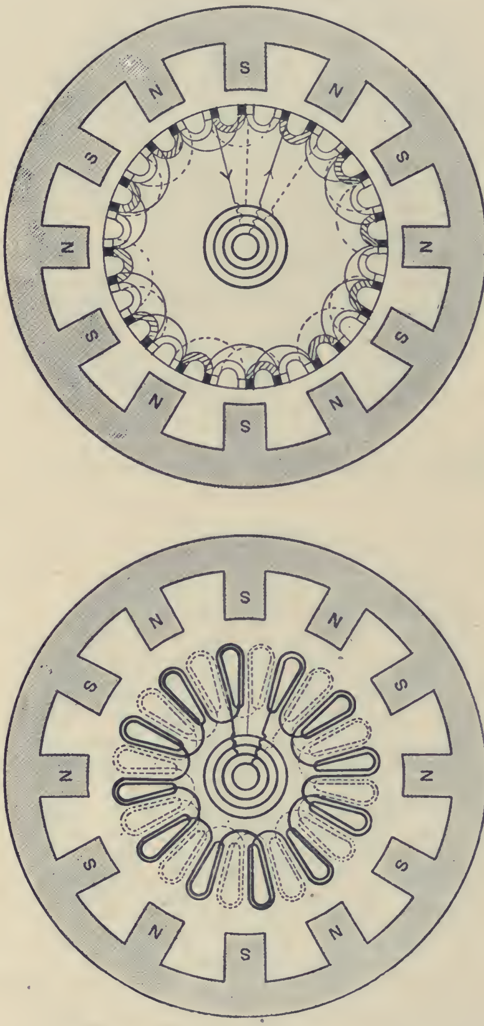


Fig. 443.—Quarter-phase non-overlapping winding.

in the winding are determined, and connections brought from these points to the four collecting rings or terminals, while similarly in the three-phase machine three equidistant points must be taken, and connection made to the three rings. Fig. 446 shows a re-entrant drum wave winding for a delta-connected six-pole armature, exactly analogous to a continuous-current wave-wound drum armature. Strictly speaking, the number of active wires or of bars should be a multiple of three in such a three-phase winding, so that if n be the number of bars in one phase and y is the average pitch, $3n = 2py \pm 2$, and the three points which are connected to the terminals are exactly equidistant, but as this condition somewhat limits the possible number of active wires, it is permissible with a large number of bars to make one division larger than the other two to the extent of a pair of additional bars.

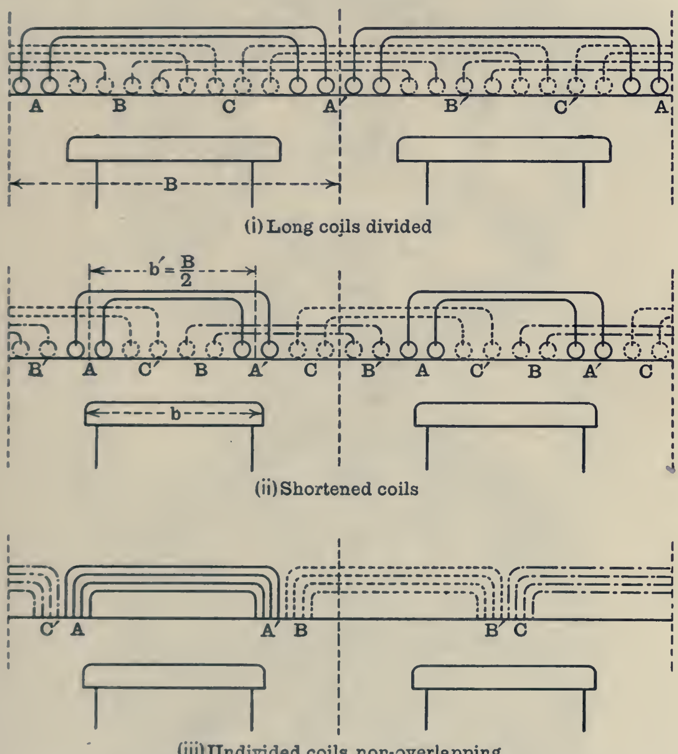


FIG. 444.—Long and shortened coils of three-phase alternator.

All the above windings, although shown in connection with a field-system of alternate poles, are equally applicable to homopolar machines with half as many poles of similar sign in each row. The only qualification that must be added is that in the case of the short-coil methods the pole-width must be equal to the pitch in order to avoid an unsymmetrical curve of E.M.F.

§ 7. **Rules for connecting up three-phase armatures.**—In order to connect the coils of a three-phase machine correctly it is convenient to consider the instant when the active wires of one phase are situated centrally under the poles, and to assume that they are then carrying the maximum current; if the machine is to be Y-connected, let one end of the phase in question be connected to the common junction,

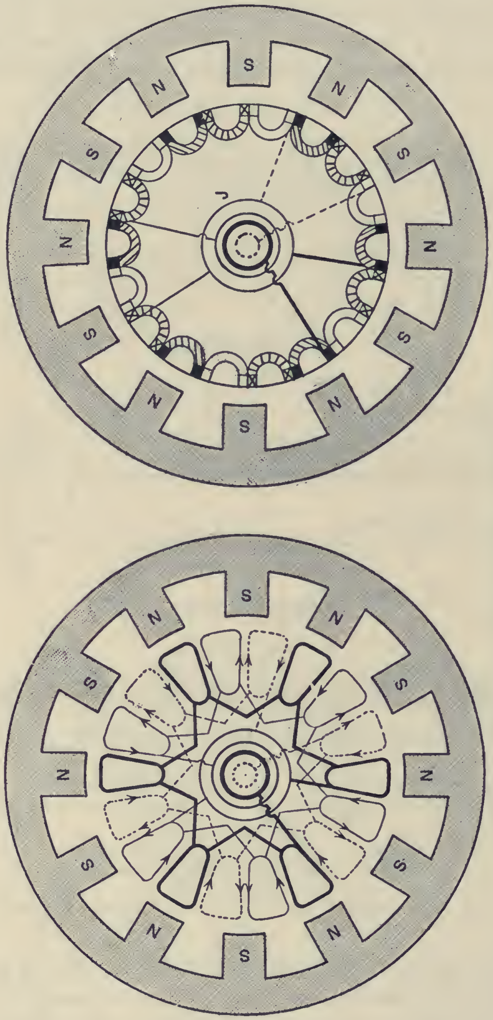


FIG. 445.—Three-phase non-overlapping winding.

and let it be assumed that the current is flowing into the common junction. The direction of the current in any group of active wires of the first phase is thus fixed, and the direction in the groups on either side in the two other phases must be the same under the same pole-piece. But the current in the two other phases must be flowing away from the common junction, so that their ends must be connected to the common junction in such a way that their E.M.F. agrees with the assumed direction of the current. If the machine is to be Δ-connected, and the ends of the first phase which is centrally under the poles are connected to two collecting rings, A and B, let it be assumed that the current flows from A to B. The current in the two other phases must also be flowing from A to B, passing through C on the way. The second phase must therefore be connected to A and C in such wise that the current flows from A to C, and the ends of the third phase must be connected to C and B in such wise that the current flows from C to B, the directions of their currents being the same under the same poles as that assumed in the first phase.

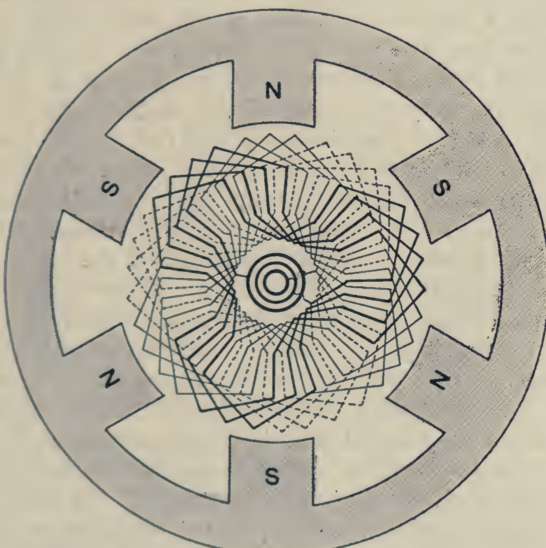


Fig. 446.—Three-phase re-entrant drum wave-winding.

§ 8. **The value of K in the E.M.F. equation.**—That the value of the factor K in the electromotive force equation of the alternator on open circuit will vary according as the winding is concentrated, grouped, or uniformly distributed, has been already shown in Chap. VIII. The effective E.M.F. induced in the armature per phase by equation (18) on open circuit or at no load, is $E_o = K \cdot 2Z_a \times \frac{pN}{60} \times$ number of active wires in series in one phase $\times 10^{-8}$ where K is itself the product of the two factors, k' the width coefficient, and k'' the form factor. In order to examine the values of either the one or the other with various ratios of pole-width and coil-width to the pitch, some assumption must be made as to the shape of the curve of instantaneous E.M.F. of one active wire, as dependent on the shape of the curve of flux from one pole. If this were sinusoidal, the resultant E.M.F. of the several active wires which are more or less out of phase with each other could be easily determined by the process of adding together their E.M.F.'s vectorially; or if the flux from each pole were strictly confined to the area of the pole-face and the length of air-gap were uniform, so that the flux would itself be uniformly dense over this area, the curve of instantaneous E.M.F. given by one wire in each half-period would be a rectangle, and the united E.M.F.'s of the several active wires or groups of wires could be easily calculated by the superposition of a number of such rectangles displaced from each other by the correct interval. Since neither assumption is true, a closer approximation is made by assuming a certain distribution of the useful fringe of lines in the interpolar gap, and by adding together the E.M.F.'s graphically. The following tables of values of K have been thus calculated on the assumption that the length of the single air-gap is $\frac{1}{20}$th and alternatively is $\frac{1}{35}$th of the pole-pitch,* and that the curve of induction in the air-gap shades off from its full value under the pole to zero on the interpolar line of symmetry, as in Fig. 264. As the ratio of the pole-width to the pitch is reduced, the effect of the fringe becomes more marked, since it forms a greater percentage of the total number of useful lines entering the armature, but for ordinary values of the pole-ratio, the results may be taken without much error even if the exact distribution of the fringe is in practice somewhat different; any reasonable allowance for the effect of the fringe gives a much closer approximation to the truth than its entire neglect.† The values given in the tables for large ratios of the pole-width to the pitch, say, from unity to 0·75, may be applied to the case of homopolar or inductor alternators with the cautions already pointed out in Chap. IX. § 4, namely, that the useful flux is the actual flux under a pole and within the arc of the pitch *minus* the flux which enters the armature in the adjoining interpolar gap and within the arc of the pole-pitch; further, strictly speaking, the instantaneous curve of a single active wire on a homopolar machine is not precisely analogous to that for a heteropolar machine, yet but little error is involved in the above identification, provided it be borne in mind that the useful flux is the difference between Z_a' and z_a.

* The values for other proportions of l_g to pitch may be interpolated between the maxima figures given below and those which are tabulated in a paper by the writer (*Electr. Review*, vol. xlvii. p. 655 ff.) for the hypothetical case of no fringe.

† For an analysis of alternator wave-shapes and their harmonics, cp. Comfort A. Adams, "Electromotive Force Wave-shape in Alternators," *Proc. Amer. I.E.E.*, July 1909.

TABLE I.—*Values of* K *for uniformly distributed Winding and long Coils with Allowance for Fringe.* $l_g = \frac{1}{20}$*th Pole-pitch.*

Ratio of width of Coil-side to Pole-pitch.	Ratio of Pole-width to Pole-pitch.						
	Alternate-pole Machines.				Inductor Machines.		
	0·4	0·5	0·6	0·7	0·8	0·9	1
0	1·25	1·18	1·125	1·08	1·05	1·025	1
0·1	1·225	1·16	1·112	1·06	1·035	1	0·968
0·2	1·19	1·135	1·09	1·05	1·008	0·97	0·932
0·3	1·162	1·11	1·075	1·025	0·983	0·94	0·897
0·4	1·125	1·083	1·05	1	0·955	0·905	0·857
0·5	1·09	1·046	1·015	0·967	0·92	0·87	0·82
0·6	1·03	1	0·963	0·92	0·87	0·816	0·78
0·7	0·976	0·944	0·91	0·87	0·82	0·772	0·731
0·8	0·906	0·88	0·843	0·803	0·764	0·72	0·682
0·9	0·841	0·812	0·78	0·745	0·707	0·67	0·633
1·0	0·77	0·745	0·716	0·685	0·654	0·617	0·578

TABLE II.—*Values of* K *for grouped Winding and long Coils with Allowance for Fringe.* $l_g = \frac{1}{20}$*th Pole-pitch.*

One slot per pole per phase .	1·25	1·18	1·125	1·08	1·05	1·025	1
Two slots per pole per phase $\frac{1}{6}$th of pitch apart . .	1·19	1·13	1·085	1·05	1·005	0·966	0·91
Two slots per pole per phase $\frac{1}{4}$th of pitch apart . .	1·15	1·102	1·058	1·02	0·965	0·915	0·866

TABLE III.—*Values of* K *for grouped Winding and long Coils with Allowance for Fringe.* $l_g = \frac{1}{35}$*th Pole-pitch.*

One slot per pole per phase .	1·317	1·235	1·162	1·105	1·058	1·023	1
Two slots $\frac{1}{6}$th of pitch apart .	1·237	1·171	1·109	1·056	1·007	0·956	0·914
Three slots $\frac{1}{6}$th of pitch apart	1·221	1·158	1·096	1·047	0·997	0·945	0·896

TABLE IV.—*Ratio of Maximum to Effective E.M.F. for uniformly distributed Winding with Allowance for Fringe.*

0	1·54	1·385	1·265	1·18	1·11	1·045	1
0·1	1·57	1·405	1·28	1·2	1·121	1·07	1·034
0·2	1·61	1·435	1·31	1·22	1·155	1·104	1·075
0·3	1·65	1·468	1·33	1·245	1·182	1·14	1·116
0·4	1·68	1·505	1·36	1·278	1·218	1·188	1·168
0·5	1·6	1·525	1·406	1·32	1·265	1·23	1·225
0·6	1·49	1·499	1·49	1·39	1·34	1·31	1·29
0·7	1·41	1·43	1·45	1·45	1·42	1·39	1·366
0·8	1·365	1·395	1·44	1·49	1·51	1·49	1·462
0·9	1·325	1·36	1·418	1·476	1·535	1·59	1·58
1·0	1·295	1·34	1·405	1·47	1·54	1·63	1·73

TABLE V.—*Ratio of Maximum to Effective E.M.F. for grouped Winding with Allowance for Fringe.* $l_g = \frac{1}{20}$ *Pole-pitch.*

Number of Slots.	Ratio of Pole-width to Pole-pitch.						
	Alternate-pole Machines.				Inductor Machines.		
	0·4	0·5	0·6	0·7	0·8	0·9	1
One slot per pole per phase .	1·54	1·385	1·265	1·18	1·11	1·045	1
Two slots per pole per phase $\frac{1}{6}$th of pitch apart . .	1·61	1·442	1·315	1·22	1·155	1·11	1·1
Two slots per pole per phase $\frac{1}{4}$th of pitch apart . .	1·64	1·48	1·35	1·25	1·205	1·174	1·155

TABLE VI.—*Ratio of Maximum to Effective E.M.F. for grouped Winding with Allowance for Fringe.* $l_g = \frac{1}{35}$*th Pole-pitch.*

One slot per pole per phase .	1·528	1·375	1·258	1·174	1·106	1·049	1
Two slots $\frac{1}{6}$th of pitch apart .	1·627	1·451	1·317	1·228	1·164	1·124	1·095
Three slots $\frac{1}{9}$th of pitch apart.	1·65	1·467	1·333	1·24	1·176	1·137	1·115

The curve of the form-factor k'' in relation to the ratio of pole-arc to pole-pitch is always concave, while the curve of the breadth-coefficient k', which is straight for a single slot, is always convex for two or more slots per pole per phase. As soon as the distance between the outer slots of a coil-side approaches the width of the interpolar gap, the curve of k' falls off rapidly; hence, in the curve of K as the product of $k'.k''$, for two or more slots there is a point of inversion where it changes from concave to convex. This point lies farther up the curve the greater the distance between the outer slots, until for distributed winding covering the whole pole-pitch the curve of K becomes entirely convex.

Any influence which causes a concentration of the flux increases K; the smaller value of the air-gap in relation to the pole-pitch, therefore, increases K, since the flux is then not so widely spread out.

It will be seen from the tables how the values of K for uniform and grouped distribution meet in the case of a concentrated winding, the hypothetical case of a single active wire of zero width giving exactly the same theoretical result as a single narrow slot in a toothed armature, while as the number of slots per pole per phase is increased, the value of K in the slotted armature becomes more and more nearly identical with the value for an equal width of coil in a smooth armature. Thus with three slots per pole and per phase $\frac{1}{9}$th of the pitch apart, the value of K is not far different from the value for a uniformly distributed winding having the equivalent width of $\frac{2}{9}$ths or 0·22 of the pitch; and the factor for a symmetrical bar winding with a large number of slots or tunnels becomes identical with that for an equivalent width of coil on a smooth armature, *e.g.*, with a three-phase closed circuit wave-wound armature, such as Fig. 446, the winding is equivalent to a uniformly distributed coil of width $= \frac{2}{3}$rds or 0·66 of the pitch.

In all cases as the winding becomes more distributed the value of K decreases; but the reduction of the E.M.F. due thereto has its full force only on open circuit or no-load. As will be more apparent later, the reaction on the field when the full current is taken out of the armature is less in the distributed than in the concentrated winding, and the inductance of the armature wires is reduced, while incidentally a better exposure of the coils to the cooling action of the air is obtained. Hence it is advantageous to employ a distributed winding even at the sacrifice of some of the possible E.M.F. at no-load, and two or three slots per pole and phase are the rule. The shape of the curve of instantaneous E.M.F. depends partly on the number of slots per pole and phase, and possesses considerable importance from its bearing upon the ratio of the maximum to the effective E.M.F. The insulation of the wires must be capable of withstanding the strain of the maximum E.M.F., but while with a concentrated winding the ratio of the maximum to the effective E.M.F. of the phase increases rapidly from unity as the pole-ratio is decreased, with a uniformly distributed winding or with a large number of slots the ratio decreases from a maximum of 1·73 corresponding to a triangular wave of E.M.F. and a pole-width ratio of unity. The two curves for concentrated and distributed winding cross at a value of about 1·33 for a pole-ratio in the neighbourhood of 0·5. Any number of intermediate cases are therefore possible, but in general it may be said that the maximum strain that the insulation of the coil has to withstand from the action of its own phase E.M.F. is about $1\frac{1}{2}$ times its effective E.M.F. In machines of high effective E.M.F. the coil must needs be concentrated within one or a few slots per pole in order not to lose too much space in the insulation of the slots; these must further be large, especially in turbo-alternators, so that there is more likelihood of higher harmonics in the E.M.F. curve.

In ordinary cases the value of K varies within the limits of 1 and 1·25, which may be compared with the value for a sinusoidal E.M.F., in a single bar or concentrated coil-side, namely, 1·11; the shape, too, of the curve usually more or less resembles a sine curve.

§ 9. **The shape of the E.M.F. curve and its delineation.**— Various methods have been devised for the purpose of delineating the curves of E.M.F. or current in alternators,* but these have been practically superseded by the oscillograph, which can be conveniently used to record the instantaneous curves of E.M.F. or current or of both together. Fig. 447 shows such an oscillogram of the potential difference curve of a 5000-volt three-phase alternator.† Many types of machines have been thus examined ‡ and their characteristics noted. How widely different may be the no-load E.M.F. curves of the alternators of different type and size that may be found in one central station appears from Fig. 448, where the curves of five single-phase alternators in one station have been sketched from oscillographic experiments.

Although for convenience the actual curves of E.M.F. and current

FIG. 447.—P.D. curve of 5000-volt three-phase alternator.

are usually assumed to be replaced by their equivalent sine waves, it must be remembered that such an assumption is only valid for calculations of power, and cannot be used in estimating the maximum E.M.F. as affecting the insulation, save only as a first approximation. Again, even if the curve of terminal E.M.F. on open circuit be a sine

* See Fleming, *The Alternate Current Transformer*, vol. i. chap. vi. §§ 2 and 3, vol. ii. chap. iv. §§ 1 and 4.

† Taken with the Duddell Oscillograph, and reproduced by permission of the Cambridge Scientific Instrument Company Ltd.

‡ For other examples see *The Alternate Current Transformer*, loc. cit.; Steinmetz, *Alternating Current Phenomena*, chap. xxx. (4th edit.); *Journ. Inst. Electr. Eng.*, vol. xxxii. in the papers of Mr. M. B. Field, and Messrs. A. D. Constable and E. Fawssett; vol. xxxiii. p. 1111 ("Properties of Alternators under Load," by A. F. T. Atchison), and vol. xxxv. p. 151 ("Notes on some Effects in Three-phase Working," by Dr. W. M. Thornton).

curve, its shape under full-load may be very different, and further, will vary widely according to the nature of the external load; *e.g.*, if the alternator be working on a non-inductive load such as a water-resistance or fully loaded transformers, the shape may differ considerably from that for the same alternator working on an inductive load such as lightly loaded transformers or motors.

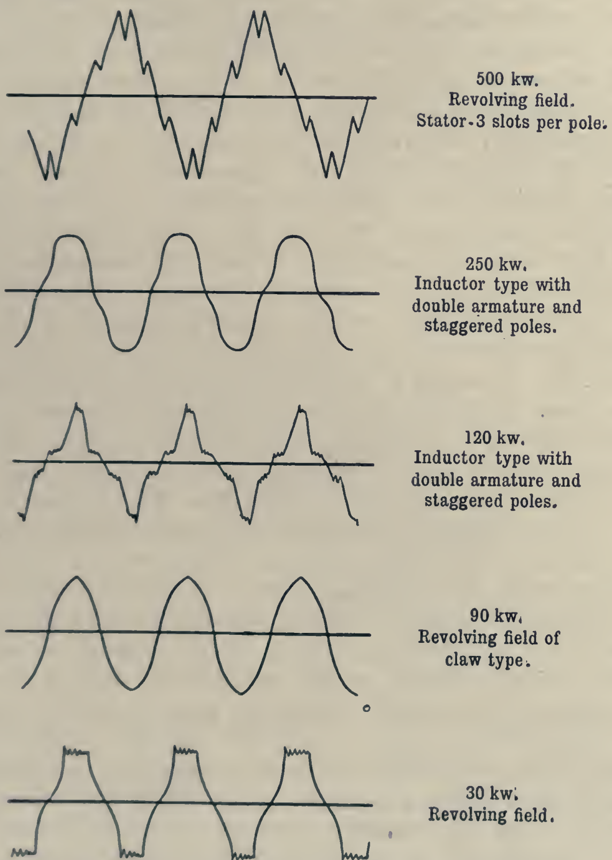


FIG. 448.—Wave-forms of five alternators in one central station.

In general it may be said that smooth-surface armatures with fairly wide coils give a curve of E.M.F. on open circuit very closely corresponding to a sine curve, although this may become much distorted under full load. In toothed armatures there are still greater deviations from a sine law both on open and closed circuit; the rhythmic passage of the teeth with their corresponding groups of active wires past the poles causes either a pulsation of the magnetic reluctance of the air-gap and therefore of the total flux, or an oscillation of its virtual centre, but apart from the minor ripples which are due thereto, the number of slots per pole and per phase has a marked influence on the shape of the curve, as also the number of phases. Thus, if for the present the ripples in the E.M.F. curve primarily due to the teeth are dismissed from consideration, a concentrated winding on open circuit reproduces in its E.M.F. curve the shape of the flux curve, but if the machine be single-phase, when carrying a considerable current it exerts a powerful distorting effect on the E.M.F. curve under load, owing to its high inductance or large armature reaction. But when the same winding is distributed among two or more slots per pole, not only is the no-load E.M.F. curve rendered more sinusoidal (cp. Figs. 87 and 88), but on full load the distortion of the field is reduced owing to the lesser reaction of the coils. Analogously in polyphase machines distribution of the winding tends to reduce the peaked form of the E.M.F. curve which is characteristic of the toothed armature. It may, in fact, be said that any comparatively flat-topped curve of flux will yield a total curve of E.M.F. of approximately sine shape at no-load, if the winding is distributed among three or more slots per pole and per phase; cf. Fig. 86, which is applicable to the case of six slots per pole and per phase. But although a near approach to a sine wave of E.M.F. naturally results at no-load, or is easily obtained by special shaping of the pole-shoes,* it is practically impossible to retain this shape under full-load, so long as the poles present salient faces entirely separate from one another, and the coils have the usual spans that are found most convenient in practice. When the distortion due to the cross-magnetising turns of the armature current is added, as will be more fully explained in Chapter XXIII., the resemblance to a sine curve is necessarily lost with projecting poles, for the following reason: the cross flux which is virtually superposed upon the main flux shows a marked depression in the gap between the pole-tips (cp. Fig. 469), and in fact almost falls to zero, yet it is just here that it should be a maximum if the cross E.M.F. is also to be sinusoidal, and this latter is the necessary condition for the resultant curve of E.M.F. under load to remain sinusoidal. This difficulty decreases as the inductiveness of the load is increased, and in consequence the cross ampere-turns of the armature are decreased, but under any fairly non-inductive load it can never be removed, unless the field-coils instead of being wound on projecting poles are embedded in the slotted surface of a comparatively smooth iron cylinder, as has been done in the case of the rotors of turbo-alternators.

By choosing certain ratios of pole-arc to pole-pitch in relation to

* On open-circuit the ripples due to the teeth may be practically eliminated by inclining the axes of the slots to the pole-edges, or *vice versa* by inclining the pole-edges to the slots by the amount of the tooth-pitch.

the number of armature slots, particular harmonics can be eliminated from the flux curve, and by choosing certain special spans for the armature coils particular harmonics can be eliminated from the E.M.F. curve. But such special spans usually necessitate an unequal spacing of the slots, which increases the cost of the stampings and is seldom convenient for the grouping of coils as required for two- or three-phase machines.*

The relative merits of various shapes of E.M.F. curve have been the subject of much discussion, and to a certain extent can only be decided after consideration of the nature of the load. For a given effective E.M.F. a peaked wave requires less magnetic flux, so that the hysteretic loss is less. The gain therefrom in the efficiency of a generator or in a synchronous motor driven by it is practically negligible, but in the case of lightly loaded transformers the lesser iron loss from hysteresis becomes appreciable and their efficiency is improved. On the other hand, the disadvantage of a peaked wave, in that its maximum E.M.F. greatly exceeds its effective value and demands better or thicker insulation, has been already noted. Further, for transmission of energy over very long distances at very high potentials, a sinusoidal curve is better, as resonance, causing a rise of voltage, is then less likely to be set up. In this connection the ripples due to the effect of the teeth on the magnetic field, to which we now return, have especial importance.

§ 10. **The higher harmonics of the toothed armature and resonance effects from capacity.**—The experiments of Mr. G. W. Worrall † have contributed greatly to the elucidation of the different results which follow from (*a*) "flux-pulsation" and (*b*) "flux-swing." ‡ Both phenomena always coexist to some extent, and in either case the frequency of the ripples primarily due thereto in the E.M.F. of a coil is the same, namely, equal to the number of teeth or slots passing a fixed point on a pole per second, $= n_s \cdot \frac{N}{60}$. But in the former case the ripples are most marked when the sides of the coil are passing through the interpolar gap (*i.e.* on the steeply rising or falling sides of the E.M.F. wave when it is passing through zero), and they die away under the pole; in the latter case the ripples are most marked when the sides of the coil are actually cutting the flux under the pole-faces (*i.e.* on the crests of the E.M.F. waves), and die away at the sides.

* See especially "E.M.F. Wave Forms," by E. A. Biedermann and J. B. Sparks, *Journ. Inst. Electr. Eng.*, vol. xxxv. p. 493, from which the above is partly derived, and F. Punga, *E.T.Z.*, vol. xxix. p. 118.

† "Magnetic Oscillations in Alternators," *Journ. Inst. Electr. Eng.*, vol. xl. p. 414.

‡ *Vide* Chap. XIX. § 24.

If there are γ slots per pole-pitch, the frequency of the pulsation of the magnetic flux in value, or of its oscillation in space in the air-gap, may also be expressed as 2γ times the frequency of the machine. This frequency must thus be an even multiple of the machine frequency, but it does not give rise immediately to such an harmonic in the E.M.F. wave of the alternator, since only harmonics whose frequencies are odd multiples can here appear. As described above, the ripples do not persist through the entire period but rise and die away again, and when the ratio of polar arc to the tooth-pitch is equal to a whole number plus one-half, there is complete discontinuity of phase in the interpolar gap.* To obtain a smooth curve of E.M.F., a polar arc which is an exact multiple of the tooth pitch is a favourable condition. A flux-pulsation or flux-swing due to 2γ teeth per pair of poles is equivalent to an alternating flux of frequency 2γ superposed upon a constant flux, and this alternating flux may itself be resolved into two fluxes, each of half the amplitude, the one rotating forwards and the other backwards, both with 2γ times the velocity of the poles relatively to the armature. Since the poles are already moving relatively to the armature with a speed corresponding to the machine frequency, the forward rotating flux gives a frequency $(2\gamma+1)$ times, and the backward rotating flux a frequency $(2\gamma-1)$ times the machine frequency. When the actual E.M.F. curve is resolved into a Fourier series of sine curves, the lowest harmonics, which may be expected to be the most pronounced owing to the presence of the teeth, will therefore be those which have $(2\gamma-1)$ and $(2\gamma+1)$ times the fundamental machine frequency.† Hence with toothed armatures and the winding of each phase concentrated in one slot per pole, a single-phase machine in which $\gamma=1$ usually shows a marked harmonic of triple frequency, and a quarter-phase machine with two slots per pole shows triple and quintuple harmonics, while a three-phase machine in which $\gamma=3$ shows harmonics of quintuple and septuple frequency; but even in the latter case a triple harmonic is usually present in the E.M.F. of one phase (although not in the line E.M.F.), due to the field itself if the slots were absent being a complex harmonic function depending upon the shape of the poles.

If a three-phase alternator is star-connected, all harmonics the frequencies of which are multiples of 3 times the machine frequency disappear in the curve of its interlinked pressure; no triple frequency current therefore flows in the lines connected to its terminals, unless the neutral points of the alternator and of a motor or transformer

* *Journ. Inst. Electr. Eng.*, vol. xxxix. pp. 218, 219.

† See Steinmetz, *Alternating Current Phenomena*, 4th edit. p. 572 ff.; M. B. Field, *Journ. Inst. Electr. Eng.*, vol. xxxii. pp. 655–659; A. Russell, *Alternating Currents*, vol. ii. p. 123; K. Simons, *E.T.Z.*, July 5, 1905, p. 631, abstracted in *Electrician*, vol. lvii. p. 581.

which it feeds are connected or grounded. When the three-phase E.M.F.'s are resolved into Fourier series, they are respectively

$$
\begin{aligned}
e_{\text{I}} &= E_1 \sin \omega t + E_3 \,.\, \sin (3\omega t + \theta_3) + E_5 \,.\, \sin (5\omega t + \theta_5) + \ldots \\
e_{\text{II}} &= E_1 \sin (\omega t - 120^\circ) + E_3 \,.\, \sin \{3 (\omega t - 120^\circ) + \theta_3\} \\
&\qquad + E_5 \,.\, \sin \{5 (\omega t - 120^\circ) + \theta_5\} + \ldots \\
&= E_1 \sin (\omega t - 120^\circ) + E_3 . \sin (3 \omega t + \theta_3) + E_5 . \sin (5 \omega t - 240^\circ + \theta_5) + \ldots \\
e_{\text{III}} &= E_1 \sin (\omega t - 240^\circ) + E_3 \,.\, \sin \{3(\omega t - 240^\circ) + \theta_3\} \\
&\qquad + E_5 \,.\, \sin \{5 (\omega t - 240^\circ) + \theta_5\} + \ldots \\
&= E_1 \sin (\omega t - 240^\circ) + E_3 . \sin (3\omega t + \theta_3) + E_5 . \sin (5\omega t - 120^\circ + \theta_5) + \ldots
\end{aligned}
$$

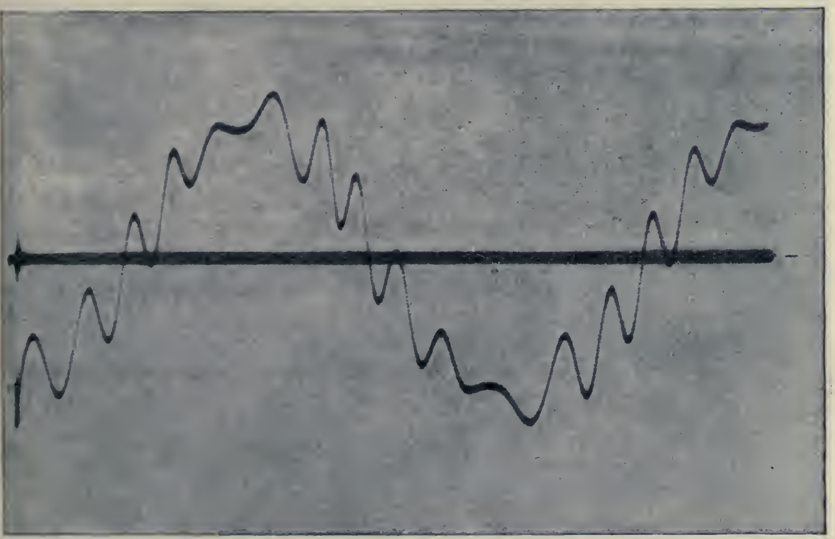


FIG. 449.—P.D. curve of alternator on unloaded cables, harmonic of 13 times machine frequency prominent.

The peculiarity of the harmonics of third order—and also of ninth, fifteenth, etc. order—is seen to be that at any instant they have the same sign and value, *i.e.*, they are all simultaneously directed either outwards from the centre of the star or inwards to the centre. They cannot therefore show on a voltmeter connected between any two lines, nor can they cause a current unless there is an additional connection to the centre of the star to form a path to or from it. E_1, E_3, E_5, etc. being the amplitudes of the components of the phase pressure, its effective value is $\sqrt{\frac{1}{2}(E_1{}^2 + E_3{}^2 + E_5{}^2 + \ldots)}$; the effective pressure between any pair of lines is then $\frac{\sqrt{3}}{\sqrt{2}} \sqrt{E_1{}^2 + E_5{}^2 + E_7{}^2 + E_{11} \ldots}$, yet between two neutral points which are earthed even in a balanced system $\frac{1}{\sqrt{2}}\sqrt{E_3^2+E_9^2+E_{15}^2+\ \ldots}$ acts. But if the armature is mesh-connected, the harmonics of the third, ninth, fifteenth, etc., order act in the same direction round the mesh, and will cause a local current to circulate therein, so that on this ground the star-connection is preferable.*

Resonance with the higher harmonics of the E.M.F. wave when the cables have capacity may seriously strain the insulation of the system and even cause its breakdown †; since it occurs when $2\pi f'L = \frac{1}{2\pi f'K}$,

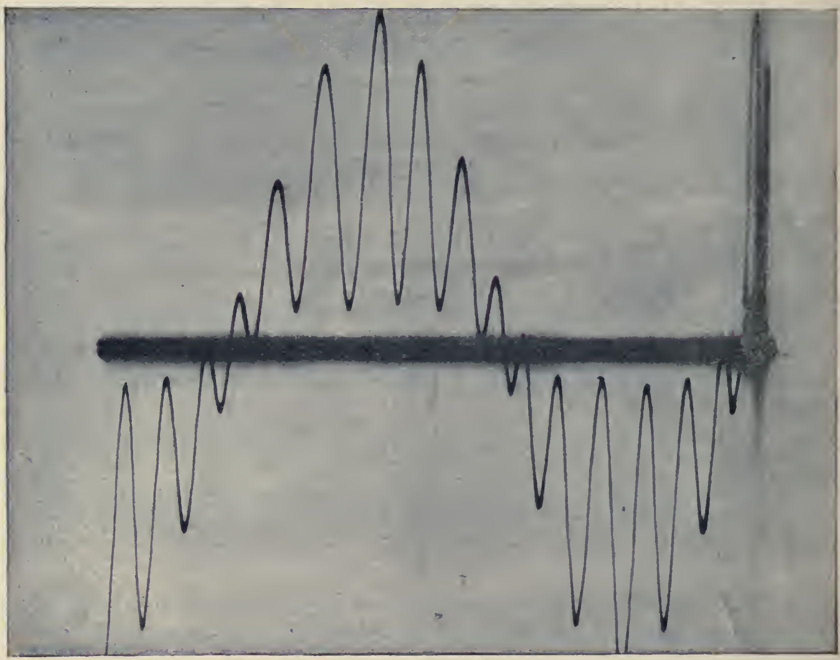


FIG. 450.—P.D. curve of alternator on unloaded cables, harmonic of 11 times machine frequency prominent.

where K is the capacity in series with inductance L and f' is the frequency of the harmonic, it is evident that the higher the

* Cp. A. G. Grier, *Electr. Journal*, vol. iv. p. 189. For the earth currents which result from the grounding of the centre of star-wound alternators, cp. J. H. Rider, "The Electrical System of the London County Council Tramways," *Journal I.E.E.*, April 1909, and correspondence in *Electrician*, April 30 and May 7, 1909.

† *Journ. Inst. Electr. Eng.*, vol. xxxii. p. 734, "Resonance in Electric Circuits," by M. B. Field; also p. 707, "Distribution Losses in Electric Supply System," by A. D. Constable and E. Fawssett, and various speakers in the discussion of both papers.

order of the harmonic, the smaller the capacity that will cause resonance.* The effect of capacity as magnifying the amplitude of ripples is shown in Figs. 449 and 450, where oscillograms † are given of alternators running on unloaded cables; the harmonic of 11 times the machine frequency is prominent in the former, and that of 13 times the machine frequency in the latter. The conclusion therefore is that in general a sinusoidal E.M.F. without harmonics is to be preferred.

The desirability of avoiding all higher harmonics when the external lines possess capacity affords a powerful argument for the superiority of hole-winding as compared with open slots. To enforce this

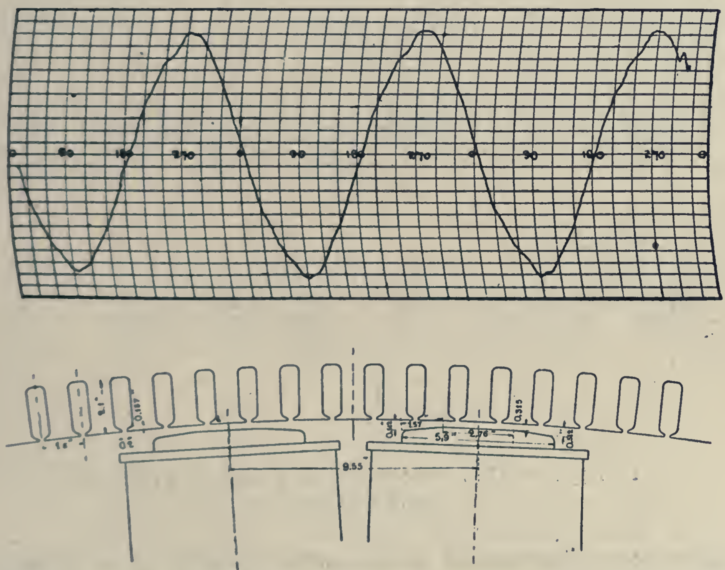


FIG. 451.—E.M.F. curve and details of three-phase alternator with nearly closed slots.

argument, Mr. A. C. Eborall has given the curves ‡ of E.M.F. under a load of 54–56 amperes as taken by the Hospitalier ondograph from two three-phase steam-driven 1200 horsepower alternators (Figs. 451

* For experimental methods of resolving the E.M.F. wave into its harmonics in order to determine their amplitude, see A. Russell, *Alternating Currents*, vol. ii. pp. 112–121, with original authorities there cited; A. F. T. Atchison, "Some Properties of Alternators under various Conditions of Load," *Journ. Inst. Electr. Eng.*, vol. xxxiii. p. 1109 and P. G. Agnew, *Electr. World*, vol. liv. p. 142.

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‡ *Electrical Magazine*, vol. i. pp. 467–68, from which by permission of the proprietors the accompanying figures are reproduced.

and 452). Both machines gave 6000 volts at 94 revolutions per minute, and were built by the same firm, the essential details being practically alike, with two slots per pole per phase in each, but in the one the slots were nearly closed, and in the other entirely open. The ripples in the E.M.F. of the latter are clearly marked.

Lastly, a practical conclusion to be drawn from the study of the

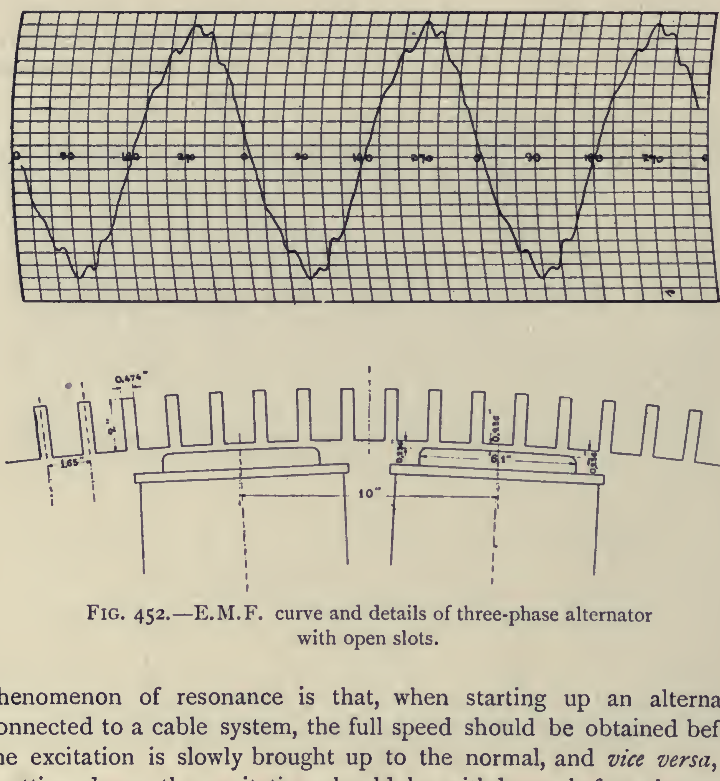


FIG. 452.—E.M.F. curve and details of three-phase alternator with open slots.

phenomenon of resonance is that, when starting up an alternator connected to a cable system, the full speed should be obtained before the excitation is slowly brought up to the normal, and *vice versa*, on shutting down, the excitation should be withdrawn before the speed is reduced; otherwise the critical speed at which resonance with some higher harmonic occurs may be passed through.*

* For even harmonics in alternating-current eircuits, see C. P. Steinmetz, *Electr. World*, vol. liii. p. 734, and J. B. Taylor, *Proc. Amer. I.E.E.*, July 1909.

CHAPTER XXIII

ARMATURE REACTION IN ALTERNATORS

1. **Regulation of alternators.** — In order to judge of the practical merits of an alternator, its behaviour under various loads with different amounts of inductance must be examined. The value of an alternator largely depends upon the degree to which it regulates for constant terminal voltage. It is therefore determined by its drop in volts, or the difference in the terminal voltages (with constant excitation and speed) at no-load and under a certain definite load formed by an external current I_e with an angle of lag ϕ_e such that $E_e\ I_e \cos\phi_e =$ the full rated watts of output of the machine. If this fall of potential exceeds a certain amount suitable to the purpose and work for which the alternator is designed, the load has passed the limit set by armature reaction, even though the heating of the armature winding may be well within the permissible limit. Thus the output of the alternator is fixed by the two considerations of heating and armature reaction, just as is that of the continuous-current dynamo, the only difference between the two being that the limit set by armature reaction in the alternator does not arise from any question of sparking, but from the point of view of the *regulation*. Since in most cases it is required that the terminal voltage on the external circuit should remain practically constant for all loads within the capacity of the machine, it is evident that if the fall of potential with unaltered excitation is great, the alternator will require considerable attention to adjust the excitation to the amount required by varying loads; or, again, the fall may be so great that no reasonable increase in the field excitation can maintain the volts under full load, and the alternator becomes useless for the work in question.

§ 2. **Value of power-factor of external circuit.**— For lighting distribution with transformers fully loaded, ϕ_e is about 10° and the *power-factor* may be taken as nearly equal to 1. For power distribution with fully loaded asynchronous motors, ϕ_e on the average is about 35°, and $\cos\phi_e$ is 0·82. In ordinary use, however, the motors will seldom all be fully loaded, and the average value of $\cos\phi_e$ may for long periods not be more than 0·6 to 0·7. Next, when a large motor is switched on, its starting current is high and lags behind the terminal voltage by a considerable angle; if the effect of this is to cause a large drop in the volts of the generator, it will seriously interfere with the steadiness of light in a mixed system of combined incandescent lamps and motors, or with motors only may cause these to stop, since their torque is proportional to the square of the terminal pressure. It is not, however, only in motor or combined light and power systems that it is necessary to consider the behaviour of the alternator under low power-factors. With a number of small transformers,* each lightly loaded as may be the case during the daytime, the power-factor of a lighting installation may fall as low as 0·3 or even to 0·2. The full current of the alternator which is run to supply the network in the daytime may then be taken up in magnetising the transformers, or it may even be necessary to run two alternators in parallel in order to keep up the voltage, although such a course is in itself uneconomical. An approximate calculation of the maximum angle of lag in the external circuit † that may be expected in the daytime may be made from the data of the magnetising current of the transformers I_m, their iron-loss current I_h, and active or useful current I_n; thence

$$\cos \phi_e = \frac{I_n + I_h}{\sqrt{(I_n + I_h)^2 + I_m^2}}$$

§ 3. **The inductance of an alternator armature.**—The reduction in the terminal voltage of an alternator under a given load of defined character as compared with the open-circuit voltage for the same excitation and speed is partly (*a*) ohmic, due to the loss of volts over the ohmic resistance of the winding, and partly (*b*) inductive, due to the effect of the alternating current as linked with a magnetic field. The "inductance" of the armature is a general term covering all effects due to the magnetic circuit or circuits upon which the alternating current acts, and it may be subdivided into three principal heads, according as it is related to (1) a real self-induced flux arising in local circuits independent of the main magnetic circuit, (2) an alteration of the *distribution* of the main field over the pole-face, or (3) an actual alteration in the *total number* of lines flowing through the main magnetic circuit for the same excitation. Although each of these three items forms one component of the total inductance of the armature, in the sense that the withdrawal of the armature current would cause a cutting of the armature wires by lines of flux and a consequent self-induced E.M.F., yet from their different origin they bear different characters and require to be very clearly distinguished. Each will therefore be shortly considered separately.

* As used *e.g.* in connection with low-voltage metallic-filament lamps.

† But for the influence of the power-factor upon the price of the machine, cp. Dr. M. Kloss, *Journal Inst. Electr. Eng.*, vol. xlii. p. 174.

In the first place, when current is flowing through the coils of the armature there arises a certain number of lines which are linked with the ends of the coils, but which do not pass through the main magnetic circuit; they are confined to local circuits chiefly outside the active length of the core. In addition, there are armature lines which are immediately closed round the active wires, passing through the iron teeth and across the tops of the slots within which the armature wires are wound. When the armature wires are situated between the poles, these lines may exist separately in closed circles. Even when the wires are situated under the pole-faces, some lines pass through the air-gap immediately above the openings of the slots without affecting the main flow of lines from the poles; they do not then actually have a separate physical existence, but show themselves in a local distortion of the main flux. They are therefore on the borderland of the second group, yet since the distortion which they cause is local, and does not extend over the pole-face, they may be grouped with the lines linked with the ends of the coils. All the above lines may therefore be regarded as *independent of the position* of the coils relatively to the poles, and together form a certain amount of flux which from the analogy of the transformer may be called the *secondary leakage*; although this leakage will not increase in strict proportion to the current, so far as its local paths are in iron and this becomes saturated, yet it is confined to circuits which are to all intents magnetically independent of the main magnetic circuit, and its amount is practically constant for any given current whatever the position of the coils.

As the current alternates, the secondary leakage alters in value and direction, and sets up a self-induced E.M.F. $-e_{sa}$ proportional to the leakage inductance L_a of each phase; on the assumption of a constant L_a and a sinusoidal rate of change, we thus have a leakage reactance per phase $x_a = 2\pi f L_a$, and corresponding thereto a leakage reactance voltage of effective value $-Ix_a$, lagging 90° behind the current in the armature, which must be counterbalanced by an equal E.M.F. impressed from the main field. This latter E.M.F., or the *voltage consumed by the leakage reactance*, is thus e_{sa} or Ix_a preceding the phase of the current by 90°, and to produce it a corresponding effect is called for from the lines of the main field, as will be described later.

But the inductance of the armature is not exhausted by the secondary leakage. It further covers a second element which from certain radical differences in its nature is to be regarded more strictly as the "armature reaction" in the proper sense of the words. This term again covers two phenomena which yield the second and third items of which mention was made at the outset. They are closely related to each other, and both as opposed to the first item will be found to depend upon the phase of the current in connection with the position of the coils relatively to the poles; *e.g.*, if any particular instantaneous value of the current, such as the positive maximum, be taken, the question of what is at that instant the position of the coil in relation to the poles has a decisive bearing upon the "armature reaction," the nature of which has now to be discussed.

§ 4. **The cross and direct ampere-turns of the armature.**—

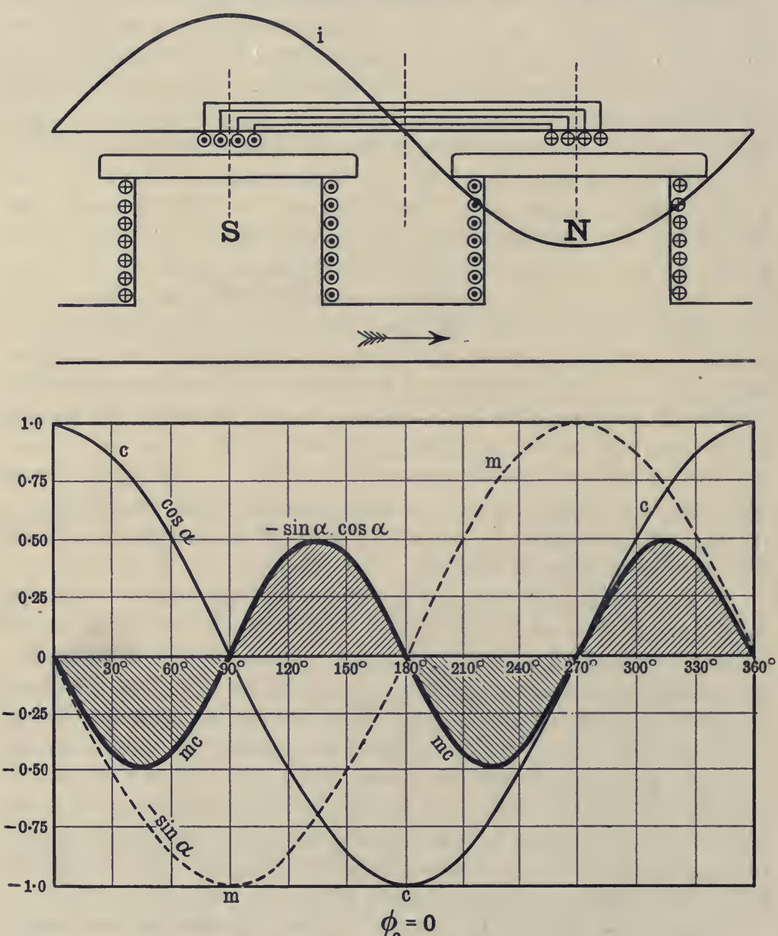


FIG. 453.—Current-vector coincident with E.M.F. or centre of pole; $\phi_0 = 0$.

Taking one phase by itself, if the paths of the magnetic circuit presented at any moment to the armature coils of the phase are traced, they will be found to be different at different positions, and the reluctance presented to the armature magnetomotive force is not constant but varies. When the centre of a group of wires forming the side of a drum coil is directly under the centre of a pole (Fig. 453), its ampere-turns act round a cross-circuit passing athwart the pole, and give a cross magnetomotive force displacing the resultant field towards one side or the other. When the same group of wires reaches a similar position relatively to the next pole its current will in an alternator be exactly reversed, and so also will be the direction of the field; its magnetomotive force will then again have the same effect, namely, entirely cross-magnetising. But when the group of wires moves away from the central position, two magnetic paths are presented to it, one of these being directly *through* a pair of poles round the main circuit of the field instead of *across* the pole. The effect of the ampere-turns is now partly *cross* and partly *direct*; in virtue of the latter they directly affect the strength of the symmetrical field, while in virtue of the former they simply displace the weakened or strengthened symmetrical field. The proportion of the directly magnetising effect to the cross or distorting effect gradually increases until the group of wires is exactly midway between the poles when there is no cross effect, and all the ampere-turns are directly magnetising (Fig. 455). Later, as the coil passes under the next pole, the effect of the armature-turns is again divisible partly into direct and partly into cross magnetisation. Thus in a single-phase alternator or in one phase of a polyphase machine considered by itself, the groups of wires have for each position which they occupy relatively to the poles a certain M.M.F., and the action of this may be separated into a cross and a direct effect.

§ 5. **Approximate calculation of direct and cross ampere-turns of single-phase alternator armature.**—So far the magnetising effect of a coil according to its position relatively to the poles has been described in general terms independently of the actual magnitude or direction of the current which may actually be flowing in it. This may be rendered more definite by considering the magnetising effect that a *constant* current of any value flowing in the same turns would have for each position. Reckoning from an initial position of space or moment of time when the axis of a drum coil is immediately in the centre of an interpolar gap, and its sides are thus under the pole-faces, the direct effect from the M.M.F. of the constant current is zero, and gradually rises to a maximum when the axis of the coil coincides with the centre of a pole, while the cross effect changes in precisely the opposite way. The gradual transition from the one effect into the other might conveniently be represented by assuming that the instantaneous direct M.M.F. of the constant current in the coil of t turns varies as $t \sin \alpha$ (where α is the angle of movement away from the initial position expressed in terms of an electrical period), and conversely that the cross M.M.F. varies as $t \cos \alpha$. Such an assumption is not rigorously exact, and is at present only put forward as a provisional hypothesis to render the action clearer; yet it is not far from being a correct representation of the facts, since, just as in a sine curve, the rate of increase of the direct magnetising effect is greatest in the earlier stages when the side of the coil first moves away from its central position under the pole, while after the emergence of the coil into the gap between the poles, as it then practically embraces the main magnetic circuit its direct effect increases much less rapidly, although reaching its maximum when the axes of coil and pole coincide. The converse is true of the cross effect, which can thus similarly be represented by a cosine curve.

Let the direction of the constant current be so chosen that it coincides with the direction of the E.M.F. that would be induced at no-load or open circuit at the moment which has been taken as the starting-point, and let its value be one ampere. Then the direct effect of the coil of t turns is at first demagnetising in reference to the field excitation, but after it has moved through 180°, and occupies a similar position in relation to another pole, it assists in magnetising the field. As acting upon a pair of poles or per magnetic circuit, it is therefore at any instant $-t \sin \alpha$, and is given by the dotted curve *mm* of Fig. 453, which is first negative and then positive.

The instantaneous value and direction of the actual alternating current in the coil for each position relatively to the poles remains to be taken into account. In a clock diagram the rotating vector of the E.M.F impressed upon the armature by the flux of the undistorted main field gives an exact clue to the instantaneous position of the armature relatively to the centre of a pole; when the impressed E.M.F. E_a passes through zero the axis of the drum coil coincides with the centre of a pole, and when it passes through a maximum the axis of a coil is midway between a pair of poles. From the same principle it follows that in a diagram of effective E.M.F.'s in their relative phases the angle between the vector of E_a due to the main undistorted field and the current vector will express in terms of an electrical period the relative distance between the centre of an interpolar gap and the position of the axis of the coil at the moment when its current is passing through a maximum, *i.e.* the temporal phase of the current can be related to the spacial position of the coils under the poles by means of the vector diagram of E.M.F.'s and current. Let it now be assumed, in the first place, that there is no angle of lag or lead between current and impressed E.M.F., *i.e.* that the maximum value of the current occurs when the axis of the coil is in our assumed initial position midway between the poles. The instantaneous value of the current from this initial moment is then $\mathbf{I} \cdot \cos \alpha = \sqrt{2} \cdot \mathrm{I} \cdot \cos \alpha$, and it is represented spacially by the curve *cc* in Fig. 453. The product of corresponding ordinates of the two curves *mm* and *cc* will then give the direct magnetising ampere-turns of the coil of t turns carrying the current of effective value I, or at any instant

$$mc = -\sqrt{2} \cdot t \, \mathrm{I} \cdot \sin \alpha \cdot \cos \alpha$$

The product of a sine and a cosine curve, back ampere-turns being plotted below and forward ampere-turns above the base line, gives a complete wave for each half-period. The shaded areas show that in this case there is a periodic weakening and strengthening of the symmetrical field, but that the two balance during each half-period, and the value of the flux is unaffected, the average direct magnetising turns being zero.

The condition assumed above would, however, be of rare occurrence. Unless the effect of the self-inductance L_a of the armature and that of the external circuit be in some way exactly balanced (as, *e.g.*, by capacity in the circuit), the phase of the current can never coincide with that of E_a, but must continue to lag behind it, or, as it may conveniently be expressed, the current vector lags behind the centre of the pole by

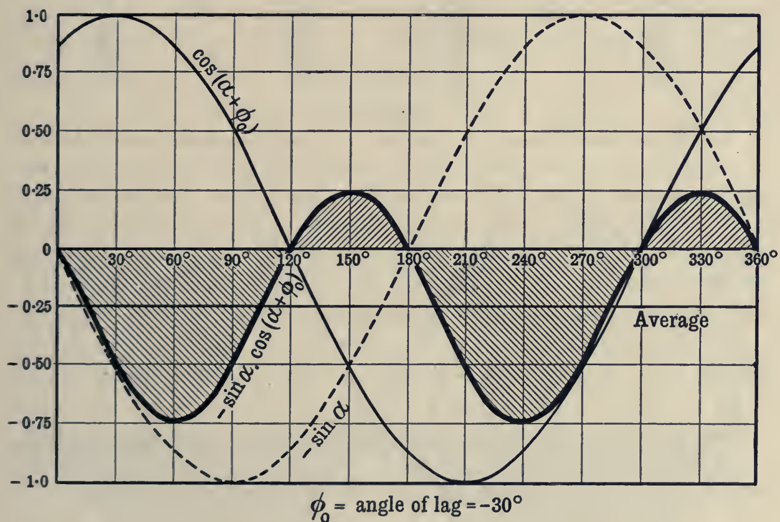


FIG. 454.—Current lagging behind centre of pole; $\phi_o = -30^\circ$.

some angle ϕ_o. In Fig. 454 an angle of lag $\phi_o = -30^\circ$ is assumed, and it is seen that the weakening or negative effect of the direct turns is greater and lasts longer than the strengthening or positive effect, so that, on the whole, there is a back effect from the armature ampere-turns. The instantaneous value of the current when expressed generally is $i = \mathbf{I}\ \cos(\alpha + \phi_o) = \sqrt{2}\mathrm{I}\ .\ \cos(\alpha + \phi_o)$, where ϕ_o may be any angle of lead (reckoned as positive) or of lag (reckoned as negative) between the current vector and the centre of the pole. The complete expression for the instantaneous direct magnetising ampere-turns of the coil is then, on the assumption that the direct effect is proportional to sin α

$$mc = -\sqrt{2}t\mathrm{I}\cos(\alpha + \phi_o)\sin\alpha$$

The relative areas of the back and forward effects in each half-period depend upon the value of ϕ_o. The two maximum values occur when $\alpha = \frac{90^\circ - \phi_o}{2}$ or $= \frac{270^\circ - \phi_o}{2}$, and are then $-\sqrt{2}t\mathrm{I}\ \sin^2\left(\frac{90^\circ - \phi_o}{2}\right)$, and $\sqrt{2}t\mathrm{I}\ \cos^2\left(\frac{90^\circ - \phi_o}{2}\right)$; the former is the largest maximum when ϕ_o is

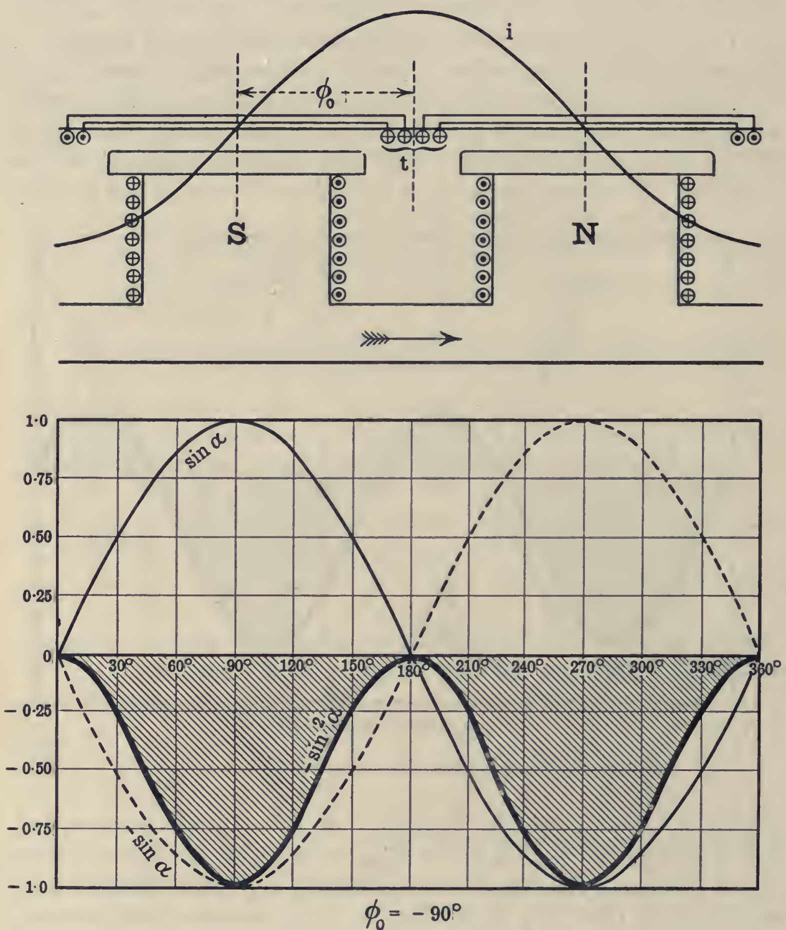


FIG. 455.—Maximum lag of current behind centre of pole; $\phi_o = -90^\circ$.

negative or an angle of lag, and the latter when ϕ_o is positive or an angle of lead. As ϕ_o increases, the area of one half wave shrinks, so that when $\phi_o = -90^\circ$ there is only a weakening effect left (Fig. 455), proportional to $-\sin^2\alpha$. This result may be at once obtained by expanding the expression for the instantaneous value of *mc* to

$$-\sqrt{2}t\mathrm{I}\ (\sin\alpha \,.\, \cos\alpha \,.\, \cos\phi_o - \sin^2\alpha \,.\, \sin\phi_o)$$

for the average value of $\sin\alpha \,.\, \cos\alpha$ between the limits of $\alpha = 0$ and $\alpha = \pi$ is zero.

Further, the average value of $\sin^2\alpha$ between the same limits is $\frac{1}{2}$. The *average* value of the direct magnetising ampere-turns for a single coil as acting upon a pair of poles or per magnetic circuit is thus in general

$$X_{DM} = \frac{1}{2}\sqrt{2}t\mathrm{I} \,.\, \sin\phi_o \qquad (163)$$

and their effect is negative or demagnetising when ϕ_o is negative and the current lags.

On the same approximate assumption that the cross effect of a single coil carrying a constant current varies as a cosine curve according to its position relatively to the poles with the same initial starting-point as before, the instantaneous cross magnetising effect when it carries an alternating current is $\sqrt{2}t\mathrm{I}\cos(\alpha + \phi_o) \,.\, \cos\alpha$. This again yields a double wave for each complete period, and may be expanded to $\sqrt{2}t\mathrm{I}(\cos^2\alpha \,.\, \cos\phi_o - \sin\alpha \,.\, \cos\alpha \,.\, \sin\phi_o)$.

If the phase of the current neither lags nor leads, and its maximum value coincides with the centre of a pole, the density of the flux at the pole-corner ahead of the centre is strengthened, while that of the pole-corner behind the centre is weakened; or if we name the pole-corners in relation to the passage of a coil through an interpolar gap, as was previously done in the case of continuous-current dynamos, the leading pole-corner is always weakened and the trailing pole-corner strengthened, although to a varying degree according to the epoch of the period. But if the maximum value of the current lags behind the centre of a pole, each pole-corner is first weakened and then strengthened, the duration and amplitude of each fluctuation depending upon the angle of lag ϕ_o, until when this $= -90°$, the strengthening is exactly balanced by the weakening and there is no net effect during a whole period, although the field so far as its distribution is concerned oscillates on either side of the centre line. If the current leads, the processes of weakening and strengthening are reversed in order of sequence at each pole-corner, and on the whole there is a strengthening of the leading pole-corner and a weakening of the trailing pole-corner, until when $\phi_o = 90°$ there is again no net effect on the field over a whole period. Since the average value of $\sin\alpha\cos\alpha$ is zero, and of $\cos^2\alpha$ is $\frac{1}{2}$, the average value of the cross turns of a single coil is

$$X_{CM} = \frac{1}{2}\sqrt{2}t\mathrm{I} \,.\, \cos\phi_o$$

Thus in a single-phase alternator or in a polyphase machine, when only one phase is considered, the direct and the cross effect of the armature ampere-turns each vary periodically both in sign and amount as the wires move relatively to the field, except in the two limiting cases of $\phi_o = 0$, and $\phi_o = 90°$, when the only variation is in amount and in synchronism with the period of the machine. The average value over a whole period is, however, in every case dependent upon ϕ_o, and the purport of the above has been to emphasise the importance of the angle ϕ_o between the current vector and the centre of the pole. If there is no angle of lag or lead, the average cross effect reaches its maximum, and there is no average direct magnetising effect; if the angular displacement of the current vector approaches its maximum possible of 90° ahead of or behind the centre of the pole, there is no average cross effect, but the average value of the direct magnetising ampere-turns is a maximum, strengthening the field in the former case and weakening it in the latter case.

§ 6. **The direct and cross ampere-turns of the polyphase alternator.**—But when the same principles are applied to polyphase machines, it is found that the instantaneous value of either the direct or the cross-magnetising effect from all the phases becomes a constant quantity. Thus in a quarter-phase machine, if a second curve of the direct effect is plotted for the second set of coils, similar to that of Fig.

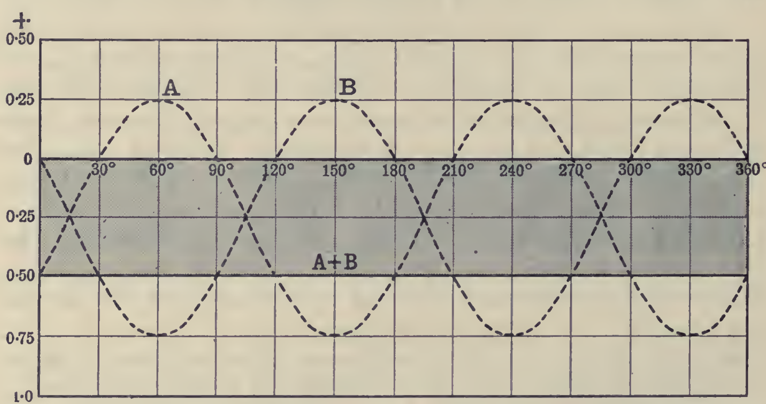


FIG. 456.—Direct-magnetising armature ampere-turns of quarter-phase alternator.

454 but displaced 90° relatively to the first, their algebraic sum (Fig. 456) yields a perfectly straight line of average height equal to twice the average height of Fig. 454. Or mathematically the instantaneous value of the direct magnetising ampere-turns is per magnetic circuit

$$-\sqrt{2}tI\,.\,\{\cos(\alpha+\phi_o)\sin\alpha+\cos(\alpha+\phi_o-90^\circ)\cos\alpha\}$$

which reduces to

$$\sqrt{2}tI\,.\,\sin\phi_o(\sin^2\alpha+\cos^2\alpha)=\sqrt{2}tI\sin\phi_o$$

Similarly, in the 3-phase machine (Fig. 457) a perfectly steady effect is reached, the instantaneous value of the direct ampere-turns being

$$-\sqrt{2}tI\,\{\cos(\alpha+\phi_o)\,.\,\sin\alpha+\cos(\alpha+\phi_o-120^\circ)\,.\,\sin(\alpha-120^\circ)$$
$$+\cos(\alpha+\phi_o-240^\circ)\sin(\alpha-240^\circ)\}$$

which reduces to

$$\tfrac{3}{2}\sqrt{2}tI\,.\,\sin\phi_o(\sin^2\alpha+\cos^2\alpha)=1\cdot5\,.\,\sqrt{2}tI\,.\,\sin\phi_o$$

In both cases, therefore, the sum of the instantaneous values is independent of the angle α, and for the same number of active wires per pole and per phase the direct magnetising ampere-turns for the quarter-phase machine are twice as great as the average value in the single-phase machine, and for the 3-phase machine are three times that of the single-phase or one and a half times that of the quarter-phase machine. Or in general

$$X_{DM} = \frac{m}{2} \sqrt{2}tI \sin \phi_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (164)$$

where m is the number of phases.

So also the instantaneous cross-magnetising effect of the several sets of coils becomes constant exactly as the direct magnetising ampere-turns, with value in the quarter-phase machine of $\sqrt{2}tI \cdot \cos \phi_o$ and in the 3-phase alternator of $\frac{3}{2}\sqrt{2}t \cdot I \cos \phi_o$. The field is then permanently displaced, and passage of a coil-side through maximum

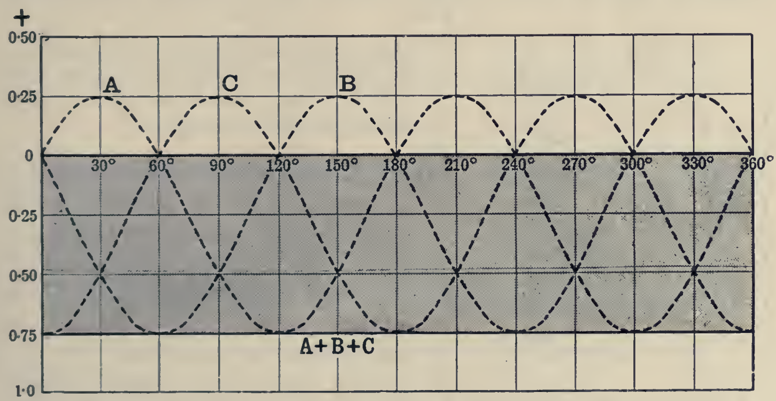


FIG. 457.—Direct-magnetising armature ampere-turns of 3-phase alternator.

field occurs later or sooner than its passage past the centre of a pole, according as the current lags or leads relatively to the centre of a pole.

§ 7. **Calculation of back ampere-turns in inductor generators.**—In the case of the homopolar or inductor alternator with single armature, a comparison of Figs. 458 and 459 shows that there is a difference in the value of X_{DM} according to the method of winding the coils. If these are not subdivided, and there are only as many as there are polar projections, the tendency of their magnetomotive force when the current lags, as shown in Fig. 458, is in the first place to cause a certain number of back lines to pass through each magnetic field, and of these the majority pass round through the main magnetic circuit by way of the second air-gap B at the other end of the armature. A smaller number pass sideways from the flanks of the poles through the interpolar spaces, following the same path as the useless lines of the main field and having the same direction. The number of back ampere-turns acting on the air-gap under a polar projection is therefore exactly the same as in the case of the multipolar machine with alternate poles, or $X_{DM} = \frac{m}{2} \cdot \sqrt{2}tI \sin \phi_o$; and owing to the greater fall of potential over the air-gap the number of useless lines passing into or out of the armature core through the interpolar spaces, or z_a, is increased. If, however, the armature coils are divided as shown in Fig. 459, and as would usually be the case, the back ampere-turns through which the useful flux must be driven, and with which it is linked, are only half the above, or $X_{DM}=\frac{m}{4}\sqrt{2}tI\sin\phi_o$. The useless flux z_a is, however, also increased by the direct assistance of the remaining half of each pair of coils, so that the E.M.F. is thereby lowered more than in the first case. Next, when the inductor alternator has a double armature and divided coils are again used as in the last case, the same effects are repeated on the second half of the armature, and in consequence the total number of back ampere-turns on the magnetic circuit as a whole is again

$$X_{DM}=\frac{m}{2}\sqrt{2}tI\sin\phi_o$$

It is evident how important is the number of the useless lines z_a in any calculation of the E.M.F. when there is lag of the current behind the centre of a pole.

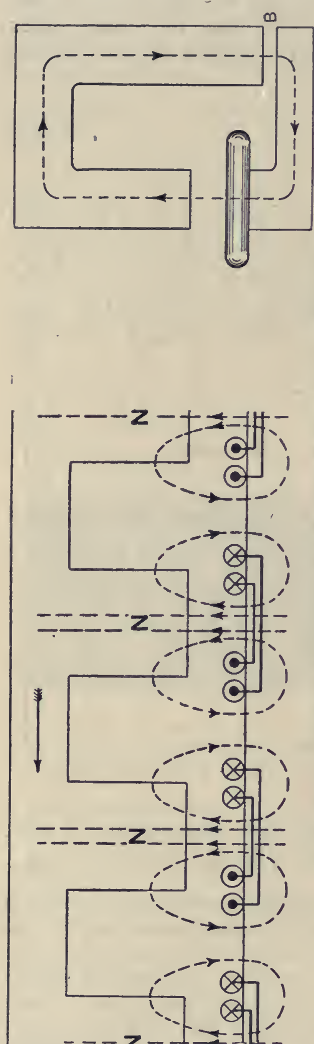


FIG. 458.—Inductor alternator with undivided coils and lagging current.

§ 8. **More accurate calculation of average value of direct and cross ampere-turns.**—The accurate determination of the equivalent direct and cross ampere-turns of the coils on an alternator armature—*i.e.* of the ampere-turns of excitation by a continuous current that would produce the same effect on the magnetic circuit of a pair of poles—is of such importance in the design and theory of alternators, that for the above simple approximations must be substituted a more rigorously exact treatment. The true expressions for either X_{DM} or X_{CM} cannot be immediately calculated from the number of turns on the armature, since they are seldom concentrated within a pair only of slots; hence, even when carrying the same current, they occupy different positions relatively to the poles—an additional consideration which has to be now introduced.*

* In the following section the writer has freely made use of the method of investigating alternator armature reaction due to Professor André Blondel, described

Taking the simplest case of a single coil concentrated in one slot per coil-side, instead of assuming that its direct and cross effect may be represented respectively by a sine and cosine curve, its true effect upon one magnetic circuit, if it were traversed by a continuous current, must first be analysed in detail.

Let B = the pole-pitch, and let b = the virtual width of the field exceeding the width of the actual pole-face by an amount which gives the equivalent of the fringe if concentrated to the same uniform density as under the pole, so that b is less than B by some amount small or great, according to the ratio of the polar arc to the pole-pitch. Let b' = the peripheral width of the coil between the axes of the two slots containing it, which may be more or less than b. The effect which the ampere-turns of the armature coil produce upon the ampere-turns over the air-gap due to the field-excitation is dependent upon the proportion of the pole-face embraced by the former; *e.g.*, if there are back ampere-turns which only include a fraction, say $\frac{3}{4}$ of the pole-face, they

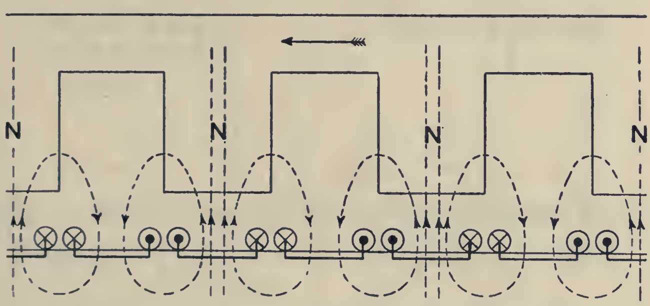


FIG. 459.—Inductor alternator with divided coils and lagging current.

may be regarded as expending their full effect upon the flux included within this area, while the remainder of the flux is unaffected, or in general if they embrace a distance q along the pole-face, their effect upon the field is to be reckoned as $\frac{q}{b}$ of their total value.

Starting with a position of the coil when its axis coincides with a line drawn midway in the interpolar gap between a pair of poles, so that the active sides are then over the poles and the direct effect on the one is neutralised by the counter effect on the other, each quarter of a period is divisible into three stages (Fig. 460). In the first place, let the width of the coil b' be less than the virtual width b of the field.

in his paper at the St. Louis Congress of 1904 (*Trans. Intern. Electr. Congress*, vol. i. p. 635), and also of the writings of C. F. Guilbert (*Electr. World and Eng.*, vol. xl. p. 738 ff., "The Armature Reaction of Alternators"), which are based upon the same general method.

During the first stage, so long as the relative displacement $x \leqq \frac{b+b'-B}{2}$, while the area of the pole-face included by one coil-side is decreasing, the area included by the other is increasing, and if in the former the M.M.F. is assisting, in the latter it is demagnetising. The *difference* of the portions of the pole-faces covered must therefore be taken, namely, $\left(\frac{b+b'-B}{2}+x\right)-\left(\frac{b+b'-B}{2}-x\right)=2x$, so that the value of the turns for a given number of amperes in relation to the width of the field or the exciting ampere-turns thereon is $\propto \frac{2x}{b}$. This lasts until the

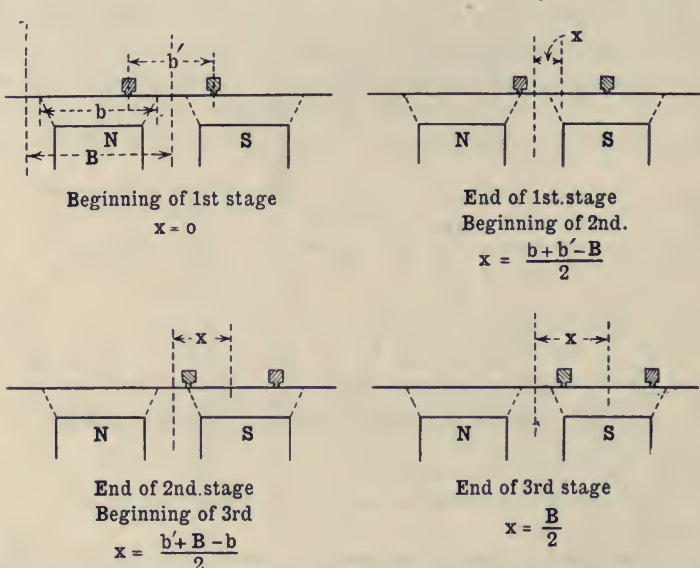


FIG. 460.—Three stages of coil-effect in relation to a pole.

trailing coil-side reaches the edge of the pole, or more strictly of the field (supposed of uniform density), *i.e.* between $x=0$ and $x=\frac{b+b'-B}{2}$.

During the second stage between $x=\frac{b+b'-B}{2}$ and $x=\frac{b'+B-b}{2}$, the trailing coil-side is passing through the interpolar gap of width $B-b$, and the area of the pole embraced by the other coil-side rises from $b+b'-B$ to b', and at any moment is $\frac{b+b'-B}{2}+x$, so that the effect is $\propto \frac{1}{b}\left(\frac{b'+b-B}{2}+x\right)$.

During the third stage, between $x=\frac{b'+B-b}{2}$ and $x=\frac{B}{2}$, the coil falls entirely within the pole-face and simply moves across it. The effect for a given number of turns is therefore constant, and $\propto \frac{b'}{b}$. The proportionate effect of a coil carrying a constant current is therefore at any moment during a quarter of a period given by a curve such as that of Fig. 461 (which has been plotted for $\frac{b}{B}=0{\cdot}8$, and $\frac{b'}{B}=0{\cdot}6$), consisting of two inclined straight lines and a straight horizontal portion, up to $x=\frac{B}{2}$; this is repeated in the next quarter of a period in the reverse order, and so on throughout a complete period. It will be seen that the sine curve of Fig. 453 is more or less correct, since in practice the corners will be rounded off.

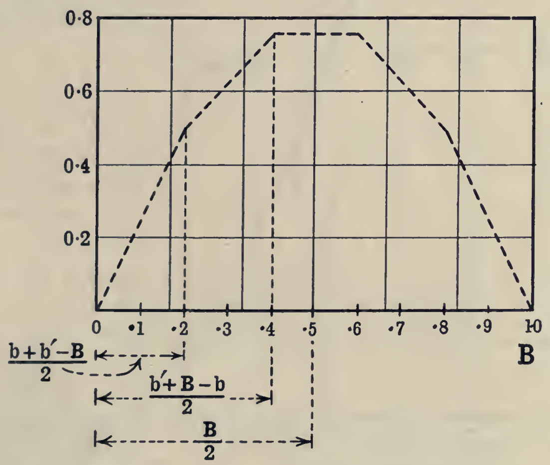


FIG. 461.—Instantaneous value of magnetising effect of coil in relation to a pole.

Meantime the current has been varying, and on the assumption that it obeys a sine law (which is necessary to render the subject amenable to simple mathematical treatment) is $\mathbf{I}\sin\left(\frac{\pi}{2}+\alpha+\phi_o\right)=\mathbf{I}\cos(\alpha+\phi_o)$. Any sinusoidal current can be resolved into two component sinusoidal currents which have in general different amplitudes, and if these amplitudes are chosen as $\mathbf{I}\,.\,\sin\phi_o$ and $\mathbf{I}\,.\,\cos\phi_o$, the two component curves must be in quadrature with each other (Fig. 462). It is then evident that the maximum ordinate of the former will occur when the axes of an armature coil and pole coincide, and of the latter when the axis of a coil falls midway between the poles. The phase of the former component will therefore synchronise with the dotted curve of Fig. 461, while that of the latter component will be in quadrature. Hence, when the product of the latter component with the dotted curve of Fig. 461 is plotted, the algebraic sum of the areas so obtained over a half-period is zero, or its net effect is nil. It may therefore be entirely neglected in calculating the value of the direct magnetising turns, and only the former component of amplitude **I** $\sin\phi_o = \sqrt{2}$ I . $\sin\phi_o$ need be considered. The instantaneous value of this is **I** $\sin\phi_o . \sin\frac{2\pi t}{T} =$ **I** $\sin\phi_o . \sin\frac{2\pi x}{2B}$, or **I** . $\sin\phi_o . \sin\alpha$, where α is the angular displacement of the axis of the coil relatively to the centre of an interpolar gap, *i.e.* the time being reckoned from the same initial position as was before assumed. The product of the instantaneous value of this current or $\sqrt{2}$ I . $\sin\phi_o . \sin\alpha$ with the dotted curve then gives the instantaneous value of the direct magnetising ampere-turns of the coil. In Fig. 462 is shown the case of $\phi_o = -30°$, *i.e.* an angle of lag of 30°; the curve of **I** . $\sin\phi_o . \sin\alpha$ is again plotted as *cc* in Fig. 463, and the product of this with curve *mm* reproduced from Fig. 461 gives the curve of direct

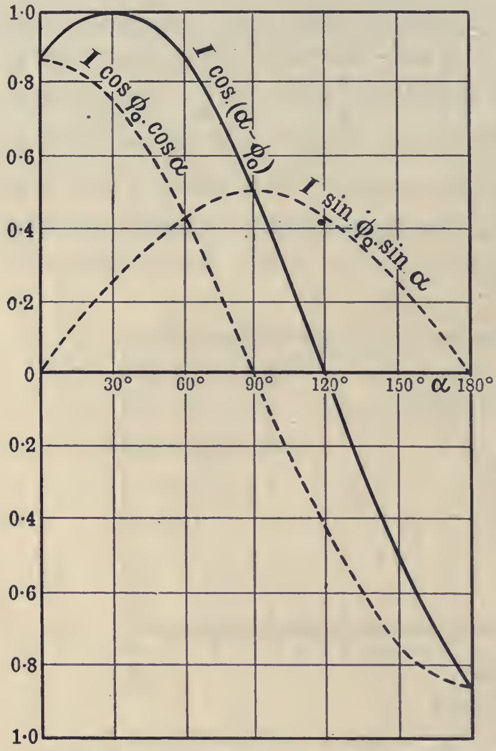


Fig. 462.—Resolution of current into direct and cross-magnetising components.

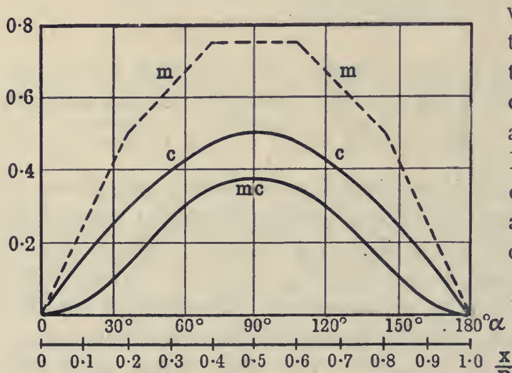


Fig. 463.—Instantaneous value of direct magnetising ampere-turns of coil.

magnetising ampere turns *mc*, which in relation to the magnetic circuit is actually negative or demagnetising.

The average effect is therefore given by the mean ordinate of the curve *mc*, or mathematically by integrating the above expressions over a quarter of a period. In order to do this, it is simplest to convert the spacial into temporal expressions, any distance x being equivalent to an angle (reckoned from the same initial position) of magnitude $\alpha = \pi \cdot \frac{x}{B}$. The mean ordinate is then $\frac{2}{\pi}$ of the area of the curve *mc* for a quarter of a period. Hence

$$X_{DM} = \frac{2}{\pi} \cdot \frac{\sqrt{2}tI \cdot \sin\phi_o}{b}\left[\int_0^{\frac{\pi}{2}\cdot\frac{b+b'-B}{B}} \frac{2\alpha B}{\pi} \cdot \sin\alpha \cdot d\alpha + \int_{\frac{\pi}{2}\cdot\frac{b+b'-B}{B}}^{\frac{\pi}{2}\cdot\frac{b'+B-b}{B}} \left(\frac{b'+b-B}{2} + \frac{\alpha B}{\pi}\right)\sin\alpha \cdot d\alpha + \int_{\frac{\pi}{2}\cdot\frac{b'+B-b}{B}}^{\frac{\pi}{2}} b' \cdot \sin\alpha \cdot d\alpha\right]$$

When the integrals within the bracket are calculated they reduce to the very simple expression

$$\frac{B}{\pi} \cdot 2\sin\frac{\pi}{2} \cdot \frac{b}{B} \cdot \sin\frac{\pi}{2} \cdot \frac{b'}{B}$$

whence

$$X_{DM} = \frac{4}{\pi^2} \cdot \frac{B}{b} \cdot \sqrt{2}tI \cdot \sin\phi_o \cdot \sin\frac{\pi}{2} \cdot \frac{b}{B} \sin\frac{\pi}{2} \cdot \frac{b'}{B} \quad . \quad . \quad . \quad (165)$$

As b' is increased relatively to b, the first and second stages are prolonged, and the third stage is shortened, until when $b' = b$ the third stage disappears and the dotted curve of Fig. 461 reaches an apex of height $= t$. When b' is increased beyond the width of the pole the second stage is altered; it now lasts from $x = \frac{b+b'-B}{2}$ to $x = \frac{b+B-b'}{2}$, and during this time the width of pole-face included rises from $b'+b-B$ to b, and at any moment is $\frac{b'+b-B}{2} + x$, so that the same expression still holds up to the new limit. But from this time, *i.e.* from $x =$

$\frac{b+B-b'}{2}$ to $x=\frac{B}{2}$, a new third stage appears during which the coil embraces the entire pole-face, so that its effective turns remain constant at t, which value can never be exceeded, *i.e.* they are $\propto \frac{b}{b}$. The mean ordinate therefore takes the form

$$X_{DM}=\frac{2}{\pi}\cdot\frac{\sqrt{2}tI\,.\sin\phi_o}{b}\left[\int_0^{\frac{\pi}{2}\cdot\frac{b+b'-B}{B}}\frac{2\alpha B}{\pi}\,.\sin\alpha\,.\,d\alpha\right.$$

$$+\int_{\frac{\pi}{2}\cdot\frac{b+b'-B}{B}}^{\frac{\pi}{2}\cdot\frac{b+B-b'}{B}}\left(\frac{b'+b-B}{2}+\frac{\alpha B}{\pi}\right)\sin\alpha\,.\,d\alpha$$

$$\left.+\int_{\frac{\pi}{2}\cdot\frac{b+B-b'}{B}}^{\frac{\pi}{2}} b\,.\sin\alpha\,.\,d\alpha\right]$$

which yields precisely the same result as already given in equation (165). When the width of the coil is equal to the pole-pitch the second stage entirely disappears, and the dotted *mm* curve consists of uniformly inclined sides joined by horizontal lines. The mean ordinate then becomes simply

$$X_{DM}=\frac{2}{\pi}\cdot\sqrt{2}tI\,.\sin\phi_o\left[\frac{2B}{\pi b}\int_0^{\frac{\pi}{2}\cdot\frac{b}{B}}\alpha\sin\alpha\,.\,d\alpha+\int_{\frac{\pi}{2}\cdot\frac{b}{B}}^{\frac{\pi}{2}}\sin\alpha\,.\,d\alpha\right]$$

the effective turns being $t\,.\,\alpha\frac{2B}{\pi b}$ between $\alpha=0$ and $\alpha=\frac{\pi}{2}\cdot\frac{b}{B}$, and thence up to $\alpha=\frac{\pi}{2}$ remaining constant at the value t. The expression then reduces to

$$=\frac{4}{\pi^2}\cdot\frac{B}{b}\cdot\sqrt{2}tI\,.\sin\phi_o\,.\sin\frac{\pi}{2}\cdot\frac{b}{B} \qquad (166)$$

$$=0.57\frac{B}{b}\,.\,t\,.\,I\,.\sin\phi_o\,.\sin\frac{\pi}{2}\cdot\frac{b}{B}$$

The expression of (165) is equally valid for a coil of which the width exceeds the pole-pitch; the actual end-connections of the active wires are immaterial and with a large undivided coil the outer turns having a width $b' > \mathrm{B}$ or $b' = (1 + k)\mathrm{B}$ may equally well be coupled up to form a short coil of width $(1 - k)\mathrm{B}$, the coil being thus divided. Precisely the same formula therefore holds good whatever relation the width of the coil b' bears to the pole-pitch.

We are now in a position to deal with the coils of a single-phase alternator, whatever may be the number of slots per pole in which they are embedded. Let t = the number of active wires per pole which form a group or sheaf, whether in one large coil or divided is immaterial, so long as the number of slots between which they are divided and the spacing of these slots is known. It is only necessary to take each component coil separately with its correct proportion of the total turns, and to add the several results. Thus in a single-phase alternator with six slots per pole of which four only are utilised, and with concentric coils, the smallest has a width of $\frac{1}{2}\mathrm{B}$ or $45°$, the next smallest a width of $\frac{5}{6}\mathrm{B}$, or $75°$, and the remaining two either repeat these values in reverse order or have a width of $1\frac{1}{6}\mathrm{B}$ and $1\frac{1}{2}\mathrm{B}$, so that their values of $\sin\frac{\pi}{2}.\frac{b'}{\mathrm{B}}$ repeat the former values. Each coil has $\frac{t}{4}$ turns, so that

$$\mathrm{X_{DM}} = \frac{4}{\pi^2}.\frac{\mathrm{B}}{b}.\sqrt{2}.\frac{t}{4}.\mathrm{I}\sin\phi_o.\sin\frac{\pi}{2}.\frac{b}{\mathrm{B}}\left(2\sin 45° + 2\sin 75°\right)$$

$$= \frac{4}{\pi^2}.\frac{\mathrm{B}}{b}.\sqrt{2}t\mathrm{I}\sin\phi_o.\sin\frac{\pi}{2}.\frac{b}{\mathrm{B}} \times 0{\cdot}836$$

This may be put in a more general form; let q_1 = the number of slots between which one side of a coil is divided, and in the first place let q_1 be an uneven number. Let b' = the mean width, *i.e.* between the central slots on either side which yield a M.M.F. proportional to $\sin\frac{\pi}{2}.\frac{b'}{\mathrm{B}}$. If x = the pitch of the slots, the widths of the two neighbouring component coils on either side of the central coil, the one large and the other smaller, are respectively $b' + 2x$ and $b' - 2x$, yielding M.M.F.'s proportional to $\sin\frac{\pi}{2}.\frac{b'+2x}{\mathrm{B}}$ and $\sin\frac{\pi}{2}.\frac{b'-2x}{\mathrm{B}}$, *i.e.* to $\sin\left(\frac{\pi}{2}.\frac{b'}{\mathrm{B}} + \frac{\pi x}{\mathrm{B}}\right)$ and $\sin\left(\frac{\pi}{2}.\frac{b'}{\mathrm{B}} - \frac{\pi x}{\mathrm{B}}\right)$ The sum of the two is therefore proportional to $2\sin\frac{\pi}{2}.\frac{b'}{\mathrm{B}}.\cos g$, where $g = \frac{\pi x}{\mathrm{B}}$ is the angular pitch of the slots in terms of a bipolar machine. In the same way the next larger and smaller coils may be paired together, the sum of their M.M.F.'s being proportional to $2\sin\frac{\pi}{2}.\frac{b'}{\mathrm{B}}.\cos 2g$, and so on. If each slot contains the same number of turns $= \frac{t}{q_1}$, the joint total effect is proportional to

$$\frac{t}{q_1}.\sin\frac{\pi}{2}.\frac{b'}{\mathrm{B}}\left\{1 + 2\cos g + 2\cos 2g + 2\cos 3g + \ldots + 2\cos\frac{q_1 - 1}{2}.g\right\}.$$

The expression within the bracket when q_1 is uneven is equal to $\dfrac{\sin q_1 \cdot \frac{g}{2}}{\sin \frac{g}{2}}$, so that equation (165) becomes

$$X_{DM} = \frac{4}{\pi^2} \cdot \frac{B}{b} \cdot \sqrt{2}tI \sin \phi_o \cdot \sin \frac{\pi}{2} \cdot \frac{b}{B} \cdot \sin \frac{\pi}{2} \cdot \frac{b'}{B} \cdot \frac{\sin \frac{q_1 g}{2}}{q_1 \cdot \sin \frac{g}{2}}$$

where b' is the *width between the central pair of slots* which forms a standard of reference. Exactly the same expression holds if q_1 is even; there is then no central coil, and b' represents the mean width of an imaginary central coil. The pair of coils on either side of the mean width and nearest to it yield M.M.F.'s proportional to $\sin \left(\frac{\pi}{2} \cdot \frac{b'}{B} + \frac{g}{2}\right)$ and $\sin \left(\frac{\pi}{2} \cdot \frac{b'}{B} - \frac{g}{2}\right)$, or when grouped together their sum is proportional to $2 \sin \frac{\pi}{2} \cdot \frac{b'}{B} \cos \frac{g}{2}$; the next pair yield M.M.F.'s proportional to $2 \sin \frac{\pi}{2} \cdot \frac{b'}{B} \cos \frac{3g}{2}$, and so on. The whole is therefore proportional to $\frac{t}{q_1} \cdot \sin \frac{\pi}{2} \cdot \frac{b'}{B} \left\{2 \cos \frac{g}{2} + 2 \cos \frac{3g}{2} + 2 \cos \frac{5g}{2} + \ldots + 2 \cos (q_1 - 1) \frac{g}{2}\right\}$, and the expression within the bracket is again equal to $\dfrac{\sin \frac{q_1 g}{2}}{\sin \frac{g}{2}}$ when q_1 is even.

In a polyphase alternator with t wires per pole and per phase, if m be the number of phases, it is simply necessary to multiply by m, for the average effect of one phase during a period has been calculated, and whatever the initial value of the other phases as compared with that of the first phase, they must yield the same average effect, which may be added to that of the first phase. Thus for the same number of active wires and slots in each sheaf per pole in each phase the direct magnetising ampere-turns of the quarter-phase alternator are double, and of the 3-phase alternator are treble the average value of the single-phase alternator.

We thus have as the general expression

$$X_{DM} = k_d \cdot \sqrt{2}tI \sin \phi_o \quad \ldots \quad (167)$$

where

$$k_d = m \cdot \frac{4}{\pi^2} \cdot \frac{B}{b} \cdot \sin \frac{\pi}{2} \cdot \frac{b}{B} \cdot \sin \cdot \frac{\pi}{2} \cdot \frac{b'}{B} \quad \frac{\sin \frac{q_1 g}{2}}{q_1 \sin \frac{g}{2}} \quad \ldots \quad (168)$$

Here b' is the width between the centres of the coil-sides and the ratio $\frac{b'}{B}$ is conveniently obtained in terms of the slots.

This, the true expression for k_d, compares with the value of $\frac{m}{2}$ in eq. (164), where the division of the coil between several slots has not been taken into account, and $b'=B$ was implicitly assumed. If, as usual, $\frac{b}{B}$ = about 0·7, it will be seen that the divergence of the approximate sine-curve assumption from the true expression is only the difference of 0·5 from 0·517.

If there are γ_1 slots in each phase per pole, and these form a single group, they are equally spaced over a width $\frac{1}{m}$th of the pole-pitch. The mean width b' of the single undivided long coil which they in effect form is equal to the pole-pitch B, and $\sin\frac{\pi}{2}\cdot\frac{b'}{B}$ is unity. Further, $q_1=\gamma_1$, and $g=\frac{\pi}{m\gamma_1}$. In this, which is the most usual case, eq. (168) becomes

$$k_d=m\cdot\frac{4}{\pi^2}\cdot\frac{B}{b}\cdot\sin\frac{\pi}{2}\cdot\frac{b}{B}\cdot\frac{\sin\frac{\pi}{2m}}{\gamma_1\cdot\sin\frac{\pi}{2m\gamma_1}}\quad\ldots\ldots\quad(169)$$

Hence in the *three-phase* alternator with long coils, if there are two slots per pole and per phase,

$$k_d=3\cdot\frac{4}{\pi^2}\cdot\frac{B}{b}\cdot\sin\frac{\pi}{2}\cdot\frac{b}{B}\times 0{\cdot}9659=1{\cdot}176\frac{B}{b}\cdot\sin\frac{\pi}{2}\cdot\frac{b}{B}$$

With three slots per pole and per phase,

$$k_d=3\cdot\frac{4}{\pi^2}\cdot\frac{B}{b}\cdot\sin\frac{\pi}{2}\cdot\frac{b}{B}\times 0{\cdot}96=1{\cdot}168\frac{B}{b}\cdot\sin\frac{\pi}{2}\cdot\frac{b}{B}$$

Thus as the number of slots per pole and per phase is increased the coil-width factor decreases from unity, but even when the coil is uniformly distributed over the whole $\frac{1}{m}$th of the pole-pitch there is no very great reduction, the minimum value of the last multiplier in the 3-phase alternator being for uniform distribution $\frac{\sin\frac{\pi}{2m}}{\frac{\pi}{2m}}=\frac{0{\cdot}5}{0{\cdot}5235}$ $=0{\cdot}953$. In the single-phase case, if y = the ratio of the width of a coil-side to the pole-pitch, the breadth factor becomes for completely uniform distribution $\dfrac{\sin\left(\frac{\pi}{2}.y\right)}{\frac{\pi}{2}.y}$; if c be the outer width of the coil this may also be expressed in terms of c, the integral of the breadth factors across one coil-side being $\cos\left(\frac{\pi}{2}.\frac{c}{B}\right) \div \frac{\pi}{2}\left(1-\frac{c}{B}\right)$.

If the coils of a 3-phase machine are divided, and the two component coils are themselves shortened to a mean width b' equal to half the width of the pole piece as in Fig. 444 ii., the γ_1 slots of each phase per pole now form two separate groups, and if uniformly spaced their angular pitch still remains the same, namely, $g = \frac{\pi}{m\gamma_1}$. But q_1 is now $= \frac{\gamma_1}{2}$ and $\sin\frac{\pi}{2}.\frac{b'}{B} = \sin 45°$. Hence

$$k_d = m.\frac{4}{\pi^2}.\frac{B}{b}.\sin\frac{\pi}{2}.\frac{b}{B} \times 0{\cdot}707 \frac{\sin\frac{\pi}{4m}}{\frac{\gamma_1}{2}.\sin\frac{\pi}{2m\gamma_1}} \quad . \quad . \quad . \quad (170)$$

With 2 slots per pole and per phase (the minimum number for this arrangement) the direct magnetising turns are thus reduced in the proportion of $0{\cdot}707$ to 1; with more slots there is but little further reduction, the last multiplier approaching $\dfrac{\sin\frac{\pi}{4m}}{\frac{\pi}{4m}}$, or in a 3-phase alternator $0{\cdot}99$ with uniform distribution.

With non-overlapping coils there is a still further reduction, but as in all cases of shortened coils it must be remembered that any advantage in respect of their lesser demagnetising effect is always accompanied by a practically equivalent reduction in the E.M.F. for a given value of Z_a.

When the same methods are applied to the calculation of the average effect of the cross magnetising ampere-turns, equally simple expressions are obtained, namely, in general

$$X_{CM} = k_c.\sqrt{2}t\mathrm{I}\cos\phi_o \quad . \quad . \quad . \quad . \quad (171)$$

where

$$k_c = m.\frac{4}{\pi^2}.\frac{B}{b}\left(1-\cos\frac{\pi}{2}.\frac{b}{B}\right)\sin\frac{\pi}{2}.\frac{b'}{B}.\frac{\sin\frac{q_1 g}{2}}{q_1.\sin\frac{g}{2}} \quad . \quad . \quad . \quad (172)$$

and b' is the width between the centres of the coil-sides.

With long coils and γ_1 slots per pole per phase

$$k_c = m \cdot \frac{4}{\pi^2} \cdot \frac{B}{b}\left(1 - \cos\frac{\pi}{2} \cdot \frac{b}{B}\right) \frac{\sin\frac{\pi}{2m}}{\gamma_1 \cdot \sin\frac{\pi}{2m\gamma_1}} \quad . \quad . \quad . \quad (173)$$

With shortened coils

$$k_c = m \cdot \frac{4}{\pi^2} \cdot \frac{B}{b}\left(1 - \cos\frac{\pi}{2} \cdot \frac{b}{B}\right) \times 0{\cdot}707 \frac{\sin\frac{\pi}{4m}}{\frac{\gamma_1}{2} \cdot \sin\frac{\pi}{2m\gamma_1}} \quad . \quad . \quad . \quad (174)$$

It is evident that the two divisions of the armature reaction only differ by the factors $\sin\phi_o$ and $\cos\phi_o$ in the special case when $b = B$, and that they may then be represented by the two sides of a right-angled triangle. But in any other case they diverge, and, as $\frac{b}{B}$ is decreased, $\left(1 - \cos\frac{\pi}{2} \cdot \frac{b}{B}\right)$ decreases much more rapidly than $\sin\frac{\pi}{2} \cdot \frac{b}{B}$.

§ 9. **Determination of direct and cross effect by resolution into a series of sine-curves.**—The same results as above may be reached by a different train of reasoning, which is also primarily due to M. Blondel, and which has been followed by Messrs. Henderson and Nicholson in a paper on "Armature Reaction in Alternators." *

Consider a coil arranged on the internal surface of an armature concentric with which is an iron cylinder representing the field-magnet system. The exact nature of

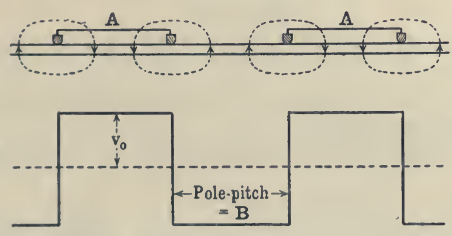


FIG. 464.—M.M.F. of coil with one slot per side.

the latter need not at present be further defined, and the justification for this preliminary assumption will appear later. If the spacial distribution of the magnetic potential in direction and magnitude along the face of the armature is plotted when a direct current is passed through the coils of phase A, and these are concentrated within a single slot per pole, a series of rectangles of width equal to the pole-pitch and of height proportional to half the ampere-turns of the coil is obtained (Fig. 464); for the two halves of each magnetic path as shown by the dotted lines are symmetrical, and half of the total M.M.F. of the coil will be expended over each air-gap, the iron being regarded as unsaturated. Or, if there are two or three slots per phase and per pole, we have stepped right-angled curves such as Figs. 465 and 466; as each

* *Journ. Inst. Electr. Eng.*, vol. xxxiv. p. 465.

slot is passed, the magnetic potential across the air-gap undergoes a sudden increase or decrease, equal to $\frac{1 \cdot 257}{2}$ times the ampere-wires within the slot.

Now, instead of a direct current, let a sinusoidal alternating current of effective value I flow through the coils each containing t turns; then the vertical dimensions of the curves will vary periodically, and in synchronism with the period of the current, between equal positive and negative maxima values, *i.e.* the rectangular curve will collapse or expand vertically up and down. The maximum value will be proportional to $\frac{\sqrt{2} \cdot tI}{2} = v_o$.

The curves so obtained can be analysed into a Fourier series of sine curves, and

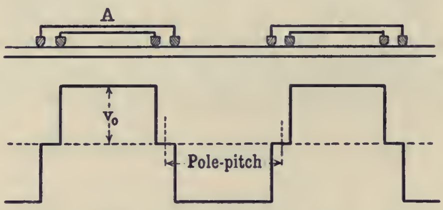


FIG. 465.—M.M.F. of coil with two slots per side.

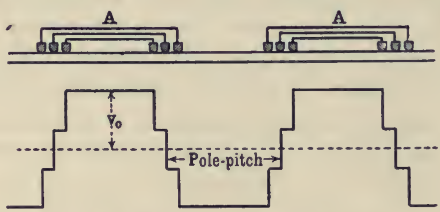


FIG. 466.—M.M.F. of coil with three slots per side.

for the typical cases of Figs. 464-66 the following amplitudes a are obtained for the fundamental sine curve, namely—
for a single coil of breadth equal to the pole-pitch, giving a series of rectangles,

$$a = \frac{4\, v_o}{\pi};$$

with two slots per pole and per phase, separated by a distance of $\frac{1}{6}$ of the pole-pitch, as in a 3-phase alternator

$$a = \frac{4\, v_o}{\pi} \times \cos 15^\circ = \frac{4\, v_o}{\pi} \times 0 \cdot 9659;$$

with three slots per pole and per phase, separated by a distance of $\frac{1}{9}$ of the pole-pitch, as in a 3-phase alternator

$$a = \frac{4\, v_o}{6\pi}\left(2 + 4 \cos 20^\circ\right) = \frac{4\, v_o}{\pi} \times 0 \cdot 96.$$

But the value of the difference of magnetic potential between armature and field-magnet due to the ampere-turns of one phase is for each particular point on the armature a periodic function not only of the space or distance in relation to the poles, but also of the time in relation to the value of the current. The space-period is equal to twice the pole-pitch or 2B, and the time-period is equal to the period T of the current. The full expression for the difference of magnetic potential (expressed in terms of ampere-turns) along the face of the armature is therefore

$$y = a \sin \frac{2\pi . x}{2B} . \sin \frac{2\pi t}{T}$$

reckoning from the spot occupied by a coil-side at the moment when the current is zero. In a single-phase alternator, since $\frac{x}{2B}$ and $\frac{t}{T}$ are equal, this gives $y = a \sin^2 \alpha$, so that its value pulsates, and this average value of $\sin^2 \alpha$ being $\frac{1}{2}$, the average value over a period is $\frac{1}{2} a$.

In the case of a polyphase machine the other phases have also to be taken into account in their relative phases of space and time. Thus in a quarter-phase machine the two component fundamentals are

$$y_{A1} = a . \sin \frac{2\pi x}{2B} . \sin \frac{2\pi t}{T}$$

$$y_{B1} = a . \sin \left(\frac{2\pi x}{2B} - 90^\circ\right) . \sin \left(\frac{2\pi t}{T} - 90^\circ\right)$$

and adding the two together,

$$y_{A1} + y_{B1} = a . \cos 2\pi \left(\frac{x}{2B} - \frac{t}{T}\right)$$

Similarly in a 3-phase alternator

$$y_{A1} = a . \sin \frac{2\pi x}{2B} . \sin \frac{2\pi t}{T}$$

$$y_{B1} = a . \sin \left(\frac{2\pi x}{2B} - 120^\circ\right) . \sin \left(\frac{2\pi t}{T} - 120^\circ\right)$$

$$y_{C1} = a . \sin \left(\frac{2\pi x}{2B} - 240^\circ\right) \sin \left(\frac{2\pi t}{T} - 240^\circ\right)$$

and adding the three together

$$y_{A1} + y_{B1} + y_{C1} = 1{\cdot}5\, a . \cos 2\pi \left(\frac{x}{2B} - \frac{t}{T}\right)$$

$$Y = A . \cos 2\pi \left(\frac{x}{2B} - \frac{t}{T}\right)$$

If we now take any point on the armature (assumed stationary) opposite a particular point on the rotating magnet at any instant t, calculate the value Y, and next consider other points on the armature which are opposite to the same point on the rotating pole at later times, $\frac{x}{2B}$ and $\frac{t}{T}$ will have been so chosen as to increase proportionately, and we always reach the same value for Y. In other words, above a given spot on the rotating pole the combination of the fundamental sine curves in the polyphase machine always gives the same difference of magnetic potential at the armature surface due to the armature ampere-turns of the several phases. This proves that the resultant fundamental curve moves forward in exact synchronism with the field poles, or is stationary over them, although moving relatively to the armature. We are therefore justified in plotting the resultant fundamental curve F_{AR} (Fig. 467) of the armature ampere-turns acting at the armature surface in direct relation to the field poles as rotating with them. Its shape is sinusoidal with an amplitude of a in the quarter-phase machine, or of $A = 1{\cdot}5\, a$ in the 3-phase machine. This maximum always stands centrally over a coil-side at the instant when the current in it is a maximum; if, therefore, the maximum value of the current in a coil-side lags behind or leads in front of the centre of the pole, the resultant fundamental is also similarly displaced by the same angle to one side or the other of the centre of the pole. The exact position of the resultant fundamental sine curve relatively to the field poles thus depends upon the power-factor of the entire circuit. If the power-factor of the external circuit were unity and there were no secondary leakage, the zero of the joint fundamental will stand over the centre of the pole, or if the power-factor of external circuit and armature is zero the maximum value will stand over the centre of the pole.

So far only the combined fundamentals have been considered. There are no harmonics whose frequency is an even multiple of the fundamental, and, applying the same process to the harmonics of triple, quintuple, etc. frequency, it will be found that, when the component harmonics of triple frequency are combined for the three sets of coils in a 3-phase alternator they cancel out, and similarly all other multiples of three disappear. On the other hand, the harmonics of quintuple frequency combine to give a resultant sine curve whose amplitude is 1·5 times that of each component, and which at any moment is proportional to $-\cos 2\pi\left(\frac{5x}{2B}+\frac{t}{T}\right)$; the joint har-

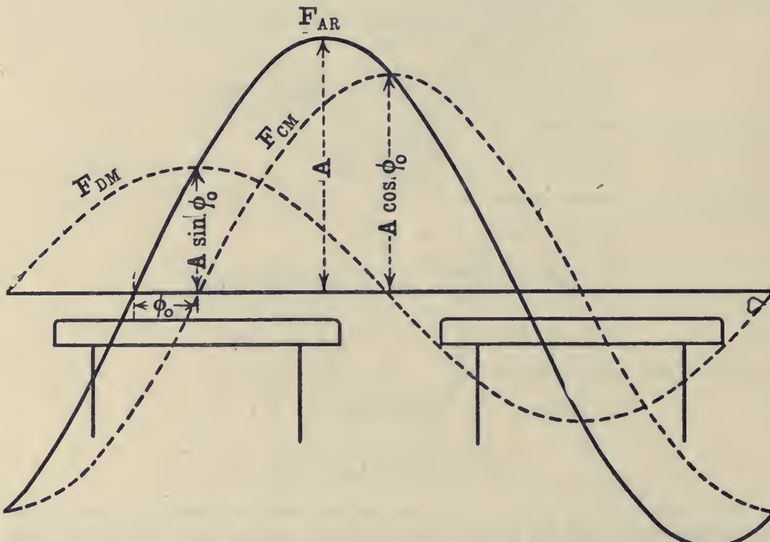


FIG. 467.—Resultant fundamental curve of M.M.F. from armature ampere-turns in 3-phase alternator.

monic of quintuple frequency therefore rotates in the opposite direction to the field poles at one-fifth of the synchronous speed, and similarly the joint eleventh harmonic moves with one-eleventh of the synchronous speed in the opposite direction to the field poles. On the other hand, the seventh, thirteenth, etc. joint harmonics are proportional to $\cos 2\pi\left(\frac{7x}{2B}-\frac{t}{T}\right)$, etc., and therefore like the fundamental rotate in the same direction as the field poles, but at their corresponding fractions of the synchronous speed. Since all the joint harmonics move relatively to the poles, they will induce in the field-coils and pole-faces damping currents, and the magnetising effect that they would otherwise have is thereby to a great extent neutralised. They may therefore be dismissed from consideration, and we return to the joint fundamental sine wave F_{AR} of magnetomotive force, displaced by an angle ϕ_o relatively to the centre of the pole.

As before in § 8, this sine wave may be resolved into two sine curves F_{DM} and F_{CM} of amplitudes A sin ϕ_o and A cos ϕ_o; the former stands over the pole centre and corresponds to the direct-magnetising ampere-turns of the armature ; the latter stands midway between the poles and corresponds to the cross-magnetising ampere-turns (as in Fig. 467, where the joint fundamental is resolved into its two dotted components F_{DM} and F_{CM}).

The numerical value of X_{DM} can now be obtained from the fundamental sine curve of F_{DM} in Fig. 467. Since this is symmetrical with the pole, the equal division of the total M.M.F. into two halves with which we started is justified, for it is expended over closed paths, each of which is divisible into two halves of equal length and reluctance. We are also able to pass from the initial assumption of a uniform cylinder to the actual pole-surface which is not continuous but broken. The curve of F_{DM} is at its maximum at the pole-centre, and thence falls to zero on a line midway between the poles. Even beyond the pole-tips it will have its proportionate effect upon the fringe lines, yet since, in order to deal with the entire flux of one field, it is necessary to concentrate the fringe to the normal density within a virtual width b which is greater than the actual width of pole-face, so the direct-magnetising ampere-turns per pole expressed in terms of their effect upon the normal density will corre-

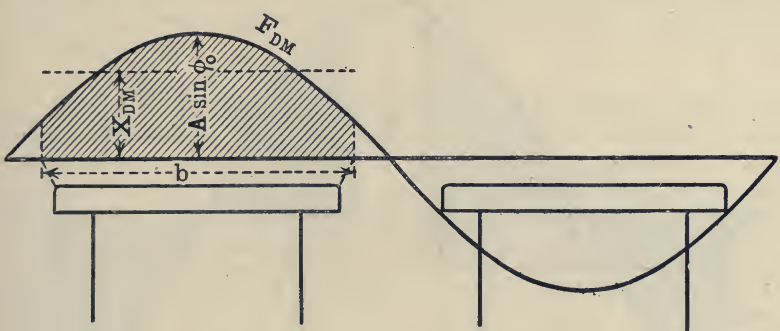


FIG. 468.—Average value of direct-magnetising armature ampere-turns.

spondingly be their average value reckoned over the virtual width of pole-face, *i.e.* over b or the shaded portion of Fig. 468. The average ordinate of this portion of the sine curve is equal to maximum ordinate $\times \dfrac{\sin \frac{\pi}{2} \cdot \frac{b}{B}}{\frac{\pi}{2} \cdot \frac{b}{B}}$.

Since the same effect is produced on the adjacent pole, both current and direction of field being reversed, the direct magnetising turns acting per pair of poles or per magnetic circuit are

$$X_{DM} = 2 \times \text{amplitude} \times \frac{\sin \frac{\pi}{2} \cdot \frac{b}{B}}{\frac{\pi}{2} \cdot \frac{b}{B}} = \frac{4}{\pi} \times \text{amplitude} \times \frac{B}{b} \times \sin \frac{\pi}{2} \cdot \frac{b}{B}.$$

Inserting then the amplitudes of p. 870 multiplied by $\sin \phi_o$, we have in the quarter-phase alternator with one slot per pole and per phase

$$X_{DM} = \frac{4}{\pi} \times \frac{4 v_o}{\pi} \sin \phi_o \times \frac{B}{b} \cdot \sin \frac{\pi}{2} \cdot \frac{b}{B}$$

or since $v_o = \dfrac{\sqrt{2} t I}{2}$

$$= 0{\cdot}812 \cdot \frac{B}{b} \cdot \sqrt{2} t I \cdot \sin \phi_o \cdot \sin \frac{\pi}{2} \cdot \frac{b}{B}$$

In the 3-phase alternator with one slot per pole and per phase

$$X_{DM}=\frac{4}{\pi}\times A\sin\phi_o\times\frac{B}{b}\cdot\sin\frac{\pi}{2}\cdot\frac{b}{B}=1{\cdot}5\times\frac{4}{\pi}\times a\times\frac{B}{b}\cdot\sin\phi_o\cdot\sin\frac{\pi}{2}\cdot\frac{b}{B}$$

$$=1.22\frac{B}{b}\cdot\sqrt{2}tI\cdot\sin\phi_o\cdot\sin\frac{\pi}{2}\cdot\frac{b}{B}$$

the numerical constant becoming with two slots per pole and per phase $=1{\cdot}176$ and with three slots per pole and per phase $=1{\cdot}168$.

The value of the cross magnetising turns is arrived at in a similar manner from the fundamental curve of F_{CM} as shown in Fig. 469. Their effect may be taken into account by assuming a certain cross flux corresponding to them, and it now follows that this local flux will be constant in its distribution and will rotate with the poles. The curve of F_{CM} is symmetrical about the centre of the pole, and as before an air-gap of uniform length, extending over the virtual breadth b of pole-face, may be assumed. The curve must then be integrated over $\frac{b}{2}$ or an angle $\frac{\pi}{2}\cdot\frac{b}{B}$, and the result doubled to

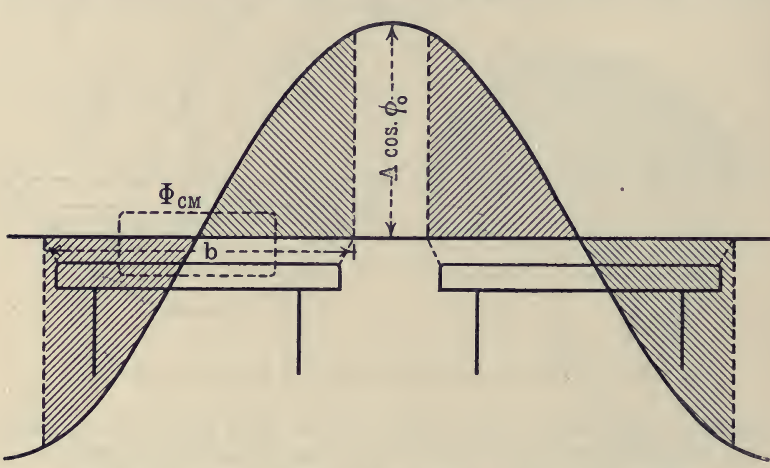


FIG. 469.—Average value of cross-magnetising armature ampere-turns and cross flux.

take into account the assisting M.M.F. over the other half of the same pole. The mean ordinate is then

$$X_{CM}=\frac{2\times\text{amplitude}}{\frac{\pi}{2}\cdot\frac{b}{B}}\int_0^{\frac{\pi}{2}\cdot\frac{b}{B}}\sin\alpha\,.\,d\alpha=\frac{4}{\pi}\cdot\frac{B}{b}\times\text{amplitude}\times\left(1-\cos\frac{\pi}{2}\cdot\frac{b}{B}\right)$$

and inserting the values of the amplitude $a\cos\phi_o$ or $A\cos\phi_o$ for the quarter-phase and 3-phase alternators the same values are obtained as in § 8, namely—

$$X_{CM}=m\cdot\frac{4}{\pi^2}\cdot\frac{B}{b}\cdot\sqrt{2}tI\cdot\cos\phi_o\left(1-\cos\frac{\pi}{2}\cdot\frac{b}{B}\right)\sin\frac{\pi}{2}\cdot\frac{b'}{B}$$

The exact correspondence of the two methods of §§ 8 and 9 is evident; it finds its reason in the underlying identity of the two processes that have been followed. The calculation in § 8 is based on the average effect of the m phases when the widths of pole and coil are taken into account, and this is exactly analogous to expressing the M.M.F. wave of each phase by a series of sine curves and isolating for consideration only the fundamentals. The harmonics which are dismissed from consideration may cause local variations of the flux, but cannot exert any net permanent effect.

The difference from the less rigorous method of §§ 5 and 6 arises from the fact that the maximum value of the assumed sinusoidal curve for a constant current coincides with the maximum height of the rectangle of Fig. 464, whereas the amplitude of the true stationary fundamental exceeds it, so that the value for X_{DM} by the simple approximate method in this respect would be less than that given by the true fundamental in the ratio of $1 : \frac{4}{\pi}$. On the other hand, this divergence is nearly equalised by the fact that the former curve has been integrated over the whole pole-pitch, whereas the latter was only integrated over the pole-face, which gives the multiplier $\frac{\sin\frac{\pi}{2}\cdot\frac{b}{B}}{\frac{\pi}{2}\cdot\frac{b}{B}}=\frac{2B}{\pi b}\cdot\sin\frac{\pi}{2}\cdot\frac{b}{B}$. If the coil is distributed, the effect of the two or more slots in each coil-side is partially taken into account in the process of obtaining the fundamental, while on the simple sine hypothesis we should have further to multiply by a breadth coefficient, which would again leave the two results in the same relative position.

§ 10. **The different nature of the effects from cross and direct armature ampere-turns.**—The different character of the effects from the cross and direct magnetising ampere-turns of the armature, which has above been partially shown, may again be emphasised. The cross turns do not increase or decrease the resultant ampere-turns acting on the main magnetic circuit, but they increase the M.M.F. acting over one-half of the pole-face and diminish the M.M.F. acting over the other half. They thus simply alter the distribution of the main flux over the pole-face without sensibly altering its average value. But precisely the same effect upon the induced E.M.F. of the armature is obtained by assuming the actual main flux to remain undistorted, and by superposing upon it a local cross flux passing through one half of the air-gap, along the iron of the pole-shoe, across the second half of the same air-gap, and so completing its magnetic circuit through the iron of the armature (Fig. 469). This local flux must be of such magnitude and distribution as to reproduce the actual distribution when compounded with the main flux.* Since the cross flux either varies in amount or, if constant, travels with the rotating poles, it will cut the armature wires, and thence arises a self-induced E.M.F. in the armature. By calculating this E.M.F. we account for one element of the armature reaction corresponding to the second item in the group of effects covered by the generic term "inductance" of the armature, and it will be clear that the effect of this item is best obtained by converting it into an electric voltage.

On the other hand, the third item of the armature reaction, or the

* For the approximate plotting of the cross flux and also of the main flux, see "E.M.F. Wave Forms," by E. A. Biedermann and J. B. Sparks, *Journ. Inst. Electr. Eng.*, vol. xxxv. p. 493.

direct magnetising effect, is essentially magnetic, and may be taken into account by adding it to or subtracting it from the interpolar excitation X_P supplied by the field-winding. If ϕ_o is negative, X_{DM} are back ampere-turns, and increase the primary leakage of the field-magnet just as X_b in the continuous-current dynamo.

§ 11. **The effective value of the cross E.M.F.**—The assumed cross flux flowing through one-half of the pole-face is equal to the M.M.F. of X_{CM} divided by the reluctance of two air-gaps in series, each of length ml_g and of area equal to the breadth of half the virtual pole-face multiplied by the axial width of the pole-face, *i.e.*

$$1.257\ X_{CM} \div \frac{2ml_g}{\frac{b}{2} \cdot L}$$

The total cross flux when the half of an adjacent pole in which it has the same direction is taken into account is double the above amount, or

$$Z_c \text{ forming one cross field} = \frac{1 \cdot 257\ X_{CM}}{\frac{2ml_g}{b \cdot L}} \quad . \quad . \quad . \quad (175)$$

As the armature wires cut this flux, the effective value of the cross E.M.F. induced per phase is

$E_{CM} = K_c .\ 2Z_c . f \times$ no. of active wires in series in one phase $\times\ 10^{-8}$ . . . (176)

In order to determine the factor K in relation to the cross flux, the accurate process will be to convert the flux curve of Fig. 469 into an E.M.F. curve in terms of the volts produced by one component coil of the armature winding in a single pair of slots; then to plot one or more such curves spaced at the correct distance apart according as there are one, two, or more slots per pole and per phase, and finally to add together algebraically the ordinates of the component curves, rounding off the sharp corners to form a resultant instantaneous E.M.F. curve. The width coefficient k' is then

$$\frac{\text{area of resultant E.M.F. curve}}{\text{area of single E.M.F. curve} \times \text{number of slots per pole per phase}}$$

Converting the curve of resultant E.M.F. plotted by rectangular co-ordinates into a polar curve, and finding the mean radius of the polar curve (Chap. VI. § 14), the form factor is

$$k'' = \frac{\text{mean radius of polar curve}}{\text{mean ordinate of Cartesian curve}}$$

and $K_c = k' . k''$.*

The effective cross E.M.F. as above obtained includes the harmonics of which that of triple frequency is usually strongly pronounced. The

* An example of the process is given in the above-cited paper of Messrs. Henderson and Nicholson, *Journ. Inst. Electr. Eng.*, vol. xxxiv. p. 472.

effect of this triple-frequency harmonic, although present in the E.M.F. of each phase, is eliminated when the terminal E.M.F. is taken between the lines connected to a star winding.

Any influence from the saturation of the teeth through the crowding of the actual flux to the trailing corner of the pole has so far been neglected. The effect would be to shift the virtual centre of the field towards the unsaturated side, as described in Chapter XVII. § 10, which would correspond to an increase of the angle ϕ_o when the current lags, and to a decrease of ϕ_o when the current leads. The effect is, however, except in extreme cases, quite negligible in view of other sources of error which are present.*

Although the shape of the cross E.M.F. curve is not sinusoidal, yet we are justified in treating its effective value as a vector for the purpose of calculation from a vector diagram, just as in the case of the effective E.M.F. E_a from the main field, which is similarly treated as a vector. The two vectors E_{CM} and E_a will be in quadrature, since the cross flux is in quadrature with the main field flux, and by combining the two at right angles to one another we obtain the effective value of the real induced E.M.F. E_i which is greater than E_a. The increase of E_i as compared with E_a, although the total value of the flux Z_a remains constant, corresponds to the more peaked shape of the resultant flux curve, which increases the form factor and the effective E.M.F. for the same value of the average E.M.F.

From E_{CM} can be calculated the equivalent cross reactance or the equivalent cross self-inductance of the armature per phase of the winding. If $\mathcal{R}_c$ be the reluctance of the cross circuit, the cross flux is $\frac{1{\cdot}257\,X_{CM}}{\mathcal{R}_c}$, and by eq. (171) and (176) the E.M.F. from it is per phase $E_{CM} = 2K_c \cdot \frac{0{\cdot}4\pi k_c \cdot \sqrt{2}\,t I \cos\phi_o}{\mathcal{R}_c} \cdot f \cdot 2pt \times 10^{-8}$. But E_{CM} is also $= 2\pi f L_c \cdot I \cos\phi_o$, where L_c is the equivalent cross inductance per phase.

Therefore
$$L_c = \frac{K_c \cdot 4k_c \sqrt{2}\,t^2 \cdot 2p}{\mathcal{R}_c} \times 10^{-9}$$

and if K_c be assumed as for a sinusoidal curve, *i.e.* $= \frac{\pi}{2\sqrt{2}}$

$$L_c = \frac{4\pi k_c \cdot t^2}{\mathcal{R}_c} \times p \times 10^{-9}$$

The value of K_c in the particular case of three slots per pole and per phase and a pole-width 0·75 of the pole-pitch was $0{\cdot}89 \times 1{\cdot}06 = 0{\cdot}945$, and in general it does not differ greatly from K in the equation for the effective E.M.F. E_a from the main field. If the two are taken as identical, and further $\mathcal{R}_c$ is identified with $\frac{2ml_{vg}}{b \cdot L}$ or the two halves of

* Henderson and Nicholson, *loc. cit.*, p. 482.

adjacent poles, $\frac{E_{CM}}{E_a}=\frac{X_{CM}}{X_g}$, and the angle between E_a and E_i is $\sin^{-1}\frac{X_{CM}}{\sqrt{X_g^2+X_{CM}^2}}$, or $\sin^{-1}\frac{E_{CM}}{\sqrt{E_a^2 \times E_{CM}^2}}$

§ 12. **The single-phase alternator.**—It has been shown that in the single-phase alternator the armature reaction or its impedance pulsates as the armature coils and poles change their relative position through rotation, and such pulsation can be rendered evident by experiment as "beats" on ammeter and voltmeter.* The pulsation affects both the direct and the cross magnetising turns, and due to this the determination of the actual effect of either the one or the other is open to much more uncertainty than in the polyphase machine. In the latter the fundamental sine curve is stationary in relation to the poles, and the deductions therefrom are only slightly invalidated by the influence of the higher harmonics. But in the single-phase alternator the alternating M.M.F. may be resolved into two equal constant M.M.F.'s, each of half the amplitude, one rotating in the same direction and at the same speed as the poles (and therefore fixed in relation to them), and the other in the opposite direction and also at the same speed. The latter causes a pulsating flux through the field-magnet of double the machine frequency. The above expressions for the average effect X_{DM} when applied to the single-phase alternator thus involve the assumption that the latter component may be completely neglected, and the method is only true if the armature is provided with a practically perfect amortisseur or damper winding. If the pole-pieces and magnet-cores are solid, no doubt eddy-currents are set up which do very appreciably limit the amplitude of the variations; and even if the iron is laminated, the exciting coils will partially act as a secondary. In the polyphase machine the same objection only applies to the higher harmonics.

In order, then, to render the method based upon average values for the direct and transverse reactions more true in the case of the single-phase alternator, it is necessary to increase the value of the self-inductance of the armature L_a as compared with its value under otherwise similar conditions in the polyphase machine. Taking the theoretical values for X_{DM} and X_{CM}, calculated as for a polyphase machine, the fluxes which they yield are respectively $\frac{1{\cdot}257\,X_{DM}}{\mathfrak{R}_d}$ and $\frac{1{\cdot}257\,X_{CM}}{\mathfrak{R}_c}$, where $\mathfrak{R}_d$ and $\mathfrak{R}_c$ are the reluctances of the direct and cross circuits. In the same way as the equivalent cross reactance and cross inductance L_c were obtained in the preceding section, so the equivalent direct self-inductance of a complete phase is

$$L_d=\frac{K\,.\,4k_d\,.\,\sqrt{2}t^2\,.\,2p}{\mathfrak{R}_d}\times 10^{-9}$$

or if K be taken as for a sinusoidal curve $=\frac{\pi}{2\sqrt{2}}$

$$=\frac{4\pi k_d\,.\,t}{\mathfrak{R}_d}\times p\times 10^{-9}$$

The *additional* parasitic inductance is then equal to the mean of the two values L_d and L_c, reduced by some coefficient c less than unity to allow for the greater or lesser amount of damping by surrounding solid masses or by actual damping circuits, so that $L_a'=L_a+c\,.\,\frac{L_d+L_c}{2}$. Finally, in proportion as c is decreased the resistance of the armature must be increased to some value greater than r_a in accordance with the energy which is lost by the eddy-currents, for this energy must be supplied by

* Atchison, *Journ. Inst. Electr. Eng.*, vol. xxxiii. p. 1085.

the armature. If the actual values of the armature inductance are measured with the magnet poles first opposite to the armature poles and then crossed, half of the mean value or $\frac{L_1+L_2}{4}$ may be taken as the equivalent of $L_a + c \cdot \frac{L_d+L_c}{2}$. The remaining half is taken account of in the average direct and transverse ampere-turns. It is evident that in the matter of its regulation the single-phase alternator is at a disadvantage as compared with the polyphase machine, since any additional self-inductance will again be prejudicial to the regulation.

§ 13. **The vector diagram of E.M.F.'s.**—The vector diagram illustrating the relation between E.M.F. and current can now be drawn, and therefrom can be determined the ampere-turns of excitation required on the field-magnet to produce a given terminal E.M.F. under load, the magnetisation curves of the alternator being assumed to be known.

The active external E.M.F. is itself the sum of the volts lost over the ohmic resistance of the external circuit and of any E.M.F. which is absorbed in doing useful work against a back E.M.F., as, *e.g.*, in the primary coil of a transformer or the armature of an alternating-current motor. The vector sum of this active external E.M.F. with the E.M.F. consumed by the self-inductance of the external circuit, the two being in quadrature, yields the terminal E.M.F. with its phase angle ϕ_e relatively to the current vector.

The fall of potential in the armature as compared with the open-circuit voltage is due (1) to the loss of volts over the ohmic resistance of the winding, (2) to non-uniform distribution of current in the armature bars, (3) to the weakening of the field through such portion of the eddies set up by the varying armature current as are in phase therewith, principally in the poles if these are not laminated, or in the armature discs and their cast-iron case, and (4) to the reaction of the armature ampere-turns upon the field.

The effects of (2) and (3) may be taken into account by assuming the apparent resistance of the armature to be higher than its real ohmic resistance. Thus, if r_a be the real resistance of one phase of the armature winding, r_a' may be assumed as say $1\frac{1}{2}$ times r_a. Any such allowance is purely empirical, and must be checked by reference to other facts. Many of the eddy-currents in the magnetic system are already present on open circuit, and then weaken the field as much as at full-load; they have no further share in causing the drop of volts between no-load and full-load. The chief effect of additional eddy-currents set up in the iron carcase by the armature current is to render the actual value of the back ampere-turns lower than the theoretical value; for they are practically wattless, and the ampere-turns to which they give rise are almost exactly opposed to the ampere-turns of the armature. They thus reduce the apparent number of armature magnetising turns or the apparent inductance, just as in a transformer or as in the amortisseur of Leblanc,* but do not call for any great increase in r_a. On the other hand, the increase due to non-uniform distribution of may become of considerable magnitude in the case of solid bars of great depth and thickness.†

Given the effective value of the terminal voltage E_e per phase as OA inclined to the current vector OI at the angle ϕ_e (Fig. 470), from A draw AB parallel to OI and of length $= Ir_a'$, or the loss of volts over the resistance of one phase, allowance being made for the effect of eddy-currents by empirically assuming $r_a' = 1{\cdot}5 r_a$, as explained above. While the ohmic loss of volts coincides in phase with the current, the internal self-induced E.M.F. due to the first portion of the armature inductance L_a, *i.e.* to the secondary leakage, will have a lag of 90°

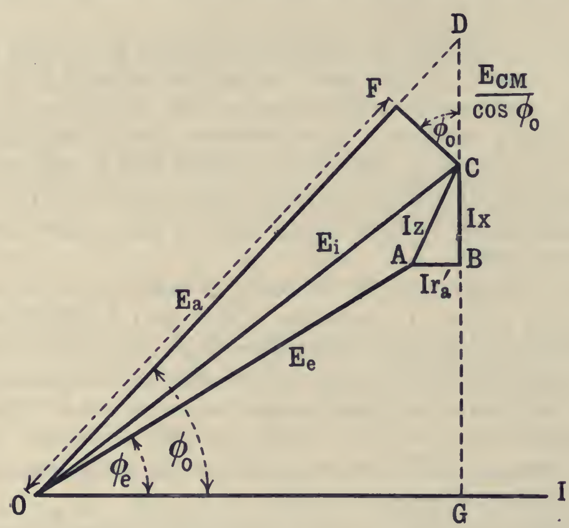


FIG. 470.—Vector diagram of E.M.F.'s of alternator per phase.

behind the current, and the voltage consumed in overcoming it, *i.e.* e_{sa} or Ix (on the assumption of a sinusoidal rate of change and a reactance x, corresponding to L_a) must be exactly opposite. At right angles, therefore, to AB draw the perpendicular BC = the effective value of e_{sa}, and the resultant of AB and BC, or AC represents the E.M.F. consumed by the armature impedance, or Iz. The resultant of OA and AC, or OC, is then the E.M.F. actually induced in the armature winding per phase E_i, and this must itself be the resultant of the E.M.F.'s from the cross flux (E_{CM}) and from the main flux (E_a), these two latter E.M.F.'s being at right angles to each other.

* Cf. Potier, *L'Éclairage Électrique*, vol. xxiv. p. 138.
† Cf. Chap. XXV. § 9.

The angle of lag of the current behind the centre of the pole is not yet known, and therefore the cross E.M.F. from the transverse reaction is unknown. It can, however, easily be determined geometrically. Although the cross E.M.F. is itself undetermined, the only factor in equations (171), (175), and (176) unknown is $\cos \phi_o$; all the remaining factors are known from the constructional data of the machine, so that $\frac{E_{CM}}{\cos\phi_o}$ is known. Hence produce BC by a length $CD = \frac{E_{CM}}{\cos\phi_o}$, and join OD; then the angle between OD and OI is the required angle ϕ_o, and the vector of the current OI lags behind the centre of the pole by the angle ϕ_o. Upon OD let fall the perpendicular CF from C; then $FC = E_{CM}$, and OF = the E.M.F. which is caused by the main field flux Z_a if this were undistorted, and to which Z_a must be proportional. That this construction fulfils all the requirements can be seen from the following considerations. If DB is produced to meet OI at G, it is evident that the triangle DOG is similar to the triangle DCF, and angle DCF = angle DOG, or ϕ_o. Hence $FC = DC \cdot \cos \phi_o = E_{CM}$, and this is at right angles to OF, or the E.M.F. due to the main flux and of which the phase coincides with the centre of the pole.

The angle ϕ_o is thus given by the relation

$$\tan \phi_o = \frac{DG}{OG} = \frac{GB + BC + CD}{OG}$$

$$= \frac{E_e \cdot \sin \phi_e + e_{sa} + \dfrac{E_{CM}}{\cos \phi_o}}{E_e \cdot \cos \phi_e + Ir_a'} \qquad \text{(177)}$$

Assuming a sinusoidal law for both e_{sa} and E_{CM},

$$\tan \phi_o = \frac{E_e \cdot \sin \phi_e + 2\pi f (L_a + L_c) I}{E_e \cdot \cos \phi_e + Ir_a'}$$

where L^c has the same value as in § 11.

The angle ϕ_o for a given value of ϕ_e is thus chiefly dependent upon the cross E.M.F. or transverse inductance L_c. The cross E.M.F., so far from having any direct effect in causing the fall of terminal voltage under load, slightly increases the voltage, the form factor being increased by the distortion; yet it has a most important indirect effect, inasmuch as it partly determines the angle ϕ_o upon the sine of which depends the value of the *back* ampere-turns.

Although the field after distortion by the cross ampere-turns (even when the distortion remains practically constant as in a polyphase machine) cannot as a matter of fact give a sine curve of E.M.F., so that no deductions as to the instantaneous values of the field or of the power can be based upon the relative positions and values of the vectors of E.M.F. and current, yet such a diagram as that of Fig. 470 does give a clue to the actual state of the case. If an instantaneous photograph of the lines of the field of the alternator when running under load could be taken, the flux crossing the air-gap from pole to armature core would be found to be represented by E_i; their number would correspond to E_a, but the effect of their non-uniform distribution or concentration under the distorting effect of the cross ampere-turns is given in E_i. The diagram shows that the maximum of the displaced field no longer coincides with the centre of the pole as given by the vector E_a, except in the special case of short-circuit when the lag is a maximum and the distortion is a minimum. Upon the lag of the current behind the actual distribution of the field depends the magnetic

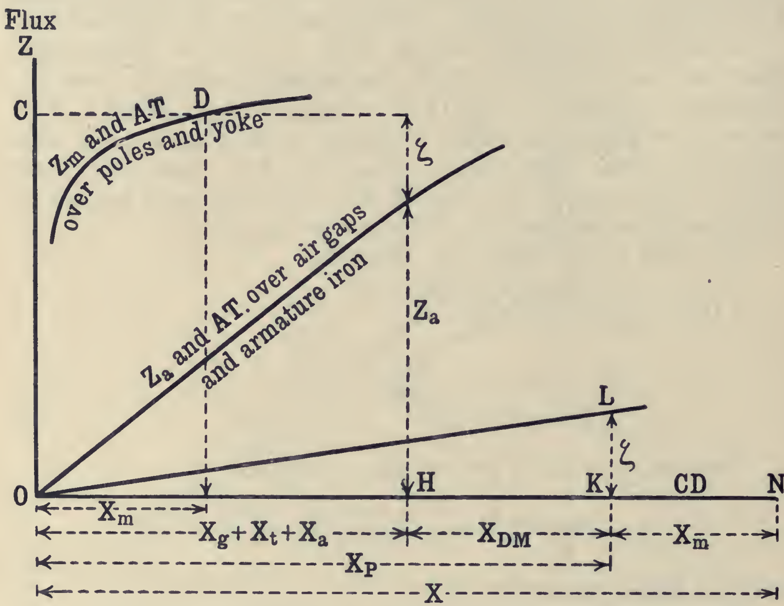


FIG. 471.—Partial magnetic characteristic curves of alternator.

pull upon the armature wires, and any deductions as to its amount would have to be based upon E_i rather than upon E_a.

§ 14. **The field excitation under-load.**—The diagram of E.M.F.'s is thus complete, and we pass to the question of the ampere-turns of excitation. In order to embrace the case of well saturated field-magnets, it is essential to employ two partial characteristics, the one connecting Z_a with the excitation required over the armature core, teeth, and air-gaps, *i.e.* over the whole of the magnetic circuit with the exception of the magnet poles and yoke, which are affected by the primary leakage, and the other a curve connecting the total flux in the magnet system Z_m with the excitation required over the poles and yoke only (Fig. 471). The useful flux Z_a to be supplied by the field-magnet within the air-gap is proportional to E_a, and is thence determined; the net ampere-turns over the interferric gaps and armature, which must be supplied by the field-magnet at the point of emergence of the lines into the air, are then found from the partial characteristic as $OH = X_g + X_a + X_t$. But the ampere-turns between the poles must also counterbalance the direct reaction. Let $HK = X_{DM}$, as above calculated in § 8. Therefore $OK = X_g + X_a + X_t + X_{DM} = X_P$, the interpolar excitation.

The primary leakage flux is $\zeta = 1{\cdot}257 X_P \times \vartheta$, where ϑ is the leakage permeance of the air-paths reduced to a quantity that may be regarded as in parallel with the armature path. The value of ζ may be conveniently shown in relation to the value of X_P measured along the axis of abscissæ as the straight line OL. For any particular value of Z_a a number of different values of $Z_m = Z_a + \zeta$, and of the density in the poles and yoke may thus be obtained according to the power-factor and consequent value of the angle ϕ_o. Exactly as in the continuous-current dynamo the total ampere-turns of excitation for a given Z_a and armature current are really dependent upon the position of the brushes and the consequent value of X_b as affecting the leakage, so in the alternator the values of Z_a and of the armature current do not determine the necessary excitation. The phase of the current relatively to the centre of the poles must also be known, and it is on this account that the total characteristic of excitation cannot be drawn once and for all. The amount $CD = X_m$ must therefore be added to X_P, and the total excitation required under the particular circumstances is $ON = X_P + X_m = X$. The only difference from the continuous-current dynamo is that the brushes in the latter usually have a fixed position for each value of the current, namely, that of sparklessness, so that, in addition to the total excitation curve for no-load, similar curves, say, for half- or full-load, can also be calculated, which under ordinary conditions will remain true, and are therefore of real service.

§ 15. **The external characteristic under load with different power-factors.**—To calculate a full-load characteristic or the complete curve connecting terminal E.M.F. as ordinate with the ampere-turns of total excitation or field-current as abscissa for any particular armature current and power-factor of the external circuit or value of ϕ_e is now a simple matter. $AB = I r_a'$, as also $BC = e_{sa}$, and $CD = \dfrac{E_{CM}}{\cos \phi_o}$ are then constant. From A (Fig. 472) a line is drawn inclined at an angle of $180° - \phi_e$ to AB, and along this line the terminal E.M.F. per phase is measured. Taking any value for this, say, AO, join OD, whence ϕ_o is found, and upon OD let fall from C the perpendicular CF. Then $OF = E_a$, to which the flux through the armature or Z_a is proportional. Another value for $E_e = AO'$ is then taken, and the corresponding E_a'' and Z_a'' determined, and so on. The calculation is as easily made analytically as graphically. From equation (177)

$$\tan\phi_o = \frac{E_e \cdot \sin\phi_e + \bar{e}_{sa} + \dfrac{E_{CM}}{\cos\phi_o}}{E_e \cdot \cos\phi_e + Ir_a'} = \frac{DG}{OG}$$

whence ϕ_o is found. Further, $OD = \sqrt{(DG)^2 + (OG)^2}$, whence $E_a = OD - \sin\phi_o \cdot \dfrac{E_{CM}}{\cos\phi_o}$.

From the lower curve of Fig. 471 the ampere-turns over the air-gaps and armature iron for each value of Z_a are found, and X_{DM} is calculated from the known value of I sin ϕ_o. The two added together give X_P, whence the primary leakage is calculated and added to Z_a. The

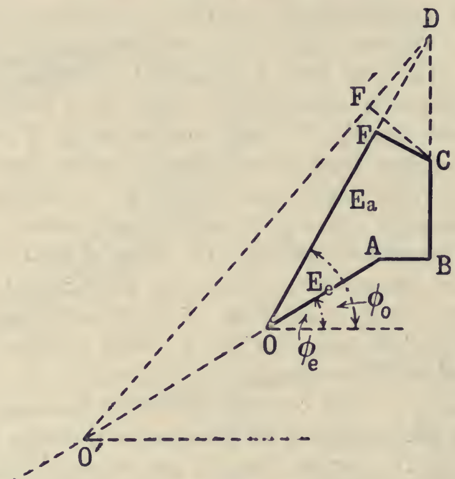


FIG. 472.—Construction to find armature E.M.F. for given current with varying external E.M.F.

corresponding value of X_m is then read off from the upper curve of Fig. 471, and when added to X_P gives the total excitation, X. A number of points having been obtained for, say, I = full-load current and $\phi_e = 0$, a curve is drawn which will fall below the no-load characteristic; below this again will fall a similarly drawn curve for $\phi_e = 90°$, and between these two limiting cases will lie the terminal voltage for all values of ϕ_e with the given current I.

An illustrative example of the process is worked out in Chapter XXV. § 6, and the curves so obtained for the three cases of unity power-factor, cos $\phi_e = 0{\cdot}85$, and zero power-factor are plotted in Fig. 473. It will be seen that for unity power-factor, as the terminal E.M.F. rises from zero the decreasing value of ϕ_o and so of X_{DM} causes the terminal E.M.F. with full armature current to gradually approach the no-load curve of E.M.F., but that as saturation of the field-magnet begins the increasing excitation required by the poles and yoke causes the two curves again to draw farther apart. With the intermediate power-factor of 0·85 the latter of these two effects is more marked, and still more with zero power-factor when the two curves throughout diverge farther from one another.

So far certain values of the terminal E.M.F. have been taken, and the necessary excitation calculated under the given conditions of current and power-factor. When it is required to solve the converse problem, namely, with a constant excitation to find the terminal voltage or the fall of volts as a function of the load, it is best to proceed

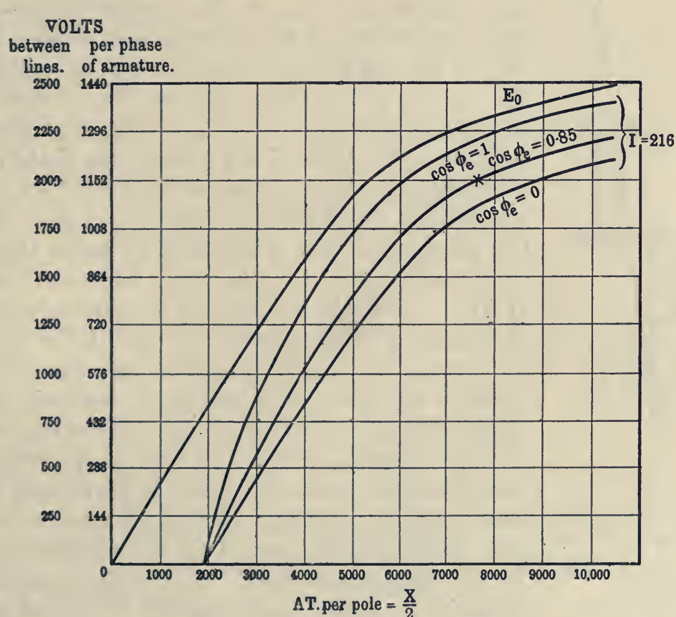


FIG. 473.—External characteristic curves of E.M.F. of alternator.

systematically by first determining the working parts of the curves exactly as above, from which the further results which are required may be deduced or interpolated.

§ 16. **The wattless current characteristic with zero power-factor.**—If $\cos\phi_e = 0$, *i.e.* if the current lags relatively to the terminal E.M.F. by 90°, and so is wattless, Fig. 474 shows that the angle ϕ_o is always large, and as the terminal E.M.F. is raised, still further increases until it nearly approaches 90°. Sin ϕ_o may therefore be at once identified with unity, and $X_{DM} = k_d \cdot \sqrt{2}tI$, or the maximum magnetising ampere-turns of the armature. $FC = E_{CM}$ practically vanishes, and E_a is nearly $= E_e + e_{sa}$. The volts required to pass the current over the resistance of the armature have practically no effect, since they are entirely out of phase with the terminal E.M.F. and the self-induced E.M.F. due to the secondary leakage.

The quantity $k\sqrt{2}t\mathrm{I}$ may be termed the *magnetising ampere-turns of the armature* as resulting from the composition of the several phases, and as affecting one magnetic circuit (without reference to the actual angle of lag or on the assumption that it is a maximum); it may be symbolised as X_a, just as X with various suffixes has been used in relation to one magnetic circuit.

Neglecting, then, the difference produced in the primary leakage, and therefore in the reluctance of the field-magnet by the addition of X_{DM} or X_a, if OB ampere-turns (Fig. 475) produce a flux corresponding to an E.M.F. BP on the open-circuit curve, a resultant excitation $\mathrm{X}_{\mathrm{R}}=\mathrm{X}-\mathrm{X}_a$ will produce the same flux, so that if we add to OB a length $\mathrm{BA}=\mathrm{X}_a$, a total excitation on short-circuit of $\mathrm{OA}=\mathrm{X}$ will produce the E.M.F. BP. From this must be deducted an amount $\mathrm{PQ}=e_{sa}=x\mathrm{I}$, so that the actual terminal voltage $\mathrm{E}_e=\mathrm{BQ}$. Since this value occurs with a total field excitation OA, BQ must be transferred horizontally across as shown at AP′. Thus P′ is one point on the curve of terminal E.M.F. for a wattless current of value I, and is seen to be virtually obtained by shifting the point P on the curve of the open-circuit voltage E_o vertically downwards by an amount $x\mathrm{I}$, and horizontally along towards the right through a distance $y\mathrm{I}=\mathrm{X}_a$. Any other point may be similarly treated as shown by the triangle KLS, where the special value of the field ampere-turns is that which gives the value I for the armature current on the short-circuit curve. The whole curve of the open-circuit E.M.F. may therefore be shifted aslant through a distance PP′, as shown by the inclined dotted lines, and will then give a characteristic of terminal E.M.F. for a given wattless current. When running as a synchronous motor, and taking a wattless current, the curve is shifted along the same inclined lines, but in the opposite direction, so that it lies above the curve of E_o. The expression for the terminal E.M.F. with a wattless current is, under the above assumptions, $\mathrm{E}_e=f(\mathrm{X}-y\mathrm{I})-x\mathrm{I}$. When the abscissæ are in amperes of exciting current A_x flowing through T_f turns wound on each pole, in the expression for the horizontal distance $\mathrm{QP}'=y\mathrm{I}$, y becomes the constant by which the magnetising ampere-turns of the armature can be converted into the equivalent current through the $2\mathrm{T}_f$ turns on each magnetic circuit; its value is then $y=\dfrac{k_d\cdot\sqrt{2}t}{2\mathrm{T}_f}$.

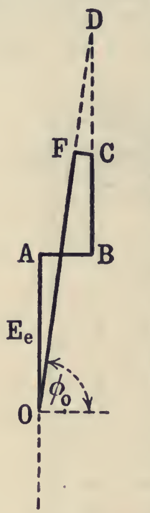


FIG. 474.—Vector diagram for wattless current, $\cos\phi_e=o$.

The above analysis of the terminal E.M.F. curve for a constant wattless current, given by M. Potier * in the first instance, and based upon the previous work of M. Blondel, will be found to yield valuable consequences in the practical work of designing, about which more will be said in § 23. The amount of error involved in the assumption that the reluctance of the field-magnet is a constant, and that the *alteration* in the primary leakage may be neglected, depends entirely upon the degree of saturation at which the magnet system is normally worked, and its extent is indicated in a particular case in Fig. 475. The more correct curve which takes into account the increased primary leakage due to

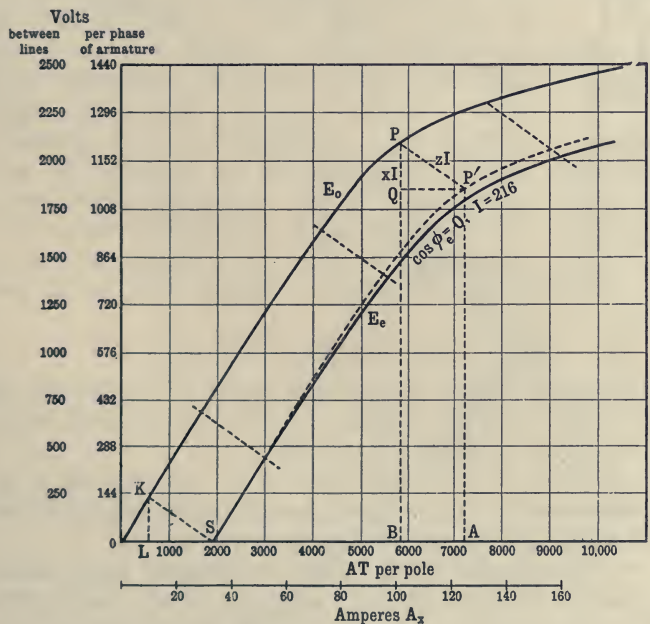


FIG. 475.—Analysis of terminal PD curve for wattless current.

the addition of X_a is repeated in full line from Fig. 473, and it is seen that the approximate curve (shown dotted) while agreeing for low values of the excitation, draws slightly away at high values.

§ 17. **The short-circuit characteristic.**—On short-circuit when $E_e = 0$, $\tan\phi_o = \dfrac{e_{sa} + \dfrac{E_{CM}}{\cos\phi_o}}{I_o r_a'}$ or $\dfrac{2\pi f(L_a + L_c)}{r_a'}$ is a constant, and $OD = I_o\sqrt{4\pi^2 f^2 (L_a + L_c)^2 + (r_a')^2} = I_o z'$, where z' is a constant. It follows that the triangle which is the form that the diagram takes is always similar to that which has been drawn in Fig. 476 for one particular value of

* *L'Éclairage Électrique*, vol. xxiv. p. 133 ff.

the current. In other words, E_a (which becomes very nearly equal to e_{sa}) is proportional to I_o, since $E_a = OD - \sin \phi_o \cdot \frac{E_{CM}}{\cos \phi_o} = I_o(z' - k)$. Further, for the low values of the flux which are now in question, the excitation over the air-gaps and armature is proportional to the flux Z_a, and so to the current. X_{DM} is itself proportional to I, so that the interpolar excitation $X_P = X_a + X_t + X_g + X_{DM}$, the leakage ζ, and $Z_m = Z_a + \zeta$ are all proportional to the current. Finally, if the field-magnet is far from saturation, X_m is itself proportional to Z_m, and so to I_o. It thence results that the short-circuit current in relation to the excitation will be a straight line, and such is usually the case, since on short-circuit there is only a comparatively small flux through the iron magnet. As a general rule, sufficient accuracy will be obtained by at once making use of the no-load flux curve, expressed as a multiple of I, and adding thereto the demagnetising armature ampere-turns, so that if X_R is the no-load total excitation for a certain value of Z_a and E_a, *i.e.* $= k'I$

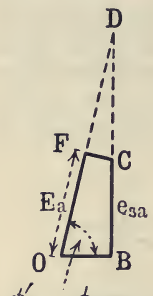


FIG. 476.—Short-circuit diagram of alternator.

$$X = X_R + X_{DM} = (k' + k'')I_o = k'''I_o;$$

this approximate method neglects the *change* in the primary leakage that will result from the addition of X_{DM}, but involves little error when the magnet is still far from saturation.

The angle ϕ_a' or ϕ_o on short-circuit is the internal angle of lag due to the armature alone; it is in good alternators usually greater than 80°, and $\cos \phi_a'$ is therefore not far different from zero.

§ 18. **Conversion into terms of fluxes, and thence of synchronous impedance.**—If the form-factor of e_{sa} and of the cross E.M.F. be assumed to be the same as that of the main E.M.F., *i.e.* if K' and K_c are both the same as K, which is approximately true, it is evident that Fig. 470 may be converted into a diagram of fluxes if the vectors are divided by $2\,K \cdot f \cdot \frac{\tau}{m} \times 10^{-8}$; or the same diagram may be retained as shown in the right-hand half of Fig. 477, the scale being appropriately altered. Further, if the reluctance of the iron armature core to the main and cross fluxes respectively is identified, and if the saturation of the pole-pieces by the cross flux may be neglected, as is usually the case, the lines OF, FC, and CD will also represent to some scale the ampere-turns $X_g + X_a + X_t$, X_{CM}, and $\frac{X_{CM}}{\cos \phi_o} = k_c \cdot \sqrt{2}tI$. A scalar addition of X_{DM} to OF will then give X_P.

But by means of an approximate assumption this conversion of the diagram into ampere-turns may be pushed a step further. If ν be the leakage coefficient and $\mathfrak{R}_m$ be the reluctance of the field magnet under the given conditions of load, the resultant ampere-turns on the magnetic circuit are $\dfrac{Z_a(\mathfrak{R}_a + \mathfrak{R}_g + \mathfrak{R}_t + \nu\mathfrak{R}_m)}{1{\cdot}257}$, and are therefore proportional to OF.

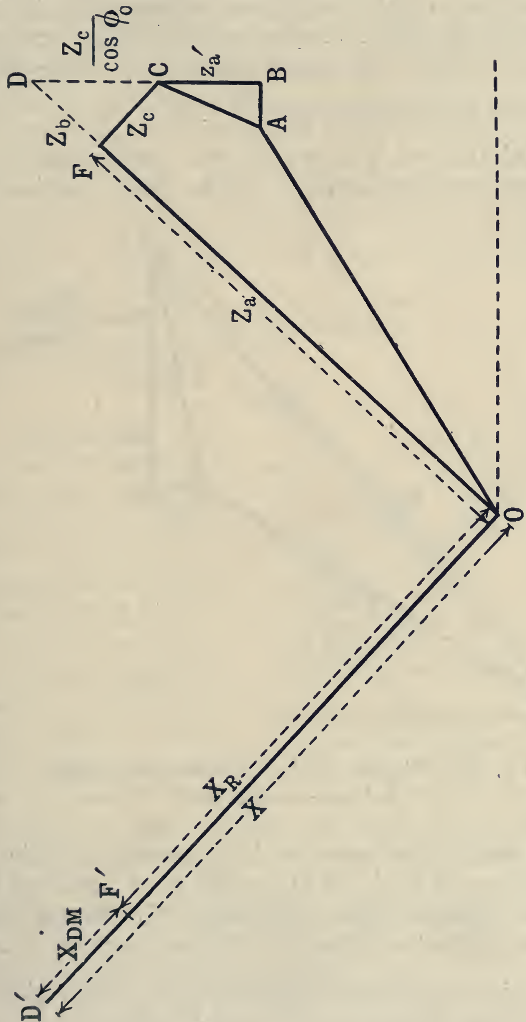


FIG. 477.—Conversion of vector diagram of E.M.F.'s into corresponding diagrams of fluxes and ampere-turns.

For the sake of clearness they may be conveniently represented (Fig. 477) as OF′ at right angles to the flux-line OF $= Z_a$. The addition of F′D′ $= X_{DM}$ to X_R will then give the total excitation X. Now, if the armature current be withdrawn the armature flux would rise to a value proportional to OD′, provided both ν and $\mathfrak{R}_m$ remained constant; and since k_d is greater than k_c when the pole-width is less than the pole-pitch, this value would be greater than the flux OD. But the increased saturation of the iron poles and increased value of $\mathfrak{R}_m$ more than outweighs the slight decrease in the real value of v, so that the product $v\mathfrak{R}_m$ increases. In consequence the armature flux is something less than in proportion to OD′, and it may approximately be identified with OD, so that the E.M.F. on open circuit E_o for the same excitation is proportional to OD. The case is then exactly represented by Fig. 478, in which the true secondary leakage E.M.F. and $\frac{E_{CM}}{\cos\phi_o}$ are added together as $OC+CD=e_{sa}+e_{sb}$ to form one quantity, which is known as the *synchronous reactance E.M.F.* When this is combined with the

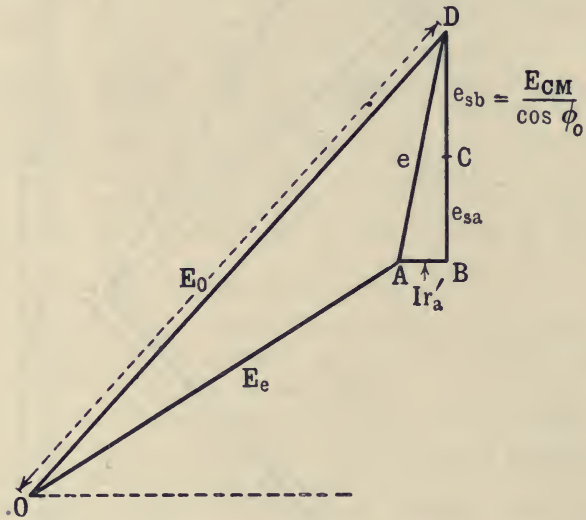


FIG. 478.—Diagram based on synchronous impedance and open-circuit E.M.F.

loss of volts over the resistance of the armature we obtain the *synchronous impedance E.M.F.*, $e=$ AD, which when combined with the terminal E.M.F. gives the open circuit E.M.F. E_o. On the assumption that for a given excitation or value of E_o, the synchronous reactance $x_a' = \frac{e_{sa}+e_{sb}}{I}$, and the synchronous impedance $\frac{e}{I}$ are constant, it is evident that the fall of volts for any given load and power factor can be easily predicted, and within wide limits the assumption is fairly true so that the method based upon it may be used in approximate calculations, and for the preliminary work of design, although for any final results it is essential to adopt the fuller method which takes into account the effect of the saturation of the field-magnet with greater accuracy.

§ 19. **Use of synchronous impedance in practical calculations.**—In Fig. 479 let OA be drawn at an angle ϕ_a' to OI, which marks the phase of the current and of length $e = \mathrm{I} \times$ impedance. From A draw AC inclined to the horizontal at the angle ϕ_e. Taking any value for E_o, describe a circle of radius E_o from O as centre, cutting the line AC at the point C; then AC represents the value of the terminal voltage for the ampere-turns or exciting current corresponding to E_o on the no-load voltage curve. The only difference from Fig. 478 is that the compounding of the E.M.F.'s is carried out in a different order. The fall of potential for the given current and power-factor corresponding to the angle ϕ_e is then $\mathrm{E}_o - \mathrm{E}_e$. It must be borne in mind that while E_e is the effective terminal voltage of one phase when the alternator is loaded, E_o is the E.M.F. induced per phase in the armature at no-load with the same excitation; it is not the E.M.F. actually induced per phase in the armature when the machine is running under the given conditions of load, and in fact never exists simultaneously with E_e.

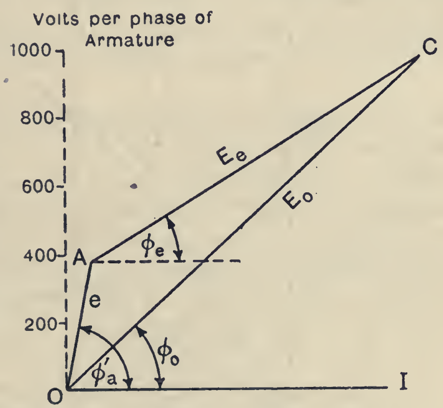


FIG. 479.—Diagram for fall of E.M.F. on synchronous-impedance method.

The experimental fact that in good alternators ϕ_a' is usually greater than 80° shows that the sum of both the ohmic loss and that by eddy-currents is small as compared with the E.M.F. consumed by the synchronous reactance of the armature, and that the synchronous impedance may be nearly identified with the synchronous reactance. When the two are identified, certain approximations may be made which are very convenient for rough calculations and in the preliminary work of designing, as will be seen later.

From the triangle of Fig. 479, $\mathrm{E}_o^2 = \mathrm{E}_e^2 + e^2 + 2\mathrm{E}_e \,.\, e\cos(\phi_a' - \phi_e)$, or $\mathrm{E}_e^2 + 2\mathrm{E}\,.\, e\cos(\phi_a' - \phi_e) = \mathrm{E}_o^2 - e^2$. Adding $e^2\cos^2(\phi_a' - \phi_e)$ to both sides and solving the quadratic equation

$$\begin{aligned}\mathrm{E}_e + e\,.\cos(\phi_a' - \phi_e) &= \sqrt{\mathrm{E}_o^2 + e^2\{\cos^2(\phi_a' - \phi_e) - 1\}}\\ &= \sqrt{\mathrm{E}_o^2 - e^2\sin^2(\phi_a' - \phi_e)}\end{aligned}$$

If we then assume $\phi_a' = 90°$, so that the synchronous reactance and impedance may be identified

$$\mathrm{E} = \sqrt{\mathrm{E}_o^2 - e^2\,.\cos^2\phi_e} - e\sin\phi_e \quad . \quad . \quad . \quad . \quad . \quad (178)$$

and for the limiting cases of a non-inductive and a purely inductive load we obtain respectively

$\phi_e = 0 \qquad \cos\phi_e = 1 \qquad E_e^2 = E_o^2 - e^2$, or $E_e = \sqrt{E_o^2 - e^2}$ . . (179)

and

$\phi_e = 90^\circ \qquad \cos\phi_e = 0 \qquad E_e = E_o - e$ . . . . (180)

It remains to indicate how the value of the synchronous-impedance voltage e may be obtained for a given value of the field-excitation. When the diagram of Fig. 479 is reduced to the case of short-circuit, it is evident that e is also E_o, and, the quotient $\frac{E_o}{I_o}$ being an alternating E.M.F. divided by current, is the apparent synchronous impedance z_a' of the armature for the given excitation. The value so obtained may

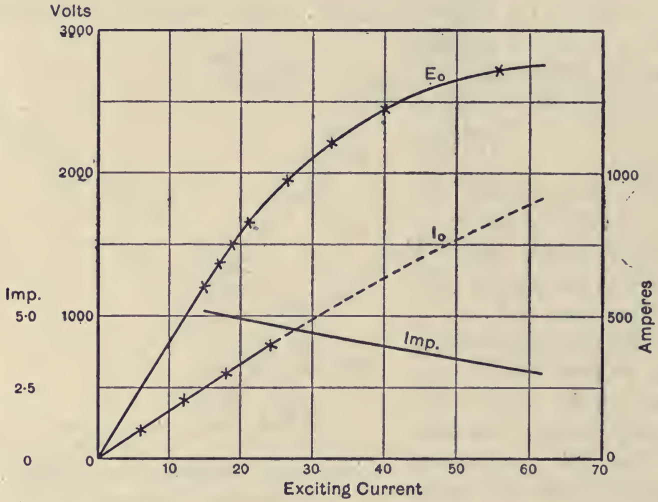


FIG. 480.—Short-circuit current and synchronous-impedance curves.

then be used to calculate the impedance voltage $e = Iz_a'$ for any other value of the current with the same excitation or no-load voltage E_o.

§ 20. **Experimental determination of short-circuit characteristic, and synchronous impedance.**—In order to determine the short-circuit characteristic of an alternator when run with its terminals short-circuited, an alternating current ammeter is placed across the terminals of the armature of a single-phase machine, or in a polyphase machine all the phases are short-circuited and a low-resistance ammeter is inserted in one phase. The values of I_o or the short-circuit current per phase at the rated speed of the alternator are then plotted as ordinates with corresponding values of the exciting current A_x or field ampere-turns as abscissæ (Fig. 480). The curve of I_o, as already mentioned, is usually nearly a straight line passing through zero if residual magnetism has been eliminated; at strong excitations it tends to bend over slightly towards the horizontal. If the teeth of the armature are highly saturated by the leakage across the slots, this secondary leakage will increase less than in proportion to the current, and with large values of I_o the tendency of the curve to bend downwards may be more than counterbalanced, and it becomes slightly concave. During the short time necessary for each observation of I_o and A_x the armature current may safely be allowed to rise to two or three times the full-load normal current. During the readings exact constancy of the speed is not of essential importance, since, as the speed varies, the reactance varies equally, and the only effect is to alter the relative amount of the eddy-currents and of Ir_a' to $e_{sa}+e_{sb}$. Since Ir_a' is small as compared with $e_{sa}+e_{sb}$, the current is almost entirely determined by the reactance, and is therefore independent of the speed for the given excitation. It is, however, important to take the short-circuit current as high as is permissible so as to reproduce as nearly as may be the full-load excitation and obtain as high a degree of saturation of the field-magnet as possible.

The curve of the terminal or induced E.M.F. on open circuit as related to the exciting current or to the field ampere-turns is taken in the same way as the similar curve of a continuous-current dynamo (cp. Chap. XV. § 12), and is in fact the no-current flux-curve expressed in terms of the E.M.F. induced at a constant speed. The quotient of corresponding values of E_o and I_o, or the synchronous impedance when plotted as in Fig. 480, is usually found to fall with increasing values of the excitation, since the curve of E_o bends over more rapidly than that of I_o.

§ 21. **Example of the synchronous impedance method.**—An actual case of a 3-phase 500 horse-power inductor generator giving 2000 volts between any pair of terminals may now be cited * as an example of the application of the synchronous impedance method under conditions favourable to its validity, the magnet system not being very highly saturated. When run at a speed of 406 revs. per min. with a frequency of 54, the open-circuit curve of E.M.F. for different values of the field-excitation was that marked E_o (Fig. 481), giving a value of 2760 volts for an exciting current of 62 amperes. When short-circuited the armature amperes for low values of the field current are given by the full line I_o (Fig. 480), which for currents above some three or four times the normal may be prolonged as shown by the dotted extension. Dividing E_o by I_o, we obtain the corresponding values of the apparent or synchronous impedance of the armature. Thus with a field current of 62 amperes the value of the short-circuit current is 920

* Taken by permission of Mr. B. A. Behrend from his paper on "The Factors which determine the Design of Monophase and Polyphase Generators," *Electr. World and Eng.*, republished in *Electr. Eng.*, vol. xxvi. p. 670.

amperes, and the apparent impedance is $\frac{2760}{920}=3$. The value of the impedance gradually rises when the field excitation is reduced as shown in the fourth column below.

A_x	E_o	I_o	Imp.	$e=$ I × Imp.	$E_o - e$	E_e observed.
62	2760	920	3	288	2472	2470
56	2710	840	3·22	309	2401	2410
54	2690	815	3·3	317	2373	2390
44	2550	680	3·75	360	2190	2230
32	2220	510	4·3	420	1800	1900
21	1650	340	4·85	465	1185	1230
15·5	1270	249	5·1	490	780	747

It is now desired to calculate the fall of potential with a highly inductive load of 96 amperes. The values of the product of the fixed current into the varying impedance is given in the fifth column, and since the load is highly inductive it may be assumed that approximately $\phi_e = 90°$. It may further be assumed that $\phi_a{}' \text{ is} = 90°$; under these

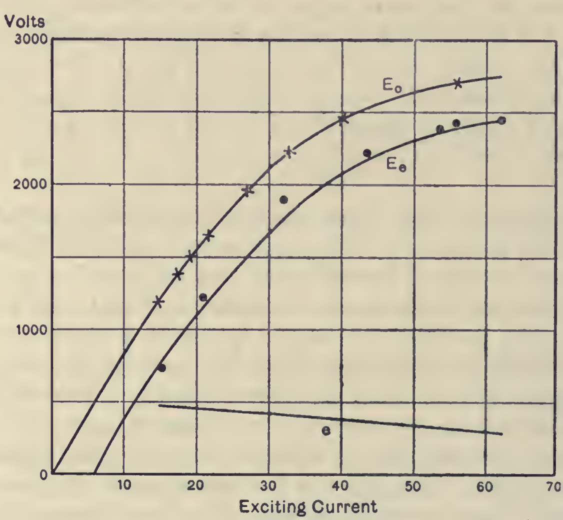


Fig. 481.

circumstances the counter E.M.F. e may be simply deducted from the value of E_o. This has been done in the sixth column, and by its side are added the observed terminal volts, from which it will be seen that the agreement is very good except in the case of the fifth observation, where there is some error probably in the reading of either speed or voltage. Neither $\phi_a{}'$ nor ϕ_e can strictly speaking be 90°, but in such a case as the present they may at least be taken as equal, and then $E_e = E_o - e$ becomes very closely true, as shown by Fig. 481.

Since for a given excitation or value of E_o, the voltage e on short-circuit $= E_o$ is measured when only a comparatively small flux is passing, and the magnet is not saturated so much as it would be under working conditions, the value which is thence deduced for the synchronous impedance, namely, $\frac{E_o}{I_o}$, is usually greater than the true quantity corresponding to it under a more normal degree of saturation. The application of the synchronous impedance method with the value of e determined from short-circuit is therefore most accurate under low power-factors, and it gradually loses its accuracy as the load becomes more nearly non-inductive. Under fairly high power-factors the terminal voltage deduced for a given excitation is thus somewhat too low, and the method of determining the drop of volts under load based upon the synchronous impedance has been in consequence called the "pessimistic" method, since the actual drop may be some 5 or 10 per cent. less.* Various corrections have been proposed to reduce this inaccuracy under different angles of lag or of power-factor in the external circuit, but they are of small value, since the method given in § 13 involves but little more calculation and is applicable to all cases, whether the magnet be saturated or unsaturated. The synchronous impedance is thus only suitable for preliminary calculations in the process of designing, as will be further explained in Chapter XXV. §§ 1, 3. The further assumption that the synchronous impedance is constant not only under different power-factors for the same excitation, but also for different values of the excitation and of E_o, leads to a still greater divergence from the truth; yet for the approximate treatment of certain problems in the running of alternators in parallel or of synchronous motors, the method remains admissible and useful within limits.

§ 22. **The diagram of short-circuit.**—From its use in the process of designing and from its importance to the theory of alternators, the diagram representing the conditions of short-circuit deserves further consideration.

On short-circuit an alternator presents many points of similarity to a transformer, the primary being represented by the field-magnet winding, and the secondary by the armature winding. The maximum value of the short-circuit current occurs when the poles formed on the armature are almost exactly opposite to the poles of the magnet-system. The demagnetising force of the armature is then nearly equal to the magnetising force of the field-winding, or, in other words, the secondary ampere-turns are nearly equal to the primary ampere-turns, and the

* Cp. Atchison, "Some Properties of Alternators under Various Conditions of Load," *Journ. Inst. Electr. Eng.*, vol. xxxiii. p. 1062.

difference between them merely corresponds to the flow of a small magnetising current and a small number of lines.

In the ideal case of a machine without ohmic resistance and in which there is no secondary leakage, X_{DM} on short-circuit would be $=X_a$, since the angle of lag of the current vector behind the centre of the pole would be 90°, giving $\sin \phi_o = 1$. These ampere-turns, embracing the main magnetic circuit, would exactly balance the interpolar excitation X_P, and there would be no resultant armature flux.

But in reality there must on short-circuit be some surplus of ampere-turns on the field to provide the flux necessary to counterbalance the secondary leakage round the end-connections of the coils and across the tops of the slots, and also to provide the loss of volts over the ohmic resistance and by eddy-currents. Of the two the former is by far the most important, since ϕ_a' although never reaching 90° usually exceeds 80°, and the total number of lines to be provided in the armature will only exceed the secondary leakage lines in a proportion slightly greater than cosec 80° : 1 or 1·015 : 1, *i.e.* by an addition of rather more than $1\frac{1}{2}$ per cent. Hence if z_a' be the secondary leakage flux and $\frac{K'}{K} . z_a'$ be the number of field lines required to counterbalance them, the actual number of field lines entering the armature is a little more than $1{\cdot}015\frac{K'}{K} . z_a'$. Neglecting the reluctance of the armature iron by comparison with that of the double air-gap, the ampere-turns required to pass these lines over the two air-gaps is $X_g = 0{\cdot}8\frac{1{\cdot}015\ K'}{K} z_a' . \frac{2ml_g}{s_g}$. X_{DM} on short-circuit is now $0{\cdot}985\ X_a$, but the slight difference from X_a may be set against the fact that the actual field lines are somewhat more than $\frac{1{\cdot}015\ K'}{K} . z_a'$, so that finally the field-excitation at the pole-faces must be very closely

$$X_P = X_a + 0{\cdot}8\,\frac{1{\cdot}015\ K'}{K} . z_a' . \frac{2ml_g}{s_g}$$

$$Z_m = \frac{1{\cdot}015\ K'}{K} . z_a' + 1{\cdot}257\ X_P \times \mathcal{S}$$

and

$$X = X_a + 0{\cdot}8\frac{1{\cdot}015\ K'}{K} . z_a' . \frac{2ml_g}{s_g} + f\left(\frac{Z_m}{a_m}\right) . l_m \quad . \quad . \quad . \quad . \quad (181)$$

Thus, instead of the ideal ratio of 1, we have $\frac{X}{X_a} = k$, where k has some value greater than unity. The short-circuit characteristic, in fact, measures the combined effect of the maximum direct magnetising ampere-turns of the armature and of the secondary leakage expressed in ampere-turns.

The expression which has been previously found for the terminal E.M.F. under a wattless load, namely, $E_e = f(A_x - yI) - xI$, must also hold on short-circuit when $E_e = 0$, and $xI_o = f(A_o - yI_o)$, where A_o is the field-current corresponding to I_o. Since the field-magnet is not strongly saturated, the volts are practically proportional to the ampere-turns of resultant excitation; in other words, the function f is given by a constant multiplier j which converts ampere-turns into volts. Hence

$$xI_o = j(A_o - yI_o) \qquad \text{or} \quad A_o = I_o\left(x \cdot \frac{1}{j} + y\right)$$
$$= I_o(x' + y)$$

from which it is seen how the field-current is balanced by the effect of the short-circuit current with its two components into which this effect is divisible, the one due to the back ampere-turns and the other to the secondary leakage. The last term in eq. (181) dealing with the loss of magnetic potential over the reluctance of the iron magnet-cores and yoke is in most cases quite negligible as compared with the first two terms, since the actual flux when the alternator is short-circuited is but small. Yet in machines with considerable primary leakage, as in inductor generators with overarching claw magnet, even the last term has some influence.

The value of k or the ratio of X to X_a on short-circuit for various types of machines and arrangements of winding has been tabulated by several observers * from experiment, and is found to differ considerably from unity. In a given machine, if the short-circuit characteristic bends over and becomes convex, k is increasing with increasing values of the excitation. For small values of the exciting current and of I_o, k may in some cases be less than unity and as low as 0·9, but this result is due to residual magnetism in the iron cores, which in effect makes the virtual ampere-turns of the field greater than the actual product of the exciting current and the field turns. The value of k is further discussed in Chapter XXV. § 2.

The relation between X and X_a as affecting one magnetic circuit may also be expressed in terms of the total number of ampere-turns on the whole field-system and armature respectively. Since X_a on short-circuit $= k_d \cdot \sqrt{2} tI_o$, and t in a heteropolar machine $= \frac{\tau}{2mp}$,

$$X = kX_a = k \cdot k_d \cdot \sqrt{2} \cdot \frac{\tau}{2mp} \cdot I_o,$$

and

$$pX = k \cdot k_d \cdot \sqrt{2} \cdot \frac{\tau}{2m} \cdot I_o$$

But pX = the total number of ampere-turns on the field-system when each pole or pair of poles is separately magnetised, and $\frac{\tau I}{2}$ = the total

* Arnold, *E.T.Z.*, 1896, p. 731; Fischer Hinnen, *Electr. Eng.*, vol. xx. p. 597.

number of effective ampere-turns on the armature. Thus on short-circuit the total AT on the field $= \frac{k \,.\, k_d \,.\, \sqrt{2}}{m} \times$ effective AT on the armature, or $\frac{AT_f}{AT_a \text{ on short-circuit}} = \frac{k \,.\, k_d \,.\, \sqrt{2}}{m}$. If there is but a single exciting coil on the field, its ampere-turns must be multiplied by p in order to obtain $pX = AT_f$ with which to compare the armature ampere-turns, and the same expressions also hold for the homopolar inductor generator with a double armature, when τ is reckoned for one armature only.

In the expression $X = k \,.\, X_a = k \,.\, k_d \,.\, \sqrt{2} t I_o$, the factors $k \,.\, k_d \,.\, \sqrt{2}$ may be grouped into one, so that $X = \lambda t I_o$. It is then sufficient for the designer to refer to tabulated values of λ which previous experiments have shown to hold for machines of similar type and of similar proportions. Thence

$$\frac{\lambda}{m} = \frac{pX}{\frac{\tau}{2} \,.\, I_o} = \frac{AT_f}{AT_a \text{ on short-circuit}} \quad . \quad . \quad . \quad . \quad . \quad (182)$$

or the ratio of the ampere-turns of the field to the effective ampere-turns of the armature on short-circuit. The value of λ forms one of the chief data in the preliminary work of design, since from it the synchronous impedance can be estimated. Thus, if its value be assumed from previous machines, or when the number of slots per pole and per phase and the pole-width ratio, etc. are known, so that k_d can be calculated, if the value of k be approximated, the short-circuit current with the full excitation is $I_o = \frac{X}{\lambda t}$, and the synchronous impedance is $\frac{E_o}{I_o}$. The smaller the value of λ, the better the regulation of the machine.

§ 23. **Separation of direct armature reaction from secondary leakage reactance by wattless-current method.**—While the value of λ takes into account both the demagnetising ampere-turns of the armature and the effect of secondary leakage, and is appropriate to the preliminary calculation of an approximate design, a more detailed analysis by which the two effects may be separated is indispensable if the terminal E.M.F. under different loads is to be accurately predicted, especially if the magnet system is strongly saturated. If the experimentally observed short-circuit characteristic is to hand, the calculated value of X_a may with more certainty be used to determine e_{sa} than *vice versa*. Yet in either case there are elements of difficulty in predetermining the exact numerical value of the two quantities concerned. The value of the magnetising ampere-turns of the armature X_a or X_{DM} on short-circuit, even when calculated with theoretical accuracy as in § 8, is itself based on a number of assumptions that may not be strictly true in practice; thus it assumes that the effect of all higher harmonics is suppressed, and the actual influence of any eddies due to the armature current it is difficult to calculate.

Some further datum based on the sure ground of experiment is therefore very desirable to replace or at least add to the knowledge that can be derived from the mere short-circuit characteristic. Such an additional datum is found in the approximate separation of the two effects of the armature back ampere-turns and the secondary leakage, which is based upon the analysis of a wattless-current curve as described in § 16. Although not completely true when the magnet is very highly saturated, yet such a condition rarely occurs with very low power-factors, so that the results obtained are found to be in close agreement with observed facts. A curve of terminal E.M.F. for a constant wattless current, *i.e.* a current of considerable magnitude but with a power-factor cos ϕ_e, say less than 0·20, is in the first place required. Such a curve may be obtained by working the alternator on a highly inductive load such as induction motors running light, or in connection with a synchronous motor under-excited so as to shift the phase of the current by a large angle of lag relatively to the E.M.F.; or lastly, the alternator may itself be operated as a synchronous motor. Even if a lower power-factor than 0·5 cannot be obtained, the drop of volts is but little less than with a truly wattless current. The field-exciting current A_x is progressively reduced while the armature current I is maintained as nearly as may be constant, until, if the machine is short-circuited, the final value of A_x is one reading of the field current in a short-circuit curve. The latter may, in fact, then be entirely dispensed with, the necessary data being all obtainable from a complete curve of the terminal voltage for different field-currents with a wattless current in the armature of constant value. Or, if only a few readings for the upper portion of the wattless-current curve are obtained, one supplementary reading giving the point where the curve cuts the axis of X may always be added from the short-circuit characteristic if this has been independently taken.

If now the open-circuit curve is drawn on tracing paper and is shifted parallel to itself in a slanting direction, a direction may be found such that it fits exactly over the lower curve of terminal E.M.F. A line joining the old and new positions of any one point, as PP′, will then give the hypotenuse of the right-angled triangle or zI, whence the two sides xI and yI are at once determined. From the former is found the leakage reactance, and from the latter the magnetising ampere-turns of the armature.

If a single reading of the correct terminal voltage E_e on a wattless-current curve is taken experimentally, and the short-circuit characteristic is known, the direct reaction may still be separated from the effect of the secondary leakage by the following construction due to M. Blondel.*

* *E.T.Z.*, vol. xxii. p. 474. For another method cf. Dr. G. Kapp, *Journ. I.E.E.* vol. xlii. p. 703.

For the same field excitation OA which corresponds to the wattless current I and terminal voltage E_e let the short-circuit current be $I_o = \alpha I$. Suppose then for the moment that the division of the effect of the short-circuit current into its kinds and their values are known; thus in Fig. 482, let $SQ = 2\pi f L_a \cdot I_o = xI_o$, and let $SA = \frac{k_d \cdot \sqrt{2}tI_o}{2T_f} = yI_o$. Join AQ, and in AQ take a point a such that $\frac{Aa}{AQ} = \frac{I}{I_o} = \alpha$. Then by similar triangles the leakage reactance E.M.F. for the current I must be aB, and the back ampere-turns or their equivalent field-current

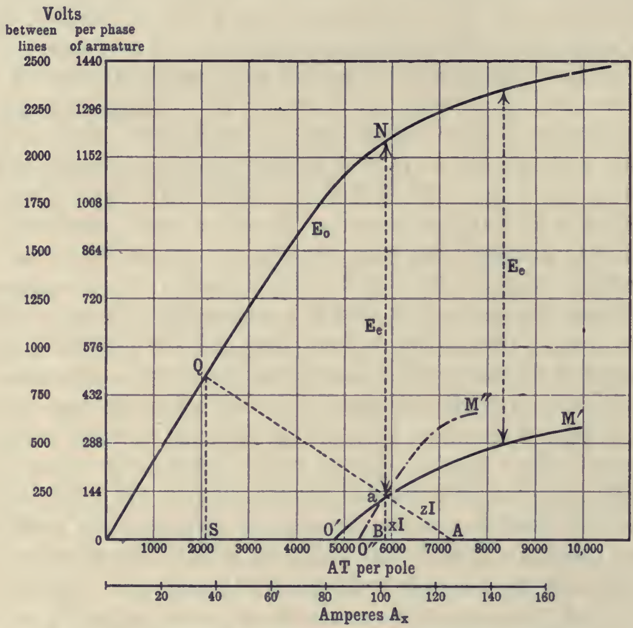


FIG. 482.—Separation of direct reaction and leakage reactance by wattless-current curve.

must be AB. In consequence the length $Na = NB - aB$ must be equal to the terminal E.M.F. E_e. The position of the point a is then completely defined by two conditions; its vertical distance from the open-circuit characteristic is equal to E_e, and its radial distance from the point A is $= AQ \times \frac{I}{I_o}$. Now, without any prior knowledge of the direction of the line AQ, the position of point a can be found as the intersection of two new curves, the one a curve parallel to OQNM at a vertical distance E_e below it, and the other a curve similar to OQNM about the point A, but with every radius reduced in the proportion $\frac{I}{I_o}$. Hence from the open-circuit characteristic deduct the fixed amount E_e, the derived curve being that shown by $O'aM'$ in Fig. 482. Drawing radii from A to the open-circuit curve E_o, reduce each in the proportion $\frac{I}{I_o}=a$, the reduced values when plotted giving the curve $O''aM''$, which is similar to OQM about the centre A. Then the leakage reactance voltage xI of Fig. 475 must be given by some ordinate to the curve $O'aM'$, while the hypotenuse to the right-angled triangle of Fig. 475 or zI must be given by some radius to the curve $O''aM''$. The intersection, therefore, of the two curves at a reproduces the right-angled triangle of Fig. 475, and

$$v=\frac{AB}{I} \text{ or } \frac{AS}{I_o}, \text{ while } x=\frac{aB}{I} \text{ or } \frac{QS}{I_o}$$

In making the test the field-excitation must be such that E_o is well above the bend, and obviously if while the terminal voltage is maintained constant at its normal amount a number of different values of I are obtained, by the repetition of the construction for the different values of I and of AO the values of x and of y can be determined with greater certainty.

§ 24. **Calculation of leakage reactance.**—In the process of designing a new alternator the value of e_{sa} must be calculated, with all the accuracy which the designer can command. The shape of the slots and the degree of saturation in the teeth have considerable effect in modifying the secondary leakage, but the general principles on which the leakage flux must be estimated remain the same as have already been employed in Chapter XVIII. §§ 21–23 in connection with the inductance of the short-circuited section of a continuous-current armature.

If $t=$ the number of active wires in a group per pole and per phase, their self-inductance may be expressed just as Chapter XVIII. § 21 through a term $\mathcal{S}$, or the equivalent permeance surrounding them, on the assumption that every line of flux is linked with all the turns; *i.e.* as

$$4\pi t^2 . \mathcal{S} \times 10^{-9} = t^2 . \Lambda \times 10^{-9} \text{ henrys.}$$

The permeance may be divided into three portions corresponding to (a) the two sides of a coil resting on or embedded in the iron of the armature core of length l cm., (b) to the straight projections of the coil-sides beyond the armature core of length (l_1-l) cm., and (c) to the ends of the coil each of length approximately equal to the pole-pitch $\frac{\pi D}{2p}$. In a complete coil there are two sheafs of wire or t turns with their end-connections, and the self-inductance of a large coil corresponding to a pair of poles becomes

$$2t^2\left\{l\lambda_a+(l_1-l)\lambda_b+\frac{\pi D}{2p} . \lambda'\right\} \times 10^{-9}$$

where λ_a, λ_b, and λ' are the three quantities corresponding to the three divisions of the permeance for one side or end of the coil. The self-inductance of a complete phase of p such coils is therefore

$$L_a = 2pt^2\left\{l\lambda_a + (l_1 - l)\lambda_b + \frac{\pi D}{2p} \cdot \lambda'\right\} \times 10^{-9} \text{ henrys} \quad . \quad . \quad (183)$$

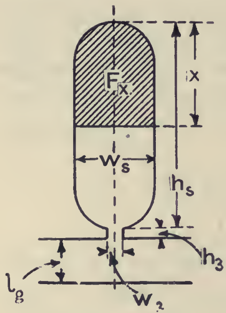


Fig. 483.—Slot inductance.

Taking a half-closed slot of shape such as that of Fig. 483, the slot-inductance may be divided into the portions corresponding to the flux within the winding space, and to the flux across the bridge or overhanging edges of the opening. Within the winding space of total cross-sectional area F the permeance of an elementary strip of width dx is $\frac{dx}{w_s} \cdot l$. If $\gamma_1 =$ the number of slots per pole and per phase, the total wires in the slot are $\frac{t}{\gamma_1} = \frac{\tau}{2mp\gamma_1}$.

If F_x be the area of the slot up to the height x, the M.M.F. acting across the elementary strip with one C.G.S. unit of current flowing in each wire is $4\pi \frac{t}{\gamma_1} \cdot \frac{F_x}{F}$. The lines in the strip are therefore $4\pi \frac{t}{\gamma_1} \cdot \frac{F_x}{F} \cdot \frac{dx}{w_s} \cdot l$ linked with $\frac{t}{\gamma_1} \cdot \frac{F_x}{F}$ wires. Hence $dL_1 = 4\pi \left(\frac{t}{\gamma_1}\right)^2 \cdot \left(\frac{F_x}{F}\right)^2 \frac{dx}{w_s} \cdot l$, and the inductance of one slot is

$$= 4\pi \left(\frac{t}{\gamma_1}\right)^2 \int_0^{h_s} \left(\frac{F_x}{F}\right)^2 \frac{dx}{w_s} \cdot l$$

Since there are γ_1 such slots for each sheaf of wires, their joint inductance

$$t^2 \cdot \frac{4\pi}{\gamma_1} \cdot \frac{l}{w_s} \int_0^{h_s} \left(\frac{F_x}{F}\right)^2 dx$$

and with straight sides to the slot and square corners, since

$$\int_0^{h_s} \left(\frac{F_x}{F}\right)^2 dx = \frac{h_s}{3}; \text{ this becomes} = t^2 \cdot \frac{4\pi}{\gamma_1} \cdot \frac{h_s}{3w_s} \cdot l \times 10^{-9}$$

The flux across the opening is $4\pi \cdot \frac{t}{\gamma_1} \cdot \frac{t_3}{w_3} \cdot l$, and is linked with all the $\frac{t}{\gamma_1}$ wires, so that for γ_1 slots another item of the slot inductance is

$$= t^2 \cdot \frac{4\pi}{\gamma_1} \cdot \frac{t_3}{w_3} \cdot l \times 10^{-9}$$

Thus for substitution in equation (183) with straight-sided slots

$$\lambda_a = \frac{4\pi}{\gamma_1}\left(\frac{h_s}{3w_s} + \frac{t_3}{w_3}\right) + \lambda_k$$

where the surface-of-the-core term λ_k must be given its correct value as dependent upon the value of γ_1.

A certain proportion of the lines which emerge into the air from the tops of the armature teeth cross over into other teeth immediately without entering the iron of the pole-pieces, and these might be calculated in the same way as the similar term for the short-circuited section of a continuous-current armature. It is, however, simpler to assume the field-magnet wheel removed so as to include none of the true cross or back lines, and to calculate λ_k for the joint flux linked with all wires integrated up to the centre-line of the coil instead of merely up to the pole-tips, the coils being embedded in the iron but half of the magnetic circuit being entirely in air. This surface-of-the-core inductance is obtained with sufficient accuracy from Maxwell's formula for two parallel cylindrical conductors. If two such conductors of length l and radius r convey equal currents in opposite directions, and are situated at a distance of b cm. apart in air, their inductance is $2l\left(4{\cdot}6 \log_{10}\frac{b}{r}+0{\cdot}5\right)$. If the conductors are embedded in iron the air-path of the lines is halved, and the permeance is thereby doubled so that their inductance is $2l\left(9{\cdot}2 \log\frac{b}{r}+1\right)$. Hence if r_q = the radius of a circle of circumference equal to the perimeter of the sheaf of t wires as they lie in their several slots with the intervening insulation and iron teeth, and reckoning the distance between the two sheafs as approximately equal to the pole-pitch $B=\frac{\pi D}{2p}$,

$$\lambda_k = 9{\cdot}2 \log \frac{\pi D}{2p \cdot r_q} + 1$$

Although such calculations yield fairly good results, the validity of the addition of the term λ_k in full is open to considerable question, which is strengthened by the fact that the values experimentally obtained for L_a when the field-magnet wheel with its poles is removed appear to be in excess of the true value that is required for the determination of the fall of potential.*

The term dealing with the wires as they project out of the slots may be calculated in the same way as that for the surface of the core, but with air only between the opposite sides, as

$$\lambda_b = \frac{\lambda_k}{2} = 4{\cdot}6 \log \frac{\pi D}{2p \cdot r_q} + 0{\cdot}5$$

For the inductance of the end-connections the permeance will be integrated right across the core from one end of the coil to the other, separated by a distance of l_1 centimetre as if the wires were entirely in air. If r^{τ} = the radius of a circle of circumference equal to the

* Cp. Hobart and Punga, *Elektrische Kraftbetriebe u. Bahnen*, vol. v. p. 613, and M. Schenkel, abstracted in *Electrician*, July 23rd, 1909.

perimeter of a section through the ends of a coil, including the spaces between the wires occupied by insulation, then if there is one large coil for each phase per pair of poles, so that all the end-connections on one side follow the same path,

$$\lambda' = 4{\cdot}6 \log \frac{l_1}{r_\tau} + 0{\cdot}5$$

But if there are $2p$ coils per phase, so that only $\frac{t}{2}$ wires follow the same path at the ends, the inductance is $\propto \left(\frac{t}{2}\right)^2 \times 2$, so that

$$\lambda' = 2{\cdot}3 \log \frac{l_1}{r_\tau} + 0{\cdot}25$$

where r_τ is the radius corresponding to the perimeter of the divided ends. The inductance of the V-shaped end-connectors of bar winding is calculated exactly as for continuous-current armatures (Chap. XVIII. § 24 (*c*)).

In a polyphase generator the mutual inductance of the other phases increases the apparent inductance of each separate phase, yet but slightly except in certain special windings where different phases occur in the same slot. An approximate allowance for this in a 3-phase generator will be made by increasing the value of L_a as calculated above by say 15 per cent.*

On the assumption that the inductance is constant and that the current change is sinusoidal, the reactance of the phase is $x = 2\pi f L_a$ ohms, and if I be the effective value of the current in amperes the effective value of the voltage consumed by the reactance is

$$e_{sa} = x\mathrm{I} = 2\pi f L_a \,.\, \mathrm{I} \text{ volts.}$$

Experimental measurements have been made of the real inductance of armature coils under different conditions and in different positions relatively to the poles, but in any deductions therefrom care must be taken to consider how far the measured inductance includes not only that due to the secondary leakage, but also that which corresponds to the cross-magnetising and demagnetising effects which are to be estimated by other methods.†

From data given by Mr. Hobart,‡ the flux of the free end-connections of the coils is of the order of 8 lines per centimetre length and per C.G.S. current-turn for one slot per pole per phase, sinking to 7, 6, and 5 respectively for 2, 3, and 4 slots per pole per phase. The flux per centimetre length of an embedded coil and per C.G.S. current-turn varies so greatly with the proportions of the slot and the grouping, that experimental values are seldom applicable.

* Cp. Parshall and Hobart, *Engineering*, vol. lxx. p. 819.

† Cp. Chap. XXV. § 7, and Dr. Kapp, *Journ. I.E.E.* vol. xlii. p. 703.

‡ *Journ. Inst. Electr. Eng.*, vol. xxxi. p. 192 ff.

§ 25. **Values of inherent regulation.**—For the purpose of estimating the degree to which different alternators tend to maintain a constant terminal voltage under varying loads, their *inherent regulation* may be compared. This latter quality may be defined in several slightly different ways; *e.g.*, as the *fall* in volts which occurs if the machine is excited to give its normal terminal voltage at no-load and full-load is then switched on while the excitation and speed remain constant. The fall, being expressed as a percentage of the no-load terminal volts, then gives some indication as to the amount by which the excitation must be altered by the attendant in order to maintain the terminal volts constant. Or it may be defined as the *rise* in volts which occurs when the full-load is removed, the speed and excitation being maintained constant at the value corresponding to the full rated output; the rise expressed as a percentage of the terminal volts at full-load then rather indicates how far the alternator under varying loads may be left without attention. Owing to the greater degree of saturation of the machine in the latter case, the second definition gives slightly smaller values than the first. Thus the inherent regulation varies according to the nature of the full external load, and the power-factor of this must be stated; or in general, the inherent regulation may be given for the two limiting values, namely, $\cos \phi_e = 1$ with a non-inductive load, and $\cos \phi_e = 0$ with a completely inductive load. In a machine with a large amount of armature reaction and correspondingly poor inherent regulation the short-circuit current may only exceed the full-load current by some 50 per cent. even with the normal excitation for full-load. It is then impossible for the machine to be very seriously overloaded or for its armature to be immediately burnt out through an accidental short-circuit. This advantage is, however, bought at the expense of the regulation, and so important is this latter quality that in modern designs it is usual to specify as low a rise of voltage as can be obtained with reasonable cost of copper and manufacture. For a few minutes a current two or three times the normal will do no serious harm, and beyond this reliance must be placed on fuses or automatic circuit-breakers to protect the machine in the case of a prolonged short-circuit. Hence, on the whole, the greater the ratio which the short-circuit current of the alternator bears to the full-load current the better is the machine, and in modern machines it is usually from 3 to 4 times the full-load current.

Apart from arrangement of the winding so as to secure a low value of e_{sa} and of the cross magnetising coefficient k_c, the only method of securing a good inherent regulation is to make the ratio of X to X_a as high as possible. The pole-pitch being usually more or less fixed by the considerations of frequency and speed, economy of manufacture dictates that it should be reasonably loaded with ampere-wires over its whole length, so that it is desired to keep X_a high. The remaining factors which can be varied are the length of air-gap and the degree of saturation of the magnet cores. The alternatives left open to the designer are therefore (1) to make X_m large by saturating the poles, or (2) to make X_g large by employing a long air-gap. The former course has the advantages of reducing the weight and cost of the iron magnet, and of reducing the length of the inner turn of the exciting coil. But it has the disadvantage of involving a great variation in the amount of the exciting current for different loads, and it cannot be pushed to an extreme without danger of failure to reach the required voltage through the materials being less permeable than calculated. It will also be seen later in Chapters XXIV. § 9 and XXV. that a small percentage rise or fall of volts, if obtained by the employment of a very high degree of saturation in the poles, may involve some loss of flexibility in meeting emergency loads, unless the alternator is designed at the start for a low power-factor; it may be then unduly large for its normal load and power-factor, or uneconomical in exciting energy.

The latter course has the disadvantage of increasing the percentage of the primary leakage, which to some extent reduces the benefit that would otherwise be due to it.

With low frequencies or very high revolutions per minute, either of which have a general tendency to increase the pole-pitch, it becomes increasingly difficult to counterbalance the effect of the large number of ampere-wires per pole, at least when the power-factor is low. This difficulty is especially felt in the case of turbo-alternators *; their secondary leakage may be low, and their regulation good on non-inductive loads, but on partially inductive loads recourse must be had to the second of the two alternatives to assist in securing a good regulation. An extreme case is found in the 5500 kilowatt Westinghouse turbo-alternators installed at Lot's Road, Chelsea, which run at 1000 revolutions per minute with a frequency of $33\frac{1}{3}$, and in which the single air-gap is as much as $3\frac{1}{4}$ inches.† Thus, in either case, economy of exciting watts as affecting the efficiency must be given its due weight, and if the alternator is to be employed to supply induction motors, a considerable overload capacity is even more important than good inherent regulation. For electric lighting the drop of volts is always more than can be permitted at the consumers' terminals, so that the excitation must necessarily be adjusted, and the amount of the adjustment within reasonable limits becomes more or less a matter of indifference.

A good polyphase generator with alternate poles and a distributed winding will regulate within about 6 per cent. on a non-inductive load with $\cos \phi_e = 1$, and within 15 to 18 per cent. on an inductive load with

* See especially Dr. M. Kloss, *Journal Inst. Electr. Eng.*, vol. xlii. pp. 163-176.

† A. G. Ellis, "Steam Turbine Dynamos," *Journ. Inst. Electr. Eng.*, vol. xxxvii. pp. 330, 331.

cos $\phi_e = 0{\cdot}8$. With concentrated windings the results obtained are not quite so good, and in the single-phase machine an equally good inherent regulation can only be obtained at greater cost of iron and copper in the field-magnet. Inductor alternators with poles of similar sign in each crown do not show such good regulation, especially on loads with low power-factor, and it is on this account that for motor work and transmission of energy they have been practically displaced by the generator with alternate poles of opposite sign. The figures recommended by the Engineering Standards Committee as maxima are 6 per cent. with cos $\phi_e = 1$, and 20 per cent. with cos $\phi_e = 0{\cdot}8$.*

§ 26. **Constant-current and constant-power alternators.**—Only in certain special cases is a high inherent regulation actually desirable, and of these two instances may be mentioned. The first is the so-called *constant-current alternator.* Historically, some of the earliest alternators such as those used for series lighting by Jablochkoff candles belonged to this class. When the alternator is applied to series arc-lighting a large amount of armature reaction is sought for, just as in the series-wound continuous-current dynamo, and the machine is worked on the drooping portion of its external characteristic in order to obtain a nearly constant current with widely varying values of the voltage. Even when entirely short-circuited, the current then hardly exceeds the normal value required by the arc lamps, so that some of their number may be cut out without affecting the working of the remainder.

The second case is that of the alternator when used for electric smelting or electro-metallurgical work, for which only the heating and not the chemical properties of the current are required, as for instance in the production of calcium carbide. For such use an approximately constant power is needed in order to maintain the temperature constant; as fusion proceeds in the electric furnace, the resistance of the circuit rapidly falls; a greatly increased current with violent fluctuations would result, but is held in check by the drop in volts which proceeds *pari passu* with the increase of the current, so that the alternator delivers an approximately constant watt output.

* Cp. H. S. Meyer, *Electrician*, vol. lii. p. 701.

CHAPTER XXIV

CONSTRUCTION OF ALTERNATOR ARMATURES AND FIELD-MAGNETS

§ 1. **The iron armature core.**—The iron core of the alternator armature must be laminated similarly to that of the continuous-current dynamo, and owing to its large size is usually built up of segmental discs. The radial depth of iron is given more lavish proportions in the alternator, in order to reduce the flux-density therein and so to keep within reasonable limits the loss from hysteresis, which would otherwise be high owing to the greater frequency. Usual values for the induction B_a in the iron below the teeth of heteropolar armatures are from 10,000 to 12,000 lines per sq. cm. for 25 cycles to 8000 for 40 cycles, 6000 for 50 cycles, and 4000 for high frequencies of 100 or more. In the homopolar drum alternator, owing to the fact that the direction of the field is never reversed, the hysteresis loss is lower (Chap. XII. § 11), and the flux-density may be increased to $B_a = 12,000$ for 40 cycles and 10,000 for 60 cycles. In all cases the discs require to be very tightly compressed by cross bolts and securely held, in order to prevent the humming noise due to the rapid variations of the magnetic field from becoming disagreeably loud. Ventilating ducts are arranged every few inches along the length of the core, and it is even more important in the alternator than in the continuous-current dynamo that the separation of the laminæ should be effective, so as to minimise eddy-currents.

§ 2. **Drum-winding.**—In the *drum armature* the two classes of wire and bar-winding—each with its subdivisions according as it is hand- or former-wound—may again be distinguished, but are not so sharply divided from each other as in the continuous-current dynamo.

The rotating drum armature has now been practically discarded except for small machines in favour of the stationary armature and rotating field-magnet. The latter can be given greater mechanical strength to withstand centrifugal force, and greater moment of inertia to secure a uniform angular velocity, while even if the exciting coil or coils revolve with the field-magnet the rubbing contacts of the collecting rings have only to transmit an exciting current of which the voltage can be made low. Further, the coils of the high-tension circuit of the armature are relieved from any strain on their coverings due to rotation.

For high voltages and moderate outputs, round wire is usually employed, and with open slots the advantages of former-wound coils in that they are easy to wind, are more regular and stiffer, and can be thoroughly insulated before being placed in the slot, are all obtained. The coils are thickly wrapped round with oiled cloth, canvas, or micanite paper, taped over, and impregnated with insulating varnish (Figs. 484 and 485); they are then driven tightly into the slots, without further protection. Or the slot may itself be lined with an open insulating trough, but in such cases the junction of trough and retaining wedge is a weak spot under high pressure. The claim that a burnt-out coil is easy to remove and replace when former-wound and inserted in an open slot is somewhat delusive, as after prolonged use the heating of

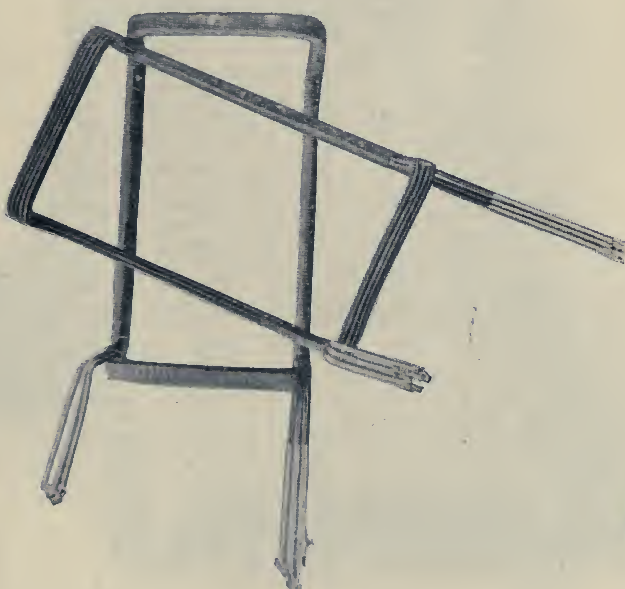


FIG. 484.—Armature coils of Bullock alternator.

the varnish causes the coils to adhere very tightly to the slots; considerable force is then required to detach them, and during the process other coils which in polyphase machines overlap may become damaged. But the chief objection to the open slot, as already pointed out, lies in the higher harmonics to which it gives rise in the E.M.F. curve. So far as loss of space by insulation is concerned, a single large slot per pole and per phase is the least wasteful, and with the open slot the pole-faces must in any case be laminated. Rectangular or oval-shaped tubes of micanite with a long scarfed opening down one side have lately been introduced, which enable the straight sides of the coils to be inserted into their insulating envelope before they are placed within the slots.

Or the scarfed opening may be arranged at the mouth of the slot (Fig. 486). If the winding is embedded in closed tunnels stamped out round the edge of the armature discs (Fig. 487 i.), the turns must be threaded through by hand,—an expensive process and somewhat wasteful of space, since plenty of room must then be allowed to the winder; the holes or tunnels are first completely lined with seamless tubes of micanite or compressed paper projecting well out beyond the ends of the armature core, and the wires must be well insulated, but need only to be just sufficiently tight in the holes to prevent chafing, since, as

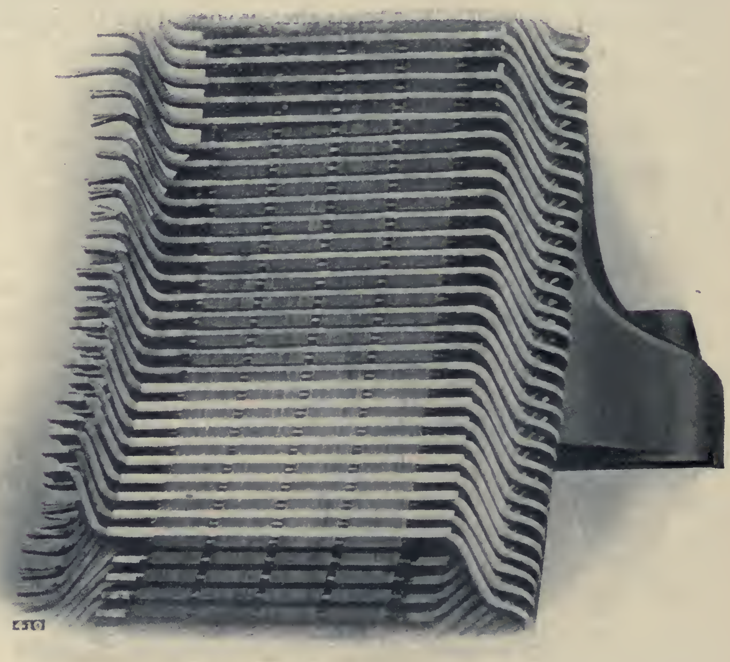


FIG. 485.—Portion of stator of 1000 kw. Bullock alternator showing method of inserting armature coils.

explained in Chapter XIII. § 21, the greater part of the mechanical stress is taken up by the iron core. Wooden formers are arranged against the inside of the stator ring, as shown in Fig. 488,* and upon these the ends of the coils are formed by the winder. A number of knitting needles corresponding to the size of the wire are placed within a tube, and as these are withdrawn one by one a turn of the wire is inserted to take their place in orderly succession, care being taken that the insulation is

* Reproduced by permission of the proprietors of the *Electrical Magazine*, from an article "On the Windings of Alternator Armatures" (A. C. Eborall).

not abraded in the process. The closed slot not only obviates ripples in the E.M.F. curve, but also avoids the necessity for lamination of the pole-shoes and slightly reduces the reluctance of the air-gap, yet it has the objection that, however thin the iron bridge across the mouth of the slot, it considerably increases the inductance of the coil and its secondary leakage. This objection is largely obviated by employing a half-

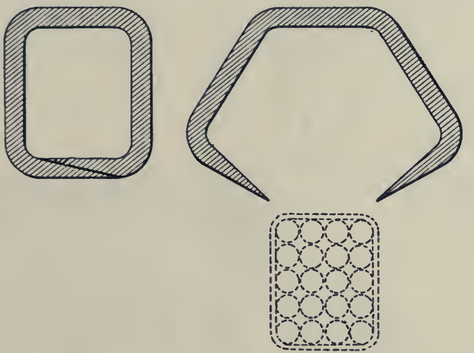


FIG. 486.—Scarfed micanite tube.

closed slot such as Fig. 487 ii. and iii.; or the narrow iron bridges left in the stamping may be sawn through subsequently, and one or other of these methods is to be preferred in almost every case to the completely closed slot. The half-closed slot is easier to wind than the tunnel, but still involves more labour than the open slot with former-wound coils, so that its advantage or otherwise as compared with the

i. ii. iii. iv.

FIG. 487.

open slot largely turns upon the price of labour. Yet, as a matter of fact, it is often possible to employ former-wound coils even with half-closed slots if the opening be not very small, and with a normal length of air-gap and a slot opening less than half its width solid poles may still be retained. If the required section of copper in the active elements becomes too great for a single round wire, a number of wires in parallel must be adopted, or a stranded cable. Rectangular wire or band copper can also be wound on shapers and used in connection with open slots. With armatures of narrow width and well-exposed ends to the coils the current-density in the armature wires is usually about 2000 to 2500 amperes per square inch.*

For low voltages or very large outputs at high voltages the transition is gradually made to the simple hand-wound bar armature with U-shaped loops pressed into the slots or finally to solid bars of rectangular or circular section, alternately long and short, threaded through insulated tunnels and bolted or riveted and soldered to end-connectors of flat copper strip bent to a double involute or forked shape (Fig. 489). Two

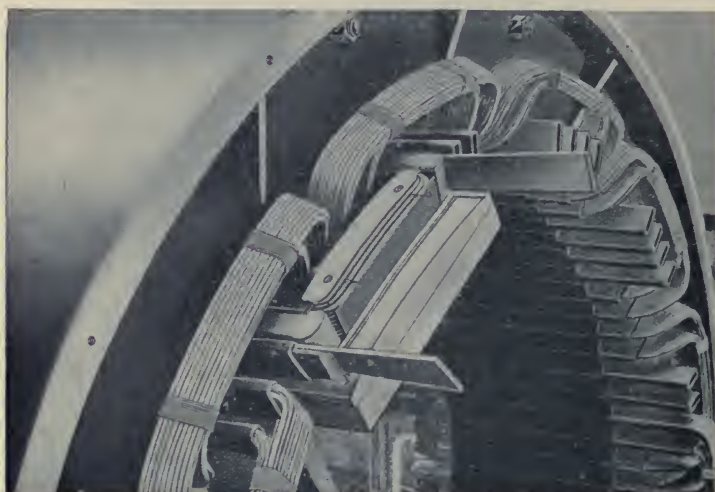


FIG. 488.—The winding of alternator armature coils by hand.

or more slots per pole and per phase can then be easily obtained, and the wave of E.M.F. becomes more nearly sinusoidal.

An important point in the arrangement of the armature coils is to secure rigidity to withstand the effects of an accidental short-circuit. When a large alternator is suddenly short-circuited under full excitation a very heavy current arises for a few seconds, which may exceed the true short-circuit current under steady conditions by 5 times or more, and the normal full-load current by 15 to 20 times. The reason for this is that for such momentary conditions the impedance of the machine is practically reduced to that corresponding to the leakage reactance of

* For measurement of the temperature of the stationary armature coils in an alternator when running, an alcohol thermometer is to be preferred if there is much stray field, as a mercury thermometer may give false readings if eddy-currents are set up in it.

the end-windings, and from local flux in the slots. The effect of a short-circuit under full-load excitation being a reduction of the full-load flux which initially exists, when the short-circuit is sudden the diminishing flux as it cuts the wires of the exciting coils momentarily raises the

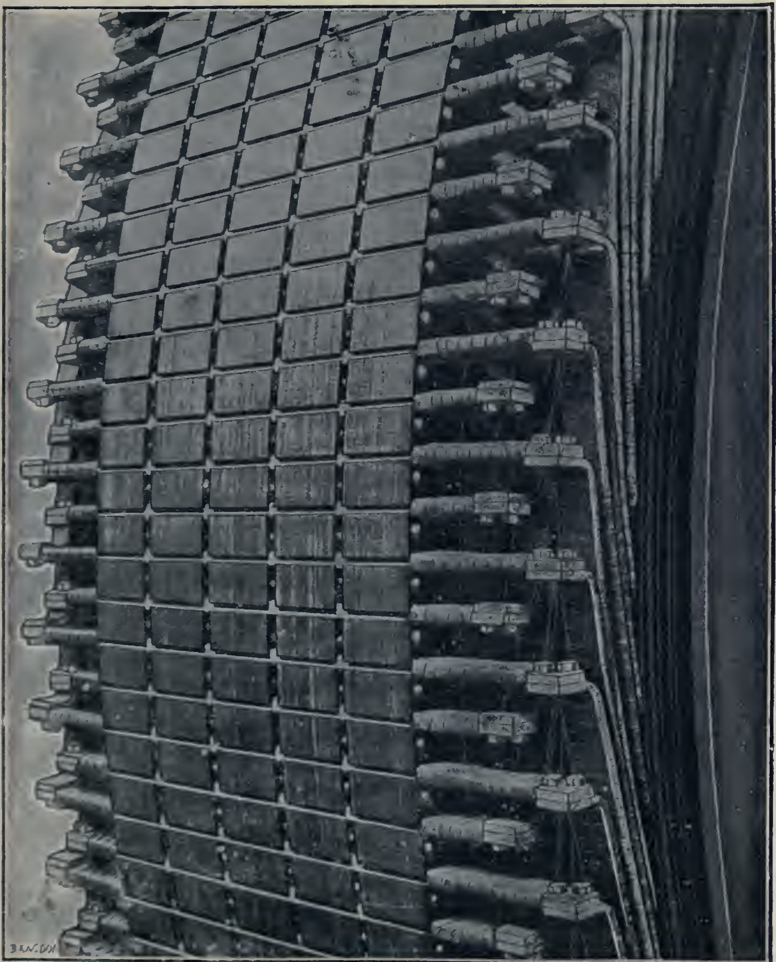


FIG. 489.—Portion of bar-wound rotating armature of Westinghouse alternator.

exciting current and so provides temporarily more than the full-load excitation. In consequence both the armature and exciting currents for a brief period rise, the former reaching such an amount that by its back ampere-turns it balances the increased excitation. After reaching a maximum both settle down, the latter to the normal excitation, and the former to the current corresponding thereto if the machine had been short-circuited gradually. The *maximum* momentary value of the short-circuit current will therefore be proportionate to a value of the flux in the magnet pole lying between the normal full-load value and that which obtains on gradual short-circuit. Thus, if the exciting current rises, say, to $3\frac{1}{2}$ times its steady value, the sudden short-circuit current will be something less than $3\frac{1}{2}$ times its steady value, for all eddy-currents in solid parts of the magnetic circuit will assist in limiting its rise.*

Owing to the overlapping of the coils of the different phases, a sudden short-circuit can thus cause an enormous mechanical force between them proportional to the square of the current, the effect of which can only be likened to that from the explosion of a charge of dynamite. Thus consequent upon a high-power surge on the distribution system of the Manhattan Railway, New York, a 5000-kw. 3-phase alternator was short-circuited and the armature bars 2 in. deep by $\frac{1}{8}$ in. thick, three in each slot, were bent edgewise for over six inches, showing that the force bending them amounted to at least 3000 or 4000 lbs.† As a general rule with slow-speed machines, the span of the coils across the pole-pitch is short, and their projection is not great, so that the end-connections have sufficient mechanical strength to prevent displacement or distortion. But the stresses increase roughly as the square of the pole-pitch, and when this reaches such figures as 100 inches, as it may in the case of turbo-alternators owing to their high speed, the end-connections require strong clamps to support them from the stator frame.‡ The same necessity arises also in the case of large high-speed single-phase alternators for low frequencies, such as 25 cycles per second for heavy railway work, where there is a likelihood of sudden short-circuits; it has been calculated § that in such cases the mechanical stress on one end of a coil may be from 2 to 10 tons, and this comes as a sudden shock.

§ 3. **Insulation and proportions of slots.**—The number of slots per pole and per phase varies; two or three slots per pole and per phase are to be preferred, but beyond this latter number the loss of space in insulation becomes too pronounced, at least with high tensions of, say, 10,000 volts. With a given number of active wires the effect of the number of slots on the secondary leakage is dependent upon a variety of complex conditions, chief among which is the width of the opening and the ratio of the width of the slot to its depth. Various shapes are in use, but in general they approximate to rectangles

* Cp. F. Punga, "The Sudden Short-circuiting of 3-Phase Alternators," abstracted in *Electr. Review*, vol. lix. p. 1015. To reduce the momentary short-circuit current in very large turbo-alternators, it is now proposed to add external reactance coils.

† *Trans. Amer. I.E.E.*, vol. xxiv. p. 301.

‡ Cp. § 9, and A. G. Ellis, "Steam Turbine Dynamos," *Journ. I.E.E.*, vol. xxxvii. p. 333, where two methods of support are shown.

§ W. L. Waters, "Single-phase Generators," *Trans. Amer. I.E.E.*, July 1908.

with a depth three times their width, and preferably the corners are rounded off (Fig. 487). From examination of a number of machines built by different makers, Mr. H. M. Hobart * found that the average thickness of slot insulation was approximately as follows :—

R.M.S. Volts of rated Output.	Thickness of Slot Insulation in Millimetres.	R.M.S. Volts per Millimetre.
500	1·3	385
1000	1·75	570
2000	2·47	816
4000	3·35	1,200
8000	4·6	1,740
12,000	5·67	2,120
16,000	6·67	2,400

A minimum thickness is necessitated by the requirements of mechanical strength to withstand handling during construction, but apart from this it is evident from the above table that in practice the same factor of safety is not obtained in machines of very high voltage as in those of moderate voltage.

Each coil after completion should be tested in place in its slots with double the working pressure between copper and iron preparatory to the final pressure test, and the application of a very high pressure should not be prolonged for more than a few minutes for fear of setting up heating of the dielectric which may cause incipient charring and subsequent break-down under a much lower voltage.

The stationary armatures of alternators of large size can be successfully wound directly for very high voltages; 11,000 volts is not uncommon, while the 1800 kilowatt alternators for the Paderno installation were built by Messrs. Brown, Boveri & Co. for 15,000 volts, and the 1500 kilowatt 15-cycle 150 revolutions per minute alternators built by Messrs. Schuckert & Co. for the Valtellina 3-phase railway have star-connected tunnel-wound armatures for 22,000 volts direct. Messrs. Ganz & Co. have built four 5200 KVA alternators for 30,000 volts pressure at the armature terminals.† The various considerations of the use of extra high-tension generators in large sizes, especially when driven by steam-turbines, their increased cost having to be set against the saving in transformers, are brought out in Mr. B. A. Behrend's paper ‡ on "The Practicability of Large Generators wound for 22,000 Volts," and the discussion thereon.

The apparent density in the teeth may be taken nearly as high as in continuous-current machines, *i.e.* from 18,000 to 21,000. A fairly high density has the advantage that it lessens the depth of the

* *Electr. Review*, vol. lvi. pp. 680 and 716, 1905.
† *Electr. Eng.*, vol. xlii. p. 691.
‡ *Trans. Amer. I.E.E.*, vol. xxvi. part 1, p. 351.

slot and reduces the weight of iron, and with it the total loss by hysteresis and eddy-currents, although the loss per unit volume of the iron is increased.

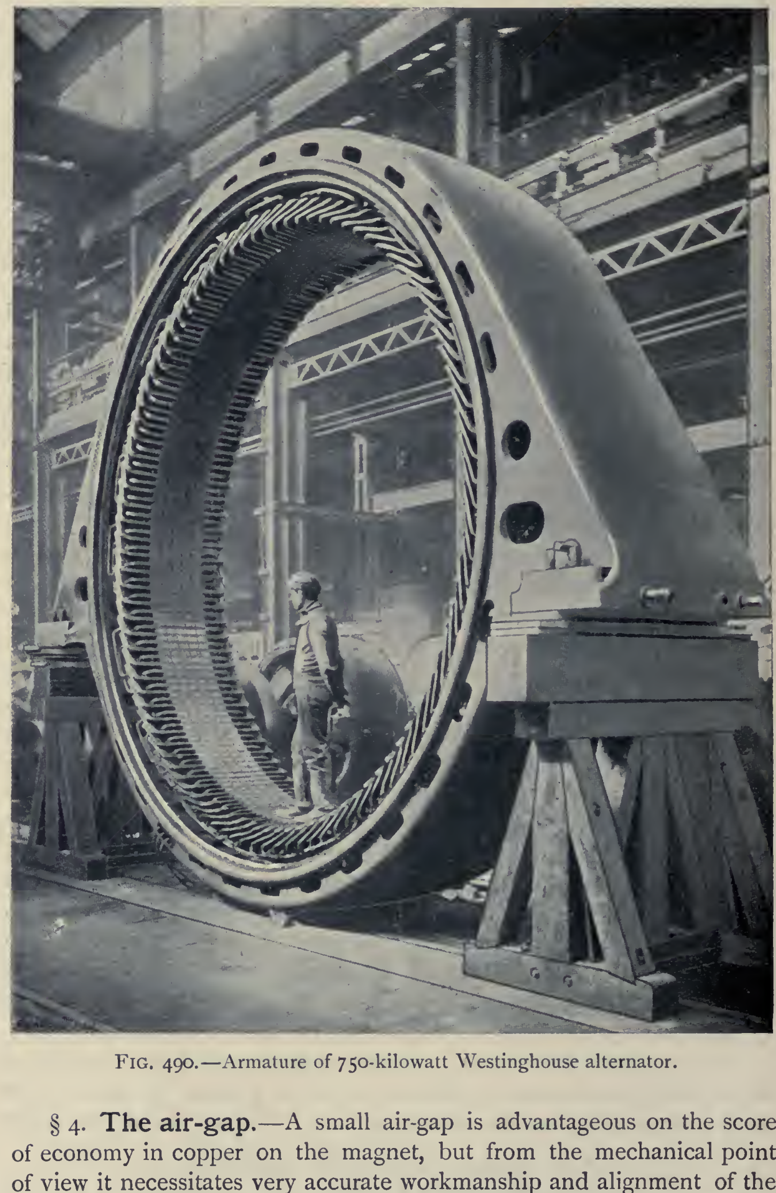


FIG. 490.—Armature of 750-kilowatt Westinghouse alternator.

§ 4. **The air-gap.**—A small air-gap is advantageous on the score of economy in copper on the magnet, but from the mechanical point of view it necessitates very accurate workmanship and alignment of the bearings in order to avoid unequal magnetic pull. The mechanical advantages of a large air-gap are further reinforced by the electrical consideration that it may render lamination of the pole-shoes unnecessary. Hence l_g seldom is less than 0·25″, except in quite small machines. Means for equalising the air-gap in case of wear of the bearings are usually provided. The densities employed in the air-gap are generally comparable with those of continuous-current dynamos, and, like the induction in the armature core, they are higher in the homopolar than in the heteropolar type. In the latter type usual values of B_g are from 8000 to 10,000 in large and 6000 to 8000 in small sizes for 25 cycles, or 6500 to 8500 in large and 5000 to 6500 in small sizes for 50 cycles, the lower and upper limits being usually accompanied respectively by a high and a low voltage.

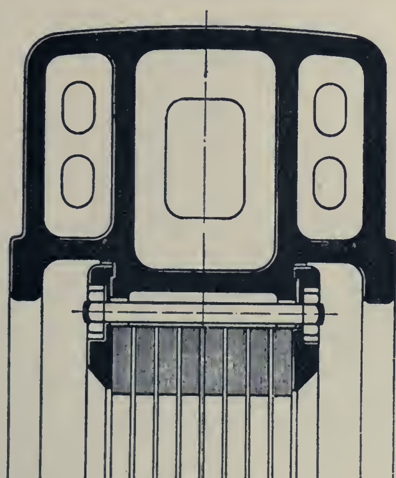


FIG. 491.—Section of stator of Manhattan 5000 KW. alternator.

With the large number of poles usual in alternators, the expression of equation (61*d*) in Chapter XIV. § 15, namely, $\frac{B_g^2 A \cdot 2p}{11,183,000} \times \frac{\delta}{c \cdot l_g}$, may be used to calculate approximately the magnetic pull in lbs. on the shaft due to a displacement δ of the magnet wheel from its true position concentric with the armature. If β = the ratio of pole-width to pole-pitch, $A 2p = \pi D L \beta$, and if $\beta = 1$ as in an inductor generator, the formula reduces to the expression for two nearly co-axial cylinders as given by Mr. Behrend (*Amer. Inst. Electr. Eng.*, vol. xvii. p. 606) save for the introduction in the denominator of the factor c.

But the deflection δ of the stator frame must be approximately estimated in the same manner as in Chapter XIV. § 17. For the weight of yoke-ring and poles must now be substituted the weight of the entire stator, discs, and frame, and the stiffening added by the internal ring of discs may be roughly taken into account by assuming the modulus of elasticity of the entire stator as that of cast iron only.

§ 5. **The external stationary armature.** — Evidently an external stationary armature ring, although free from centrifugal strains, yet requires to be of great mechanical strength to resist deflection. If

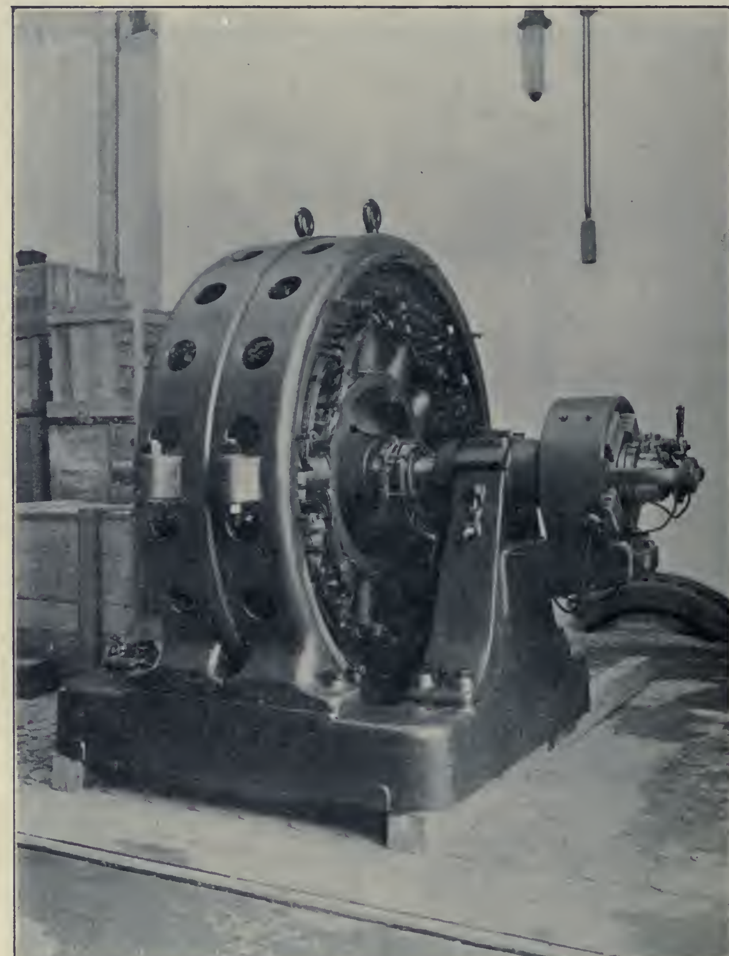


FIG. 492.—Alternator of Brown, Boveri & Co.

the cast-iron case be round when bored out in a horizontal position, it naturally suffers a certain deformation when raised to a vertical position, and supported by brackets on either side; and this deformation then becomes further reinforced by very powerful stresses due to any unbalanced magnetic pull when its internal surface is not perfectly concentric with the magnet wheel, or by reason of any inequality in the strength of the poles. Up to diameters of 30 feet the requirements of mechanical strength may be met by the employment of a deep cast-iron casing of inverted U-shape with ventilating holes in its sides (Fig. 490); round the inside of this the segmental laminations, assembled so as to break joint, are in the process of building threaded over numerous

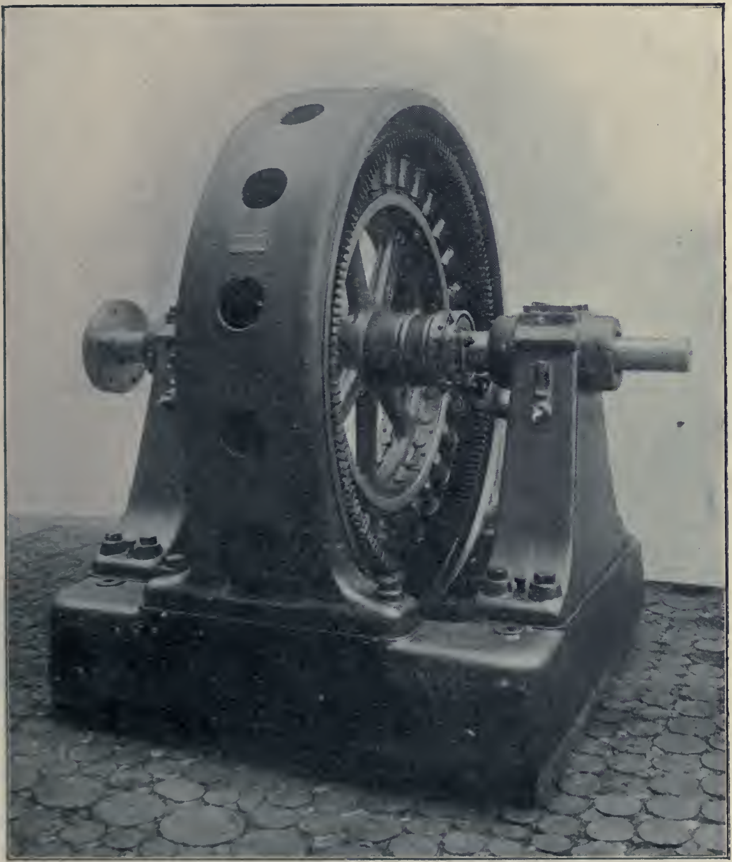


FIG. 493.—Alternator of Brown, Boveri & Co.

transverse bolts, and are clamped tightly together by plates engaging in a turned rim on the armature case. Or the segments are dovetailed into wedge-shaped grooves in the cast-iron stator frame, the transverse bolts passing through clearance spaces at the back of the discs (Fig. 491 showing section of Westinghouse Manhattan alternator). The back or sides of the armature casing are usually pierced with a number of large openings to allow of the exit of the air driven through the ventilating ducts of the core. Fig. 492, of a 106-H.P. alternator built by Brown, Boveri & Co. for the St. Petersburg Polytechnic, and giving 230 volts at 50 cycles and 215 revolutions per minute, illustrates the standard type adopted by that firm for sizes up to about 150 H.P., while Fig. 493 shows their standard type for larger sizes up to about 500 H.P. In order to render the inside of an armature with its numerous coils readily accessible for cleaning, it may in small sizes

FIG. 494.—120 KW. generator of British Thomson-Houston Co. with direct-connected exciter, showing method of sliding armature for inspection.

be arranged to slide horizontally along an extension of the baseplate by means of rackwork or screws worked by hand (Fig. 494).

With very large diameters the prevention of the mechanical deformation of the armature ring becomes more difficult, and several supporting feet (Fig. 495) * may be required. Fig. 496 shows the lower

* From the brochure, "Der Aufbau und die planmässige Herstellung der Drehstrom-dynamomachinen," by Herr O. Lasche of the Allgemeine Elektricitäts Gesellschaft, to whom we are indebted for permission to reproduce many of the accompanying figures and data illustrative of the construction of alternators. An English translation appeared in *Engineering*, 1901, vol. lxxii., p. 173 ff.

half of a 3000-kilowatt alternator built by the Allgemeine Elektricitäts Gesellschaft of Berlin, and more fully described in Chapter XXVII. ; it gives an idea of the size to which the armature casing may attain, and it will be seen that there are three pairs of feet, the chief part of the weight being taken by the middle pair halfway up the lower semicircle,

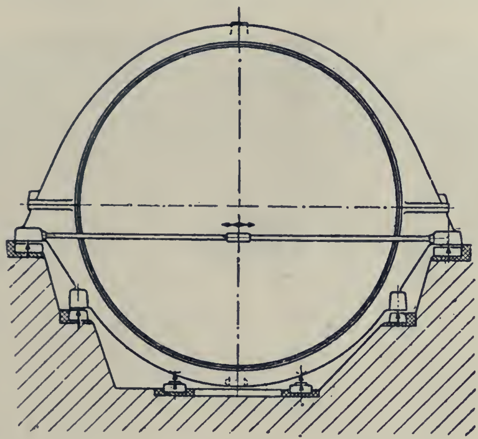


FIG. 495.

while the upper pair are given considerable proportions to satisfy the eye in the matter of appearance. In some cases greater stiffness is imparted by radial tie-rods terminating in a ring, concentric with but free from the shaft (Fig. 497). A convenient and advantageous

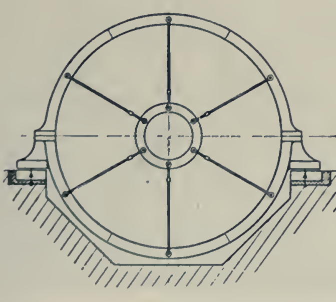


FIG. 497.

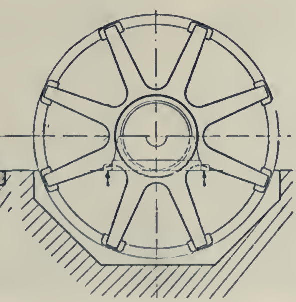


FIG. 498.

method is that devised by Mr. C. E. L. Brown, and employed by Brown, Boveri & Co. ; the armature case is mounted on a pair of star frames, one on either side, with massive arms which terminate in rings embracing trunnion journals on the plummer blocks that carry the revolving magnet (Fig. 498). The armature is thus maintained in accurate concentricity with the magnet wheel, without increase of its external diameter or of the dimensions of the foundations, while the further advantage is gained that when the bolts which fix the armature to the concrete foundation and prevent it from turning are withdrawn, the whole armature can be barred round on the trunnion bearings so as

FIG. 496.—Lower half of 3000 KW. alternator of Allgemeine Elektricitäts Gesellschaft.

to bring any coil into a convenient position for inspection or cleaning. Fig. 499 shows the large armature wheel of one of the Frankfort single-phase 1500-H.P. alternators during construction; the core is divided into similar segments, which are seen in process of being fixed between the two side star frames. The outside diameter is 26 feet, and the weight of the complete armature is 53 tons, that of the whole machine which runs at 86 revolutions per minute being 110 tons. A similar method of stiffening the armature is employed in the 15,000 volt 3-phase alternators of the Paderno-Milan transmission,

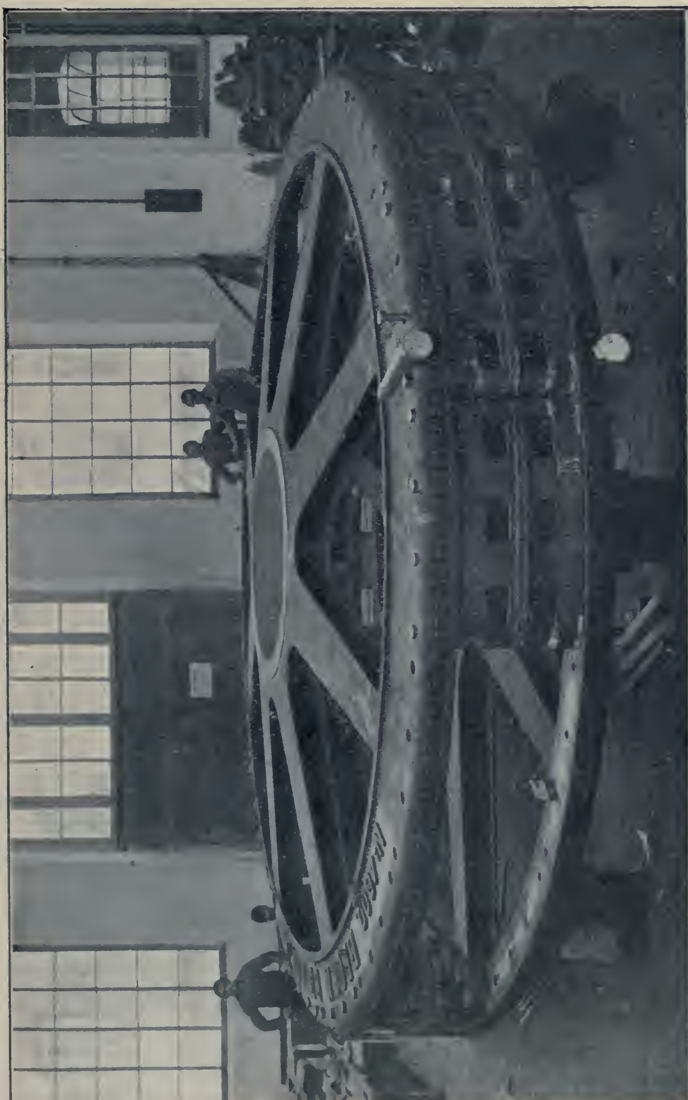


FIG. 499.—Armature wheel of 1500 H.P. single-phase alternator. (Brown, Boveri & Co.)

and also in the 3000-volt 650-H.P. 3-phase generator of Fig. 500, which shows clearly the side frame and holding-bolts. The latter machine is one of two directly coupled to blast-furnace gas engines at the Düdelingen Iron Works, and it is of interest to mention that no difficulty has been experienced in running them in parallel.

Fig. 584 shows the method adopted by the Westinghouse Company

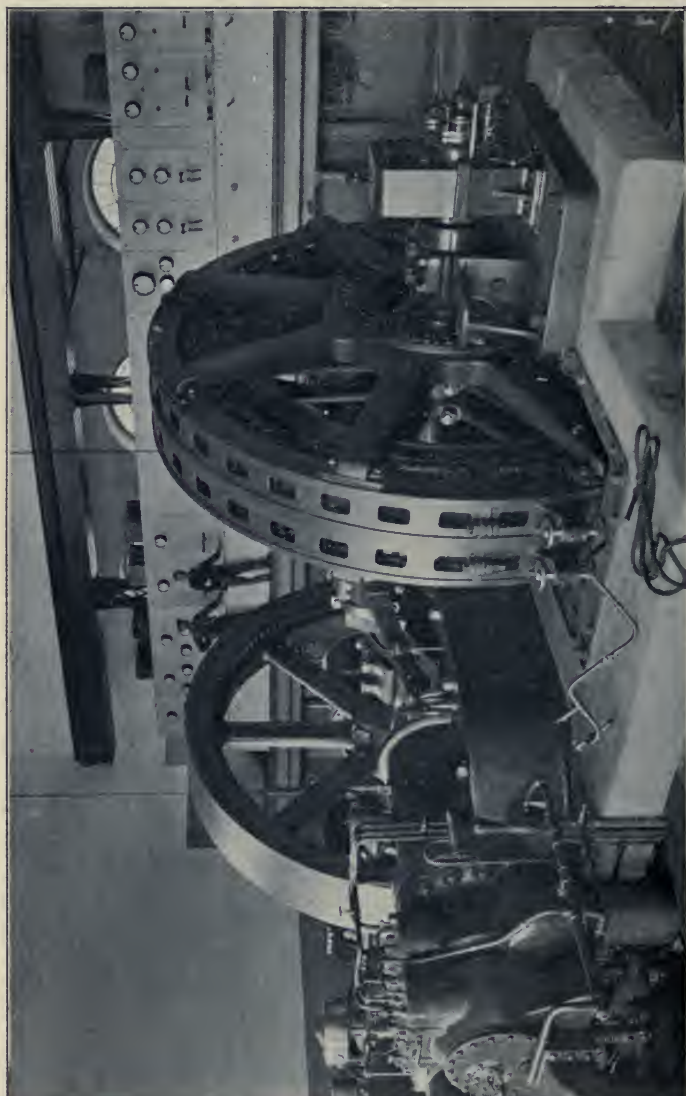


Fig. 500.—650 H.P. three-phase alternator. (Brown Boveri & Co.)

for stiffening the large armature casing of the 5000-kilowatt generators supplied to the Manhattan Railway Company; as will be more fully described in Chapter XXVII., the outside diameter is about 42 ft., and the weight of the stationary part of the machine is 234 tons, although the laminated iron core itself only weighs some 40 tons.

The mechanical problem has been met in a drastic and novel manner in a type of alternator brought out by the Allgemeine Elektricitäts

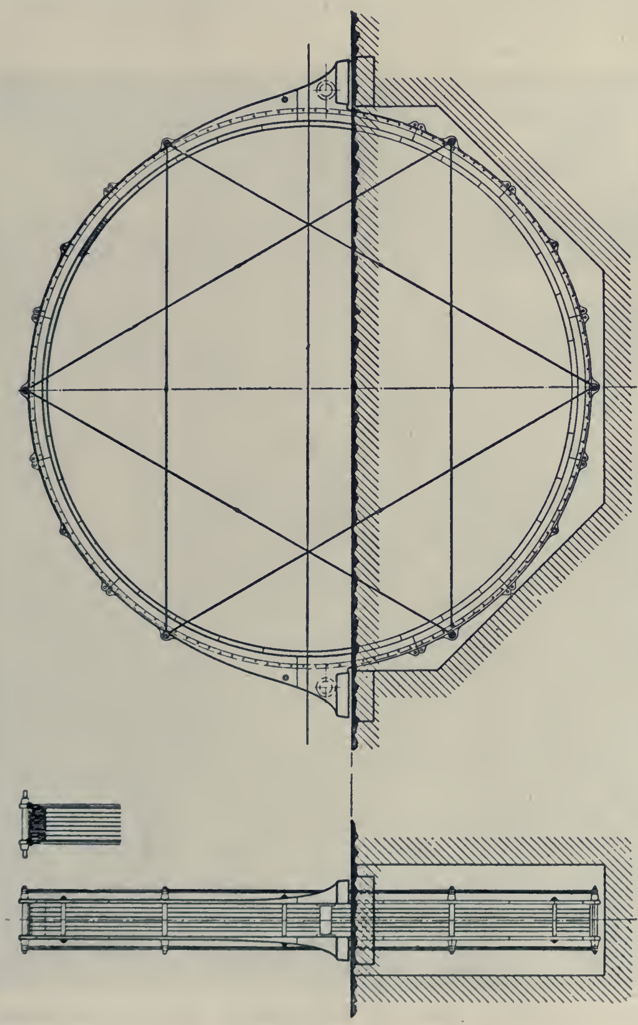


FIG. 501.—Stiffened armature of the Allgemeine Elektricitäts Gesellschaft.

Gesellschaft. The cast-iron casing is here entirely abandoned, a triangulated system of tie-rods fastened to the side-cheeks of the armature (Fig. 501) being used in its place. Fig. 502 shows such an armature as built up in the factory. The great advantage that is gained by the method is the noteworthy saving of weight, since in very large machines with the usual cast-iron case as much as four-fifths of the total weight of the stationary armature is employed in giving constructional stiffness, and only the remaining fifth is magnetically active. The method has not, however, been widely adopted, owing to the greater care required in the erection and other reasons which offset the lesser cost of the material.

FIG. 502. —Stiffened armature of the Allgemeine Elektricitäts Gesellschaft.

§ 6. **The magnet system of inductor generators.**—The general nature of the magnetic systems of alternators of various types have been diagrammatically illustrated in Chapters VIII. and IX. They may be divided into the two classes, namely, those of heteropolar alternators in which poles of alternate sign occur in succession in one ring round the armature, and those of inductor or homopolar generators in which in any one ring drawn round the armature there are only poles of one sign, those of the opposite sign forming a second ring spacially displaced.

The latter class is illustrated by Fig. 503,* which shows a number of inductor wheels of different forms in their unmachined state as cast in steel by the Skoda Works. In the foreground is seen the inductor of a double-armature alternator corresponding to that shown diagrammatically in Fig. 92, and in the background is a larger ring of the

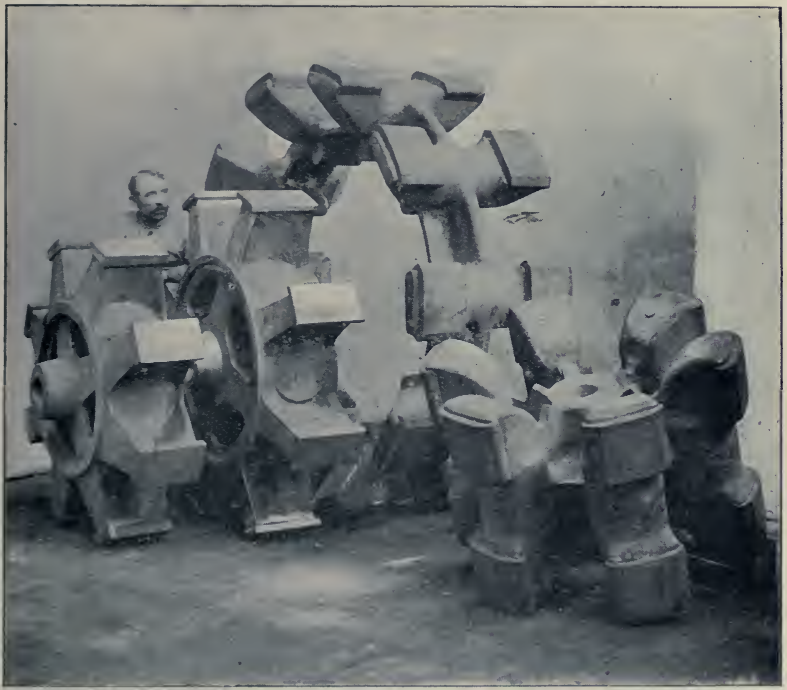


Fig. 503.—Steel inductor castings. (Skoda Works.)

same kind. In Figs. 504 and 505 the poles are staggered as in Fig. 93, but the finished machines are designed to give two phases. Owing to the unfavourable nature of the strains on the bolts carrying the overhung inductors of Fig. 94, the method of construction there shown has now been abandoned.

The use of the polyphase generator for transmitting energy obtained from waterfalls has led to the development of alternators with a vertical

* Reproduced together with Figs. 504 and 505 by permission of the Skoda Works of Pilsen, Bohemia.

axis of rotation specially suitable for direct coupling to the vertical shaft of slow-speed turbines on low falls, and in the design of such "umbrella" types Mr. C. E. L. Brown was the pioneer from 1891

FIG. 504.—Cast-steel inductor wheel, 11½ feet diameter. (Skoda Works.)

onwards. The six two-phase generators, each of 300 H.P., installed by Brown, Boveri & Co. in the Olten-Aarburg central station are directly coupled to the vertical shafts of their turbines, and are probably unique from the extremely slow speed at which they run, namely, 28·5 revs. per min. Fig. 505 shows the inductor wheel of one of these generators; its diameter is 15 feet, its weight nearly 8 tons, and the 84 poles forming

FIG. 505.—Cast-steel wheel of inductor alternator for Olten-Aarburg electricity works.

one crown give a periodicity of 40. Fig. 506 shows in section half of a double-armature 3-phase inductor generator with vertical shaft built by the Allgemeine Elektricitäts Gesellschaft for the large water-power installation at Rheinfelden; the five-armed star has bolted to it a ring carrying 55 pairs of laminated polar projections arranged in two tiers, the outside diameter of the inductors being 18 ft. 9 in. The stationary exciting coil of T section has a mean diameter of nearly 20 ft., wound with 270 turns of 0·315″ wire, and is placed within the two halves into which the armature casing is divided on the horizontal plane; the section of iron in the magnetic circuit is shown black. The single air-gap is only 5 mm. = 0·197″. The armature is wound with 110 coils of 8 turns each, and gives 6800 volts between the leads and 61 amperes

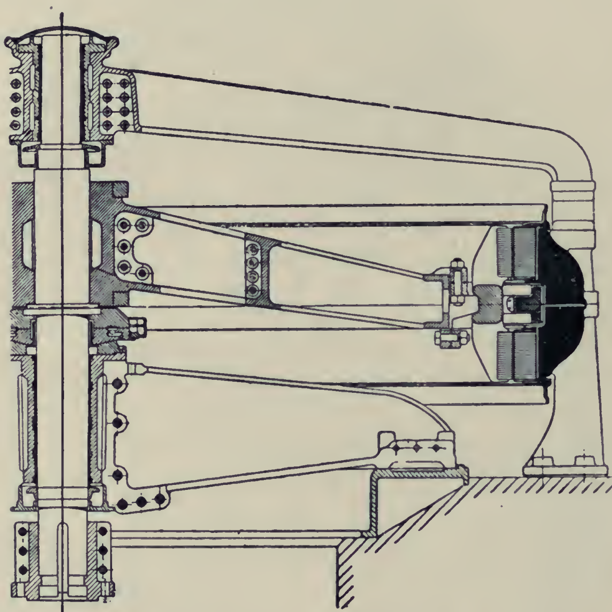


FIG. 506.—Inductor Generator of Allgemeine Elektricitäts Gesellschaft at Rheinfelden.

per phase, *i.e.* 575 kw. with cos $\phi_e = 0·8$, at 55 revs. per min. and a periodicity of 50.

The inductor generator has certain defects inherent to the type, chief among which is the flux of lines which must necessarily pass into the armature core between each pair of adjacent poles of the same sign. Although not to be called leakage in the proper sense of the word, yet any such flux is doubly disadvantageous. Not only does it assist in saturating the field-magnet and so call for more ampere-turns of excitation, but it directly causes a back E.M.F. in the armature, so that, as explained in Chapter IX. § 4, the useful flux is the difference between that which enters the armature under a pole (Z_a') and that which enters within the pole-pitch between a pair of poles (z_a). If the flux under the pole-face and between the poles be measured separately and the latter be deducted from the former, $Z_a' - z_a$ as so determined agrees closely with the value of the useful flux as deduced from the open-circuit characteristic curve, and in a machine so tested the proportion of the harmful flux z_a to the useful flux ($Z_a' - z_a$) averages about 15 per. cent., but varies with the ratio of the air-gap to the width of the pole, and further, is dependent upon the absolute value of the pole-pitch.* Thus, if an increase in frequency is required with the same speed, and it is not permissible to increase the diameter of the armature so as to increase the pole-pitch proportionately, the absolute value of the latter is reduced, and the machine for the higher frequency gives an open-circuit characteristic appreciably below that for the lower frequency.

Next, in the inductor generator there is considerable leakage from the flanks of the inductor and from the shaft and its pedestals into the armature case. If the harmful flux through the armature be added to the flux which does not enter into the armature and is therefore leakage in the strict sense of the term, and the ratio of their sum to the useful flux be compared with the leakage factor of the machine with poles of alternate sign, the latter will be found to have a decided advantage. Thus, in an inductor machine with an air-gap of $\frac{1}{16}''$ and a pole-pitch of $9''$, the leakage and harmful fluxes together amount to 20 per cent. of the useful flux, but with a pole-pitch of $5''$ and an air-gap of $\frac{1}{8}''$ the ratio rises to the high figure of 45 per cent. It is therefore very important to keep the percentage of the useless flux in the inductor generator low, and as this involves working with very small air-gaps any eccentricity of the revolving part is liable to throw great stresses on the bearings and shaft from the unequal magnetic pull; due precautions must then be taken to meet such stresses by employing an exceptionally large diameter of shaft, and by frequent taking up of any wear in the brasses of the bearings.

Finally, the secondary leakage of lines immediately round the armature coils across the tops of the slots is proportionately greater; in the homopolar generator not only the wires which at any moment are active, but those also which at the moment are merely connectors, are embedded in iron and possess considerable leakage inductance, while in the alternate-pole machine the end-connectors are comparatively far removed from any iron. Thus, although the single exciting coil is in itself an efficient means of obtaining the necessary numerous fields and the absence of any rotating copper is an attractive recommendation, yet experience has shown that the inductor type is only suitable for high speeds and low frequencies which allow of the pole-pitch being

* Behrend, "The Factors which determine the Design of Monophase and Polyphase Generators," *Electr. Eng.*, vol. xxvi. p. 598.

made large. Especially is it unsuitable for motor work, since it is not adapted to meet the problem of large lagging currents owing to its poor regulating properties. Even with the more widely extended use of steam-turbines running at very high speeds it has not again come into favour.

§ 7. **The magnet-system of heteropolar generators.**—Turning to heteropolar alternators, the advantages of the single exciting coil may still be retained by adopting the type of field used by Mr. C. E. L. Brown in the design of the Lauffen generators, in which overarching claws of alternate sign are made to spring from a single central magnet core (cp. Fig. 249). In this type the primary leakage reluctance is but small owing to the necessarily large area of the adjacent surfaces of opposite polarity, so that again it is ill adapted for inductive loads. Further, the single circular coil being concentrated at the base of the overarching poles is so situated that its entire magnetomotive force is in the best position for causing a large leakage flux. Lastly, if the coil revolves, it is subjected to strains due to the heating and cooling of the wire which are prejudicial to the life of the insulation, while if stationary all advantage of the peripheral speed in cooling it is lost. The type has therefore passed out of favour, and there has been a general return to the simple form of revolving field with internal yoke-ring and radial poles of alternate sign, each separately magnetised (cp. Figs. 84, 492, and 493). In exceptional cases, where the machine is small and the number of poles is high, every other pole may be left unwound; but this arrangement is seldom advantageous, as the mean length of a turn is increased, the leakage is greater, and the two poles, the one wound and the other unwound, are not of equal strength, so that the two half-waves of a period are unsymmetrical.* Hence the form with half the poles unwound has now been practically abandoned.

When once the secondary leakage of an armature has been reduced to the minimum that the circumstances of the design allow, the rise of volts when the full current is removed can only be brought to a small value by the methods described in Chapter XXIII. § 25; the primary leakage must be kept as small as possible, and a high saturation must be employed in the poles—as high, in fact, as is compatible with reasonable certainty of obtaining material of the estimated permeability. A small coefficient of leakage is given by the use of very short poles, but on the other hand this must not be pushed to such an extreme that the thickness of the copper winding unduly increases the mean length of a turn and the consequent weight of copper, while the cooling surface is reduced. Ingot iron or steel is more uniform in permeability than cast steel, gives greater exactness of section, and is free from the danger of blow-holes; yet with cast-steel poles, if carefully tested, the average

* Eborall, *Journ. Soc. of Arts*, vol. xlix. p. 754, and Rothert, *L'Éclairage Électrique*, vol. xxix. p. 319.

flux-density may be carried as high as $B_m = 18{,}000$ when a small rise of volts is specially required, or say B_m at the outer end of the bobbin $= 16{,}000$, and at the base $= 18{,}500$. With such high densities the yoke-ring must itself also be of cast steel, since with cast iron the lines would be throttled at the point of junction between pole and yoke, or the pole must be dovetailed into the ring; and in any case it is safer to allow a certain small number of ampere-turns for the reluctance of the joint. Thus the advisability or otherwise of a high degree of saturation in the magnet turns upon the conditions of the design and the nature of the service for which the alternator is intended. As a close degree of inherent regulation is always expensive to obtain, it is often advisable to adopt a cheaper design and to employ a Tirrill automatic regulator * to obtain a constant potential. In the highly saturated machine the value of the armature reaction must be accurately known or very carefully estimated at the outset, and unless it be designed for a low power-factor, an overload, or even the normal load with a smaller power-factor, may not be able to be carried without an impossible increase in the field excitation. In generators of low frequency for the supply of rotary converters a moderate degree only of saturation is to be recommended.

The yoke may be given considerable thickness and be worked at a lower density, since lavish proportions are here not disadvantageous, in so far as they increase the fly-wheel effect and the steadiness of the running.

The leakage coefficient of the field-magnet with alternate poles varies with the ratio of the pole-width to the pitch, with the absolute value of the pitch as determined by the frequency, and with the length of the poles, and is also affected by the load. It averages from 1·2 to 1·4, becoming larger with small poles, but, as in the case of continuous-current dynamos, unless it has been experimentally ascertained from machines of similar proportions and similarly saturated, even an approximate calculation of the leakage permeance $\mathcal{P}_l$ is to be preferred, and such calculation should be made with the greatest possible accuracy.

The use of the field-magnet as the sole fly-wheel in the case of alternators directly coupled to their steam engines is very frequent, yet is only to be recommended when the most favourable diameter of armature and of fly-wheel naturally approximate to each other, and this is usually the case only with machines of large size. In other cases, if the fly-wheel is given the dimensions which would best suit the alternator, an unduly large weight of cast iron in the yoke-ring may be necessitated and a larger shaft to carry it, while if the diameter of the alternator be increased to suit the fly-wheel requirements the armature frame must be of greater depth and size to give it sufficient rigidity.†

* For Tirrill automatic regulator, see *E.T.Z.*, 1907, December 12, 19, and 26, and *Electrical World*, vol. li. pp. 150 and 305.

† Rothert, *L'Éclairage Électrique*, vol. xxix. p. 313. See especially the brochure

With six or more half-closed slots per pole, *i.e.* two per phase in a 3-phase machine, solid pole-faces may be employed if the width of opening of the slots is not more than 1·5 l_g, but with wider openings

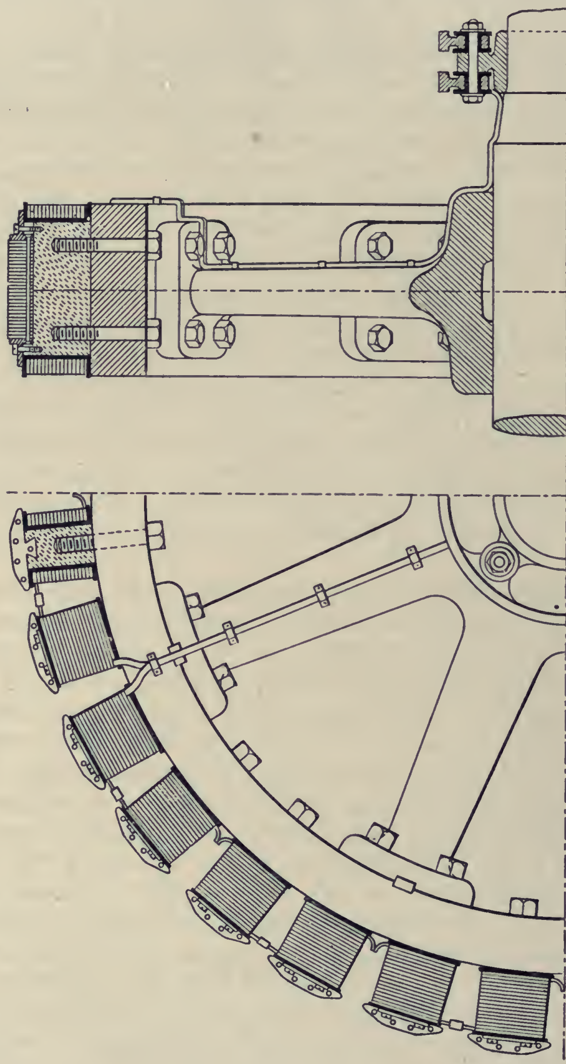


FIG. 507.—Portion of magnet-wheel with solid poles and laminated pole-shoes.

it is advisable to laminate at least the pole-shoes (Fig. 507) in order to avoid the eddy-currents arising from the variation of the flux as it

above-cited, "Der Aufbau und die planmässige Herstellung der Drehstrom-Dynamomaschinen," where various arrangements of magnet and fly-wheel are shown diagrammatically and discussed.

sweeps over the pole-faces.* Although the eddies do not penetrate far into the mass of the pole, it is frequently the practice to laminate

FIG. 508.—Laminated pole-pieces of Bullock alternator.

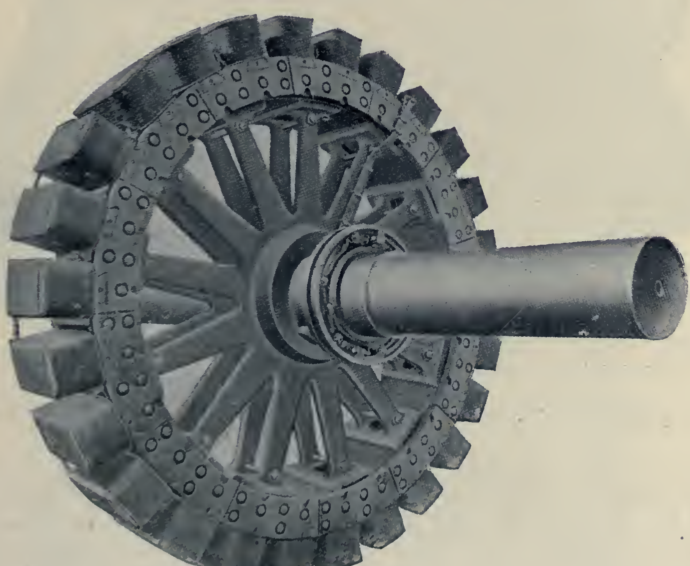


FIG. 509.—Rotor of 3000-KW. Bullock alternator.

the entire pole. This construction is employed partly from its cheapness in manufacture, since it is not easy to dovetail or fasten the

* For references, *vide* Chap. XXV. § 9.

laminated pole-shoes into solid poles, and partly also because the connection of the two usually involves a considerable radial depth of pole-shoe, so that the primary leakage between the pole tips is increased and the exciting coils do not approach close to the ends of the poles. An objection to the laminated pole-shoe as also to the laminated pole is that it does not conduce to easy parallel working; if there are periodic fluctuations in the flux or speed, the solid pole-face itself plays the part more or less of an amortisseur winding (Chap. XXV. § 25) owing to the

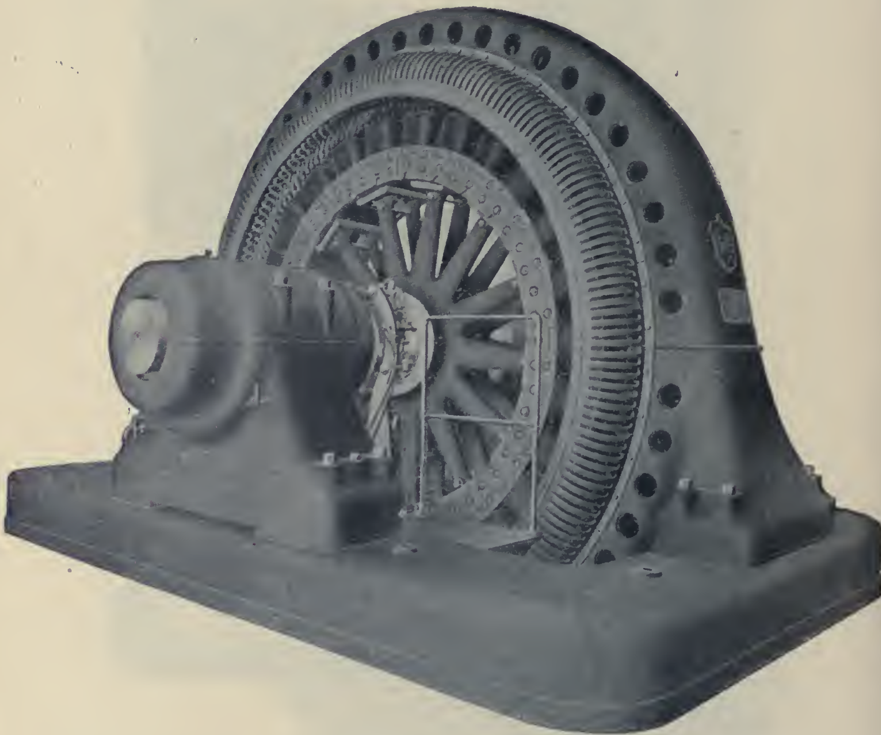


FIG. 510.—3000-KW. Bullock alternator. (Kern River Power Company's Plant, California.)

eddy-currents set up in it. Laminated poles are bolted up to or are cast into the yoke-ring, and in some cases are dovetailed into the rim of the yoke-wheel (Figs. 508–10), a method which has the advantage that the pole may then be drawn out sideways for examination or removal of the bobbin without shifting the armature or magnet as a whole. The section of the pole if laminated must be rectangular, but with solid cast steel poles, although it is seldom possible to employ the most economical section of a circle owing to the necessity of arranging a large number of poles round a wheel of moderate diameter, yet at least an approximation to the best form may be made by giving the poles an oval shape. A considerable saving of copper is thus obtained, and the bobbins are themselves easier to wind. For high peripheral

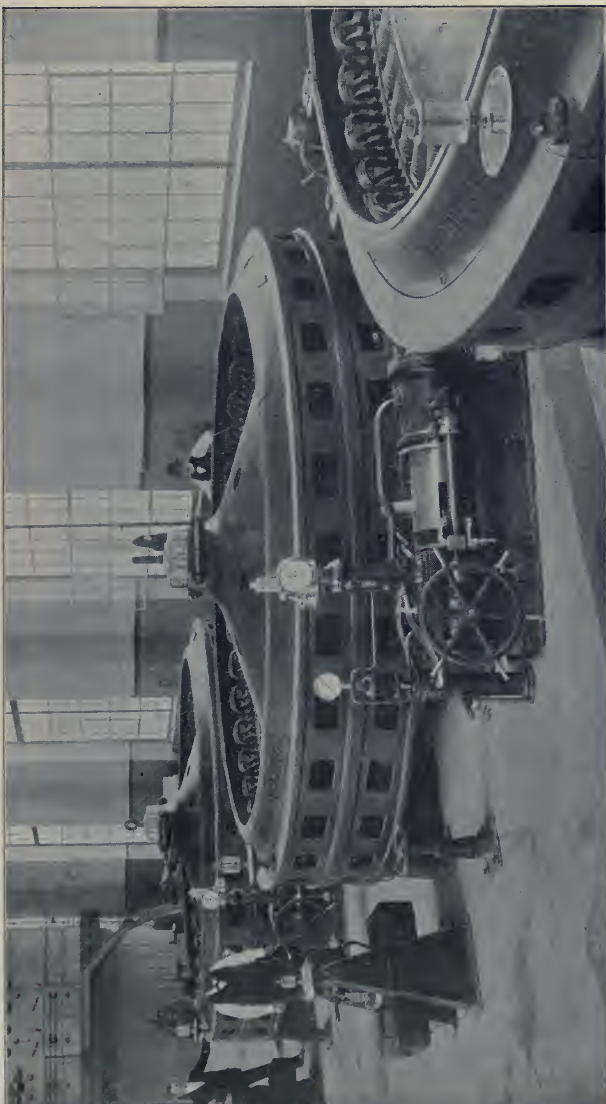


FIG. 511.—Hagneck Electricity Works. (Brown, Boveri & Co.)

speeds the sheet-steel pole has the advantage that the material is mechanically reliable, and can be tested in small packets. The same is also true of its magnetic qualities, as it is very constant in permeability, and hence the use of laminated poles enables a high density to be employed in the magnets with considerable confidence.

The revolving-field heteropolar type when adapted to the vertical shaft of a low-speed turbine is illustrated by Fig. 511, which gives a general view of the 3-phase generators of "umbrella" type by

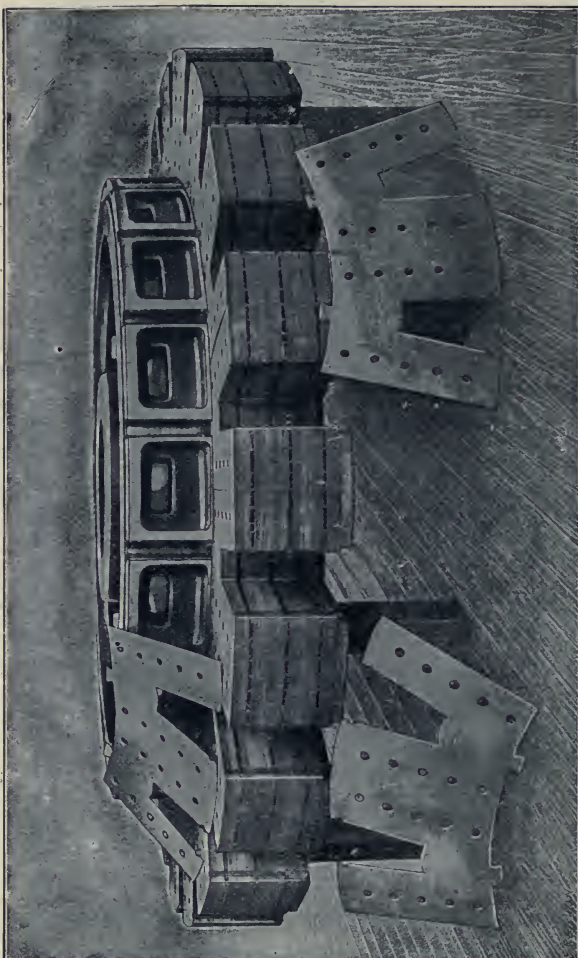


FIG. 512.—Laminated rotor of Westinghouse alternator in course of building up.

Brown, Boveri & Co. in the central station of Hagneck, near Biel. Each of the four machines is directly coupled to a vertical turbine of 1300 H.P., and gives at 100 revs. per min. a voltage of 8000 and a periodicity of 40, the number of poles being 48. The armature casing rests on a foundation having a circular opening of 8 ft. 6 in. diameter, through which when the magnet-wheel has been removed the parts of the turbine can be raised from below.

FIG. 513.—Laminated rotor of Westinghouse alternator, ready to receive the coils.

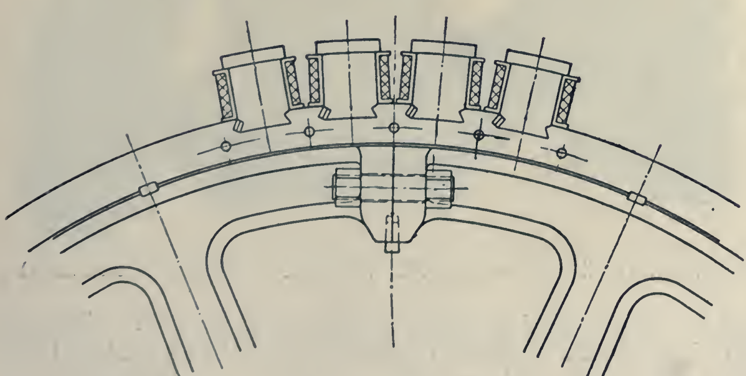


FIG. 514.—Magnet-wheel construction of Allgemeine Elektricitäts Gesellschaft.

While a peripheral speed of 6000 ft. per min. is permissible in the case of a plain cast-iron fly-wheel with solid rim and arms well rounded into the rim, such a limiting speed must be considerably reduced to, say, 4000 ft. per min., if the poles are bolted to the wheel and the

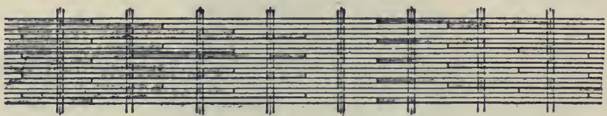


FIG. 515.—Portion of laminated rim of magnet-wheel.

further centrifugal force due to their mass and to the exciting coils which they carry is added to that of the rim itself. Especially must this be the case when the rim of cast iron is divided and has to be locked together. The employment of a cast-steel wheel is then neces-

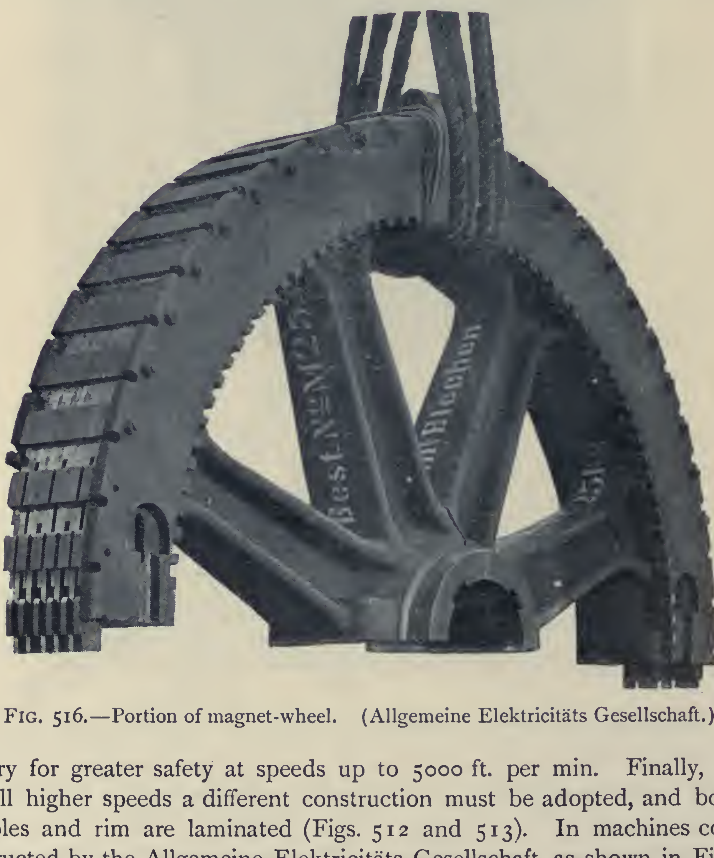


FIG. 516.—Portion of magnet-wheel. (Allgemeine Elektricitäts Gesellschaft.)

sary for greater safety at speeds up to 5000 ft. per min. Finally, for still higher speeds a different construction must be adopted, and both poles and rim are laminated (Figs. 512 and 513). In machines constructed by the Allgemeine Elektricitäts Gesellschaft, as shown in Figs.

514 and 516, the rotor takes the form of an interlocked ring of laminated sheet-steel plates wedged on to the arms of the cast-iron centre-wheel. Each packet of segmental plates is held by several bolts (Fig. 515), and the whole ring is free to expand under the action of centrifugal force without undergoing deformation. The further advantage is gained that the tensile quality of the materials of the plates and the shearing quality of the bolts can be experimentally ascertained, while the stresses in each case admit of close calculation. The pole-pieces are dovetailed into the yoke ring and are fastened by double wedges, these latter being themselves locked by a wire passing through them and bent over in opposite directions at the two ends.

§ 8. **The exciting coils.**—In the case of revolving field-magnets,

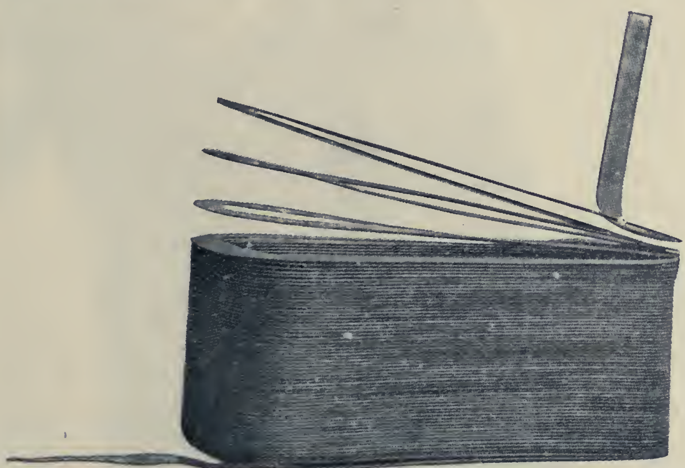


FIG. 517.—Field-coil of flat strip wound edgewise. (Bullock Electric Manufacturing Company.)

owing to their high peripheral speed (4000 to 6000 ft. per min.), the exciting coils are best formed of rectangular copper strip wound on edge with insulation coiled round between the turns (Fig. 517); the effect of the centrifugal force is then only to compress the whole spiral towards the periphery and the insulation does not suffer by chafing. Copper band wound on the flat is not so economical, since a number of turns of thin sheet nearly equal in width to the length of the poles implies a considerable amount of space lost in insulation. Round wires should never be employed on rotating magnets, owing to the danger of the wire slipping on itself and damaging the insulation. Coils of rectangular or square wire necessitate insulating bobbin cases, and are not so self-supporting as spirals of copper strip. The coils or their cases are held in place by the overhanging edges of the pole-shoes, or by special gun-metal bridges, which also serve in some measure to steady the machine for parallel working by their damping action (Fig. 518). The current-density in the field wire usually ranges from 1300 to 1900

FIG. 518.—Portion of rotating field of Westinghouse alternator showing coils in place.

amperes per sq. in., or higher than in continuous-current dynamos, owing to the greater cooling action of the rotating magnet. With rotating field-coils the influence of the peripheral speed is more than in Fig. 389, though less than in Fig. 390, owing to the greater depth of winding. The power of the peripheral speed which should be employed is, however, nearly the same as in Fig. 390, equation (145), but with a lesser coefficient of say 0·35 instead 0·7. The coefficient giving the mean rise of temperature of an ordinary bobbin wound with d.c.c. wire in a number of layers, and therefore with considerable depth of winding, may approximately be given in terms of degrees Fahrenheit and sq. ins. of cooling surface as

$$k = t^{\circ}\mathrm{F} \times \frac{S_c}{W} = \frac{260}{1 + 0{\cdot}35\left(\frac{v}{1000}\right)^{1.3}}$$

For v must be taken the mean peripheral speed at the centre of the height of the bobbin, and the cooling surface is to be reckoned exactly as in Chapter XIX. § 9, including the end-flanges. At 4000 ft. per min. a ratio of about 1·1 to 1·4 sq. ins. per watt will give a mean rise of 75° to 60° F. With flat copper strip presenting bare edges to the air, the initial coefficient can be considerably lowered. The voltage of the exciting current varies according to circumstances from 50 to 100 volts, yet good insulation of the entire circuit of coils and exciter is necessary, since owing to the great inductance of the circle of bobbins there is a considerable strain on the insulation when the circuit is broken; indeed, on this account the opening of the circuit should be effected through a non-inductive shunt and carbon-break switch.

§ 9. **Turbo-alternators.**—Many of the considerations alluded to in Chapter XX. § 9 in connection with continuous-current turbo-dynamos apply with equal force to turbo-alternators, but, owing to the absence of the commutator in the rotor and the easier design of the exciting coils with their comparatively low voltage, an even higher peripheral speed may be employed. A maximum limit is 17,000 ft. per min., which is rarely exceeded. Every lb. at the limiting speed of 17,000 ft. per min. and at 1 ft. radius, as might occur on the surface of a rotor of 2 ft. diameter revolving at 2700 revs. per min., exerts a centrifugal force of 2500 lbs., or over a ton. Or, again, by formula (29) a field pole and coil weighing 500 lbs., rotating at 1500 revs. per min. with a mean radius of 1 ft., exerts a centrifugal force of 17 tons, which must be withstood by its attachment to the hub or central core. Such figures as the above illustrate the forces that may have to be dealt with, so that the proper support of the poles and their exciting coils is evidently of the first importance.

The rotating element is universally the field-magnet structure, and two well-defined types of construction may be distinguished. In the one there are separate projecting poles, each with its magnetising bobbin, much after the fashion of the slow-speed alternator, but with special precautions for the firm holding in place of the coils. In the other the cylindrical rotor has a distributed field-winding embedded in a number of slots running along the core, so that when finished a perfectly smooth surface is obtained. For very high speeds the latter type, which was first introduced by Mr. C. E. L. Brown, and largely employed by Messrs. Brown, Boveri & Co., is to be preferred, owing to its more silent running and to its subdivision of the centrifugal force of the field-winding between a number of teeth; the solidity of the structure renders it simple and safe, and permanency of balance is more certainly secured than is the case with definite projecting pole-pieces and external field-bobbins.

Various methods of fixing the poles and coils of the first type are illustrated in *Journ. Inst. Electr. Eng.*, vol. xxxvii. pp. 336–338 (A. G. Ellis, "Steam Turbine Dynamos"), and an example of the calculation of the mechanical stresses in the poles and hub is given (p. 339). The pole-shoe may require to be relieved of some portion of the centrifugal force from the field coil by subdividing the latter into sections, the centrifugal force from the inner sections being transmitted directly to the pole-core. In the definite-pole construction formerly employed by the General Electric Company of Schenectady, N.Y., and by the British Thomson-Houston Company, the centrifugal force from the ends of the field coils is taken by wrought-iron straps embracing the coil and anchored to the hub, with gun-metal angle brackets to hold the exciting bobbins fast sideways. The constructional details with mechanical, electrical, and magnetic data of a 1500-kw. alternator at 1000 revs. per min., manufactured by the British Thomson-Houston Company, are given in *Electrician*, vol. lvi. p. 499.*

An intermediate type is found in the 4-pole magnet of the British Westinghouse Company, in which a solid ingot-steel magnet with four projecting poles has a number of slots milled along the edges of the poles wherein a flat copper strap is wound.

The smooth-core construction of rotor as employed in recent large turbo-alternators by the General Electric Company of Schenectady is illustrated in Fig. 519, which shows the field-magnet for a 4-pole machine giving 8000 kw. at 750 revs. per min. and 25 cycles, supplied to the New York Edison Company. The core is again a sheet-steel laminated structure, but the field-coils are embedded in slots formed by sliding in dovetailed wedge sections, as seen in Fig. 520, which shows the rotor of a 5000-kw. 4-pole 750 revs. per min. turbo-alternator in process of construction. The end-connections of the distributed field coils are held in place by hoops bolted through to the end-cylinder of the hub. Fig. 521 shows in part section a 9000-kw. 4-pole alternator running at 750 revs. per min. and 25 cycles with vertical shaft for direct driving by a Curtis steam turbine; the outside diameter is 10 ft. 10 in.,

* H. S. Meyer, "Design of Turbo-Alternators," to which the reader is especially referred.

and of the rotor is 6 ft. $3\frac{1}{4}$ in., the bore being approximately 6 ft. 6 in. The over-all length of the cylindrical body of the stator is 8 ft. $1\frac{1}{2}$ in. Four such machines have been supplied by the General Electric

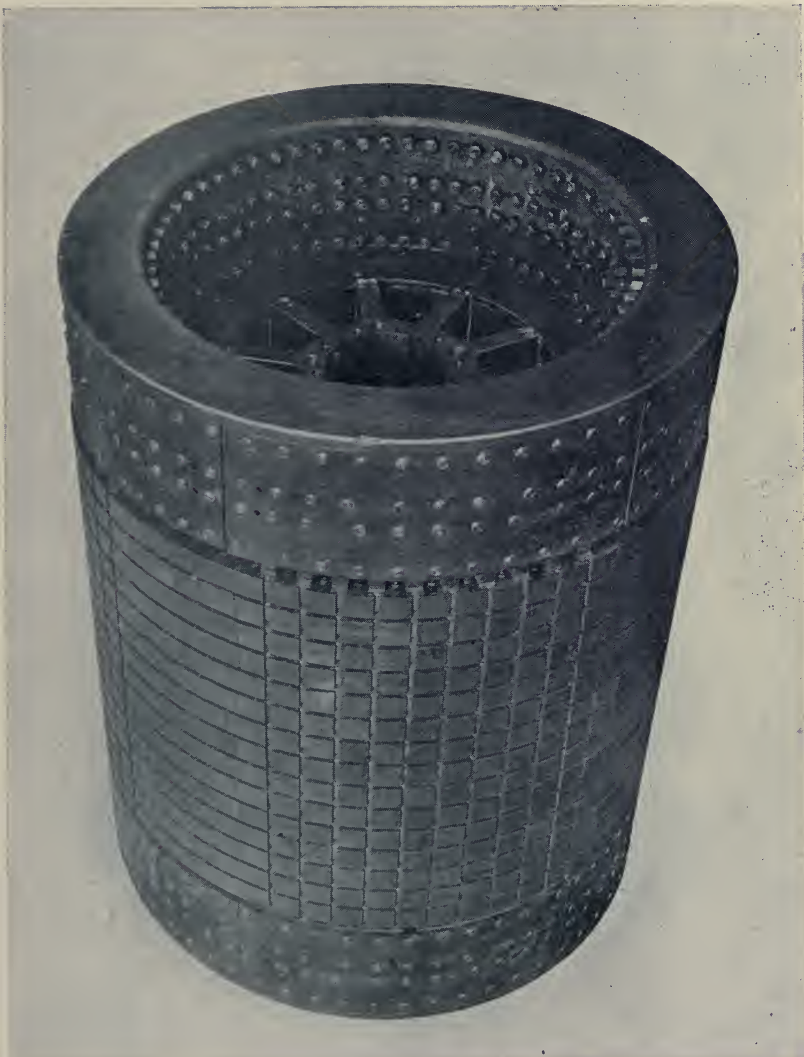


FIG. 519.—Smooth-core rotor of 8000-kw. turbo-alternator of General Electric Company of Schenectady, N.Y.

Company to the Fisk Street Station of the Commonwealth Electric Company of Chicago. The outside casing of the stator is here a closed casting for the purpose of deadening the sound of running, and at the same time of directing the flow of air drawn through by the rotor along the required channels formed by the ventilating ducts, until it is discharged outwards. The importance of mechanically supporting the end connections of the armature coils of the stator against the effect of a

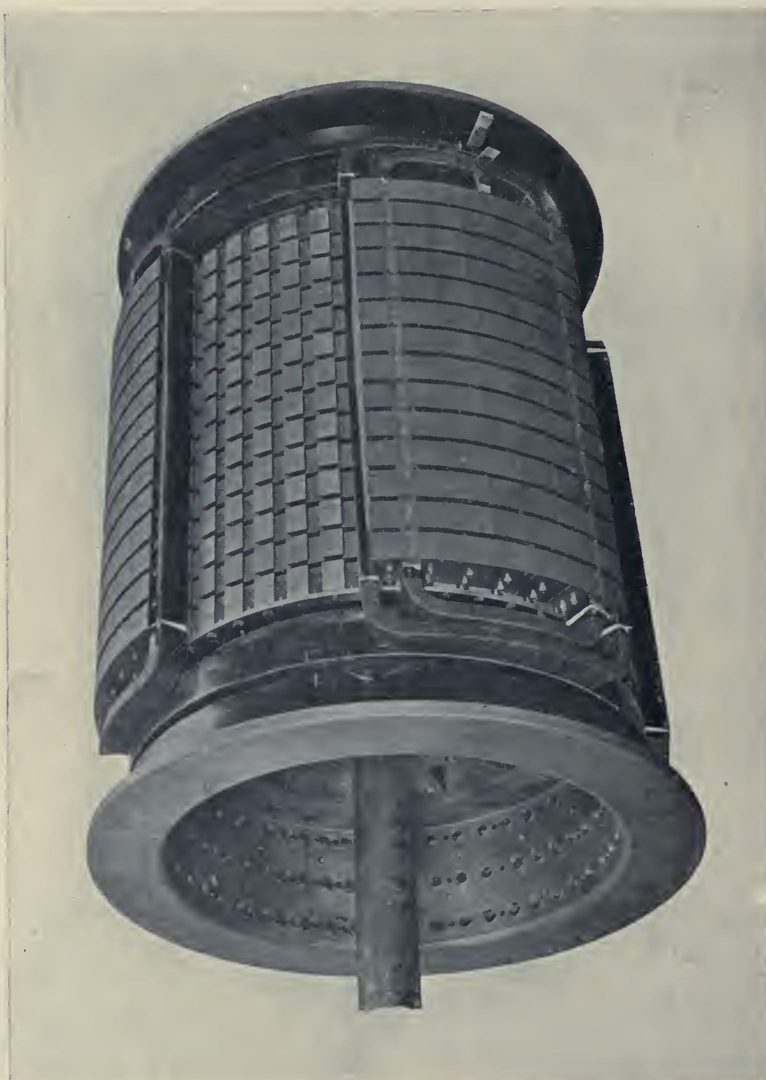


Fig. 520.—Smooth-core rotor of 5000-kw. turbo-alternator during construction. (General Electric Company of Schenectady, N.Y.)

short-circuit has already been emphasised in the case of turbo-alternators in which the coils have a wide span. Fig. 522 shows part of the stator of a 4-pole 3000-kw. 25-cycle 750 revs. per min., 13,200 volt 3-phase alternator built by the General Electric Company for the Chicago and Milwaukee Electric Railroad Company, from which it will be seen that the end-connections are bound down to an internal ring by means of an external strap.

The mechanical support of the end-connections of the armature coils

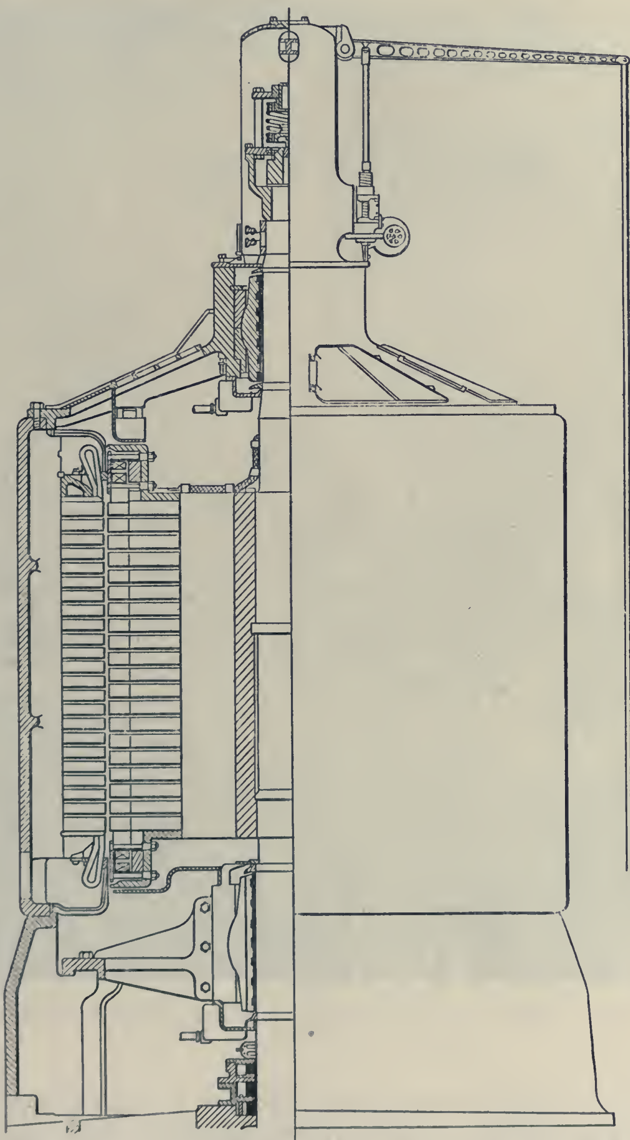


FIG. 521.—9000-kw. alternator, driven by vertical Curtis steam-turbine. (General Electric Company of Schenectady, N.Y.)

is again illustrated in Figs. 523 and 524, which show the armatures (with end-shields removed) as constructed by Messrs. Brown, Boveri & Co. for turbo-alternators, the former being that of a 3-phase

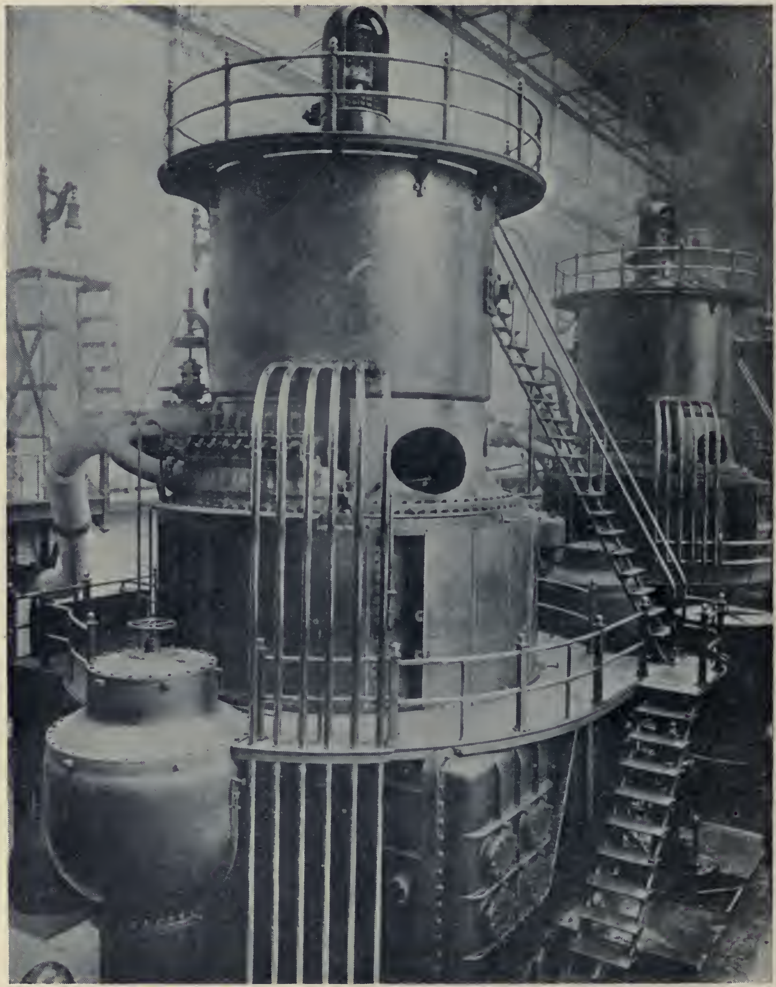


FIG. 521A.—9000-kw. alternator and Curtis turbine of General Electric Company. (Fisk Street Station, Chicago, U.S.A.)

machine of 5000 kw. 10,500 volts, 50 cycles at 1000 revs. per min., built for the Oberspree Electricity Works at Berlin. The smooth-core rotor of Messrs. Brown, Boveri & Co. is shown in Figs. 525–7, the former being a solid steel cylinder with slots milled in its surface for the reception of the field-coils in a small machine, while in larger machines the core is built up out of steel plates. The field-coils are of rectangular copper strip insulated lightly between neighbouring turns by press-spahn

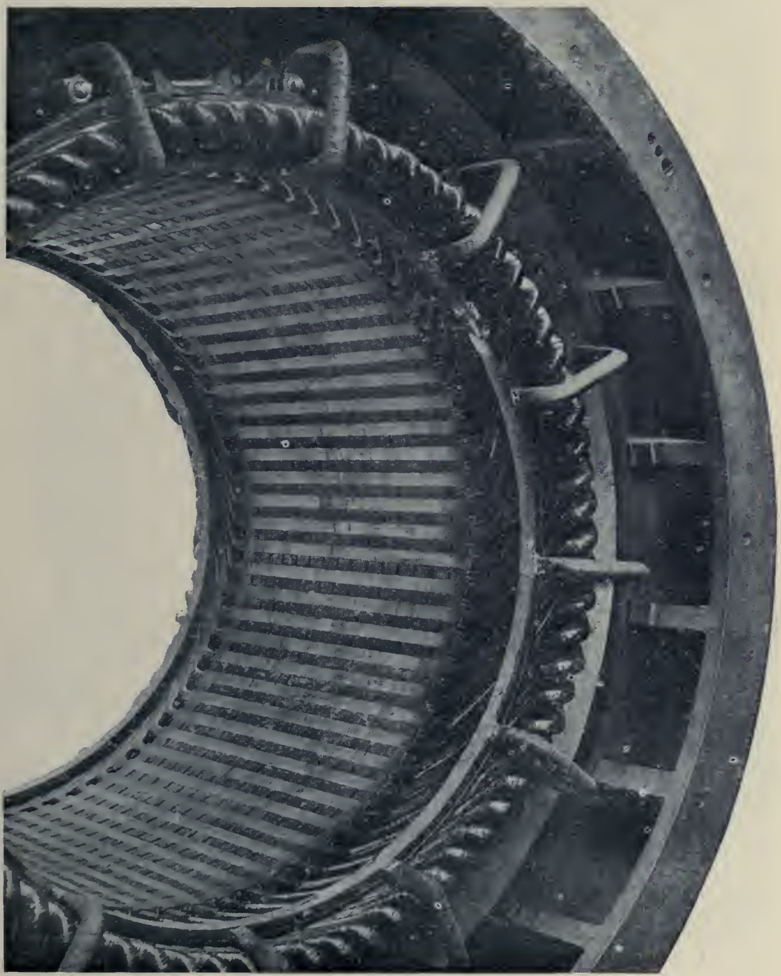


FIG. 522.—End view of armature of 3000-kw. turbo-alternator (General Electric Company of Schenectady, N.Y.)

or cotton covering with heavy insulating envelope within the slots, and fastened in place with metal wedges. Strong end-shields confine the end-windings, and a permanent balance of the whole is obtained. Total enclosure of the machine is adopted (Figs. 528 and 529), and a definite system of ventilation * is secured by arranging an air inlet at the bottom, and discharging the air after passing through the rotor and round the

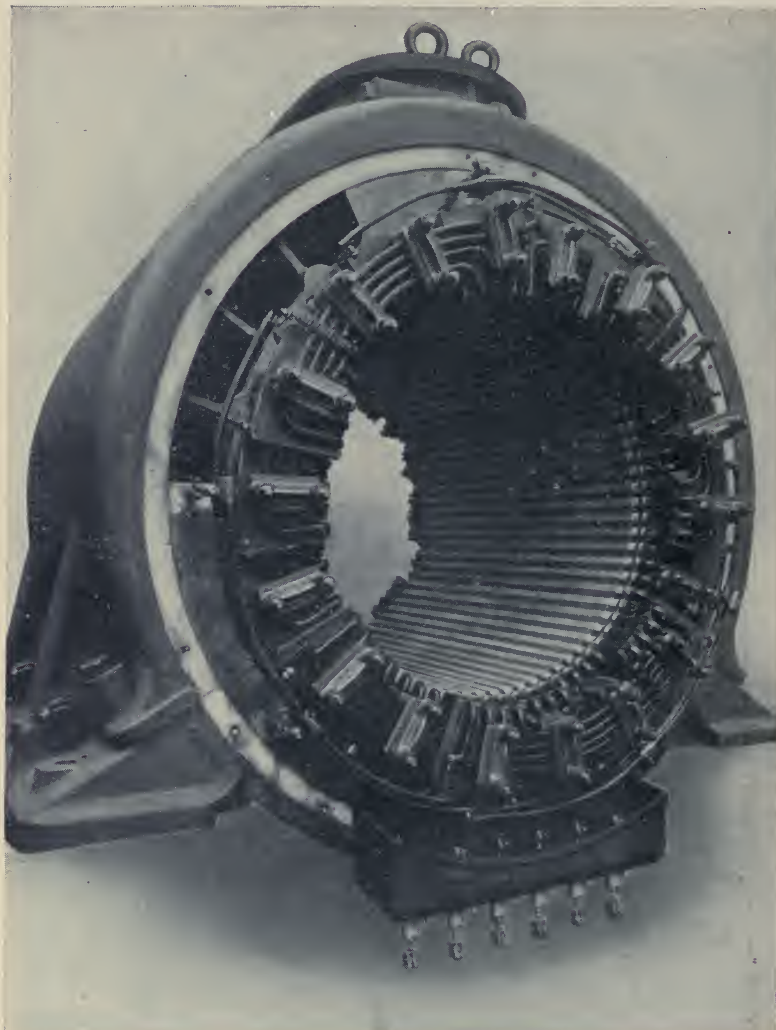


FIG. 523.—End view of armature of 5000-kw. turbo-alternator of Brown, Boveri & Co.

stator coils at a single chimney opening at the top. In Fig. 523 the large 6-pole machine has the casting of the stator frame divided into

* For the need of artificial ventilation in connection with turbo-alternator, see Dr. M. Kloss, *Journal Inst. Electr. Eng.*, vol. xlii. pp. 157–162. Cp. the descrip-

three pieces, so that by removing the bolts the side portions can be swung away for examination of the windings of field and armature. The longitudinal grooves in the hub by which the air is drawn through the numerous ventilating ducts of the rotor are seen in Fig. 527. The peripheral grooves in the core serve to bring the air into intimate contact

FIG. 524.—End view of armature of Brown, Boveri & Co.'s turbo-alternator.

with the field-windings, so that an even temperature rise of the whole results. The slip rings for the exciting current are disposed one at each end of the field-magnet, and are of cast steel.

tion of a 3200 KVA turbo-alternator by Messrs. Kolben & Co. (*Electr. Eng.*, vol. xliii. p. 438, from *E.T.Z.*, Feb. 18, 1909), which was artificially ventilated by forced draught.

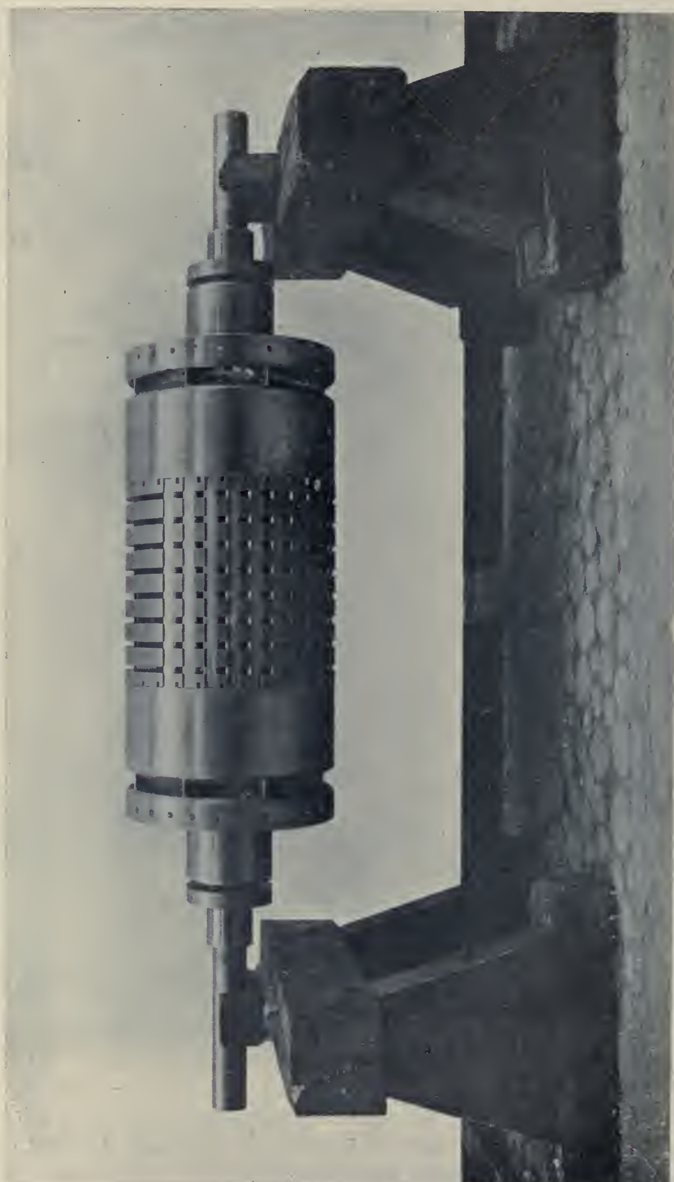


FIG. 525.—Solid smooth-core rotor of Brown, Boveri & Co.

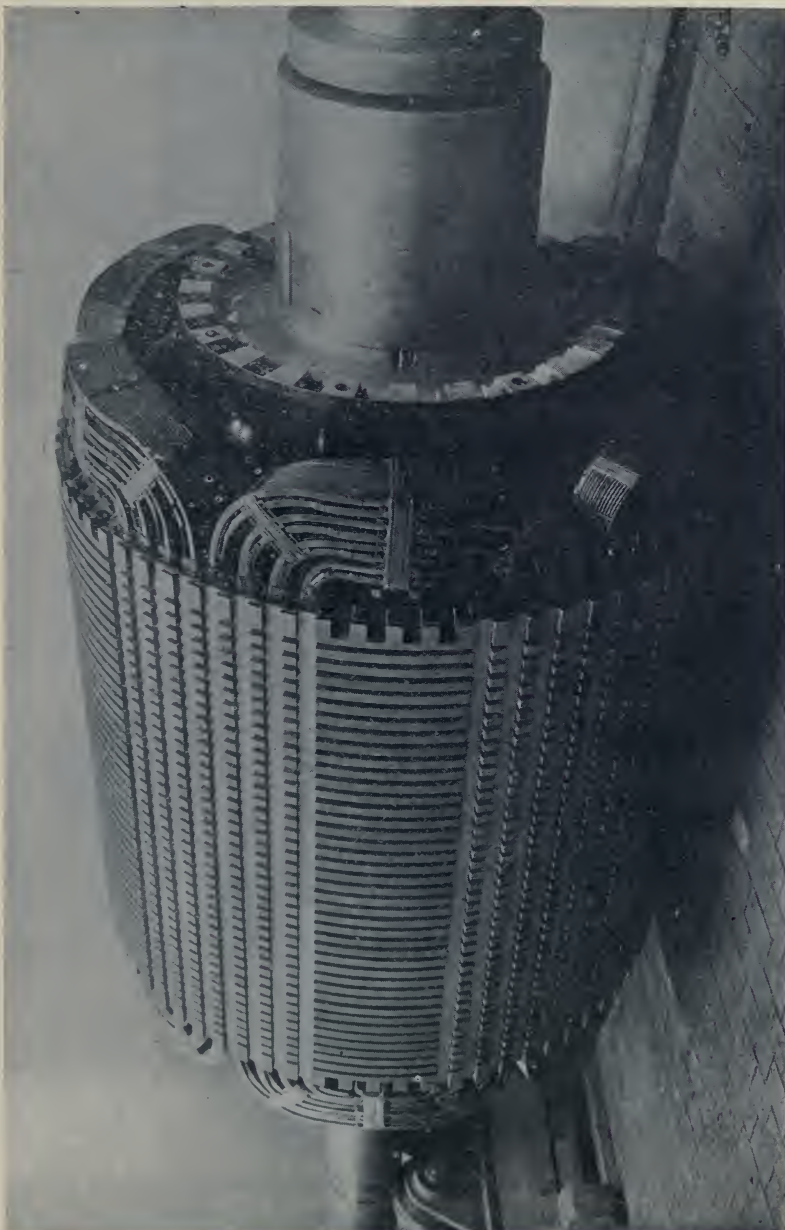


FIG. 526.—Rotor of large turbo-alternator (with end-shields removed) of Brown, Boveri & Co.

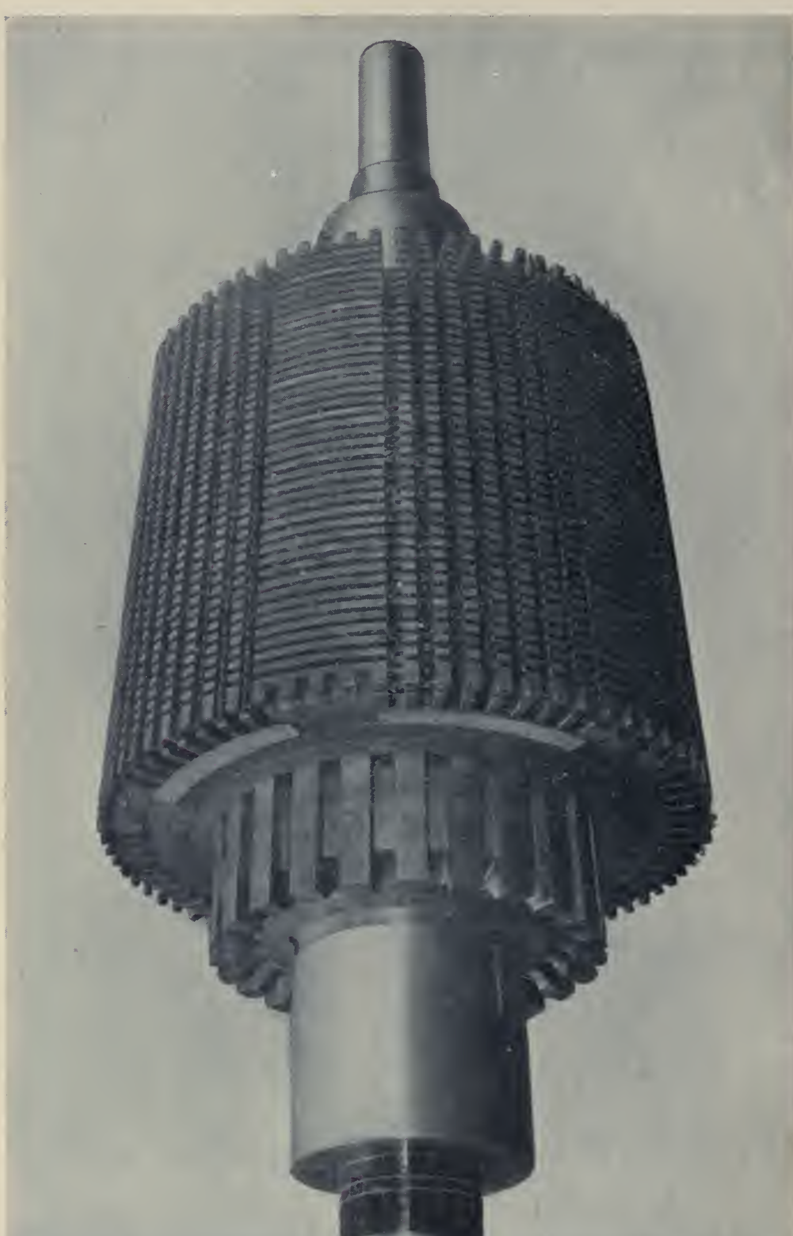


FIG. 527.—Rotor of large turbo-alternator before winding.

A similar totally enclosed construction is employed in the turbo-alternators of Messrs. Siemens Bros. Dynamo Works, with air drawn

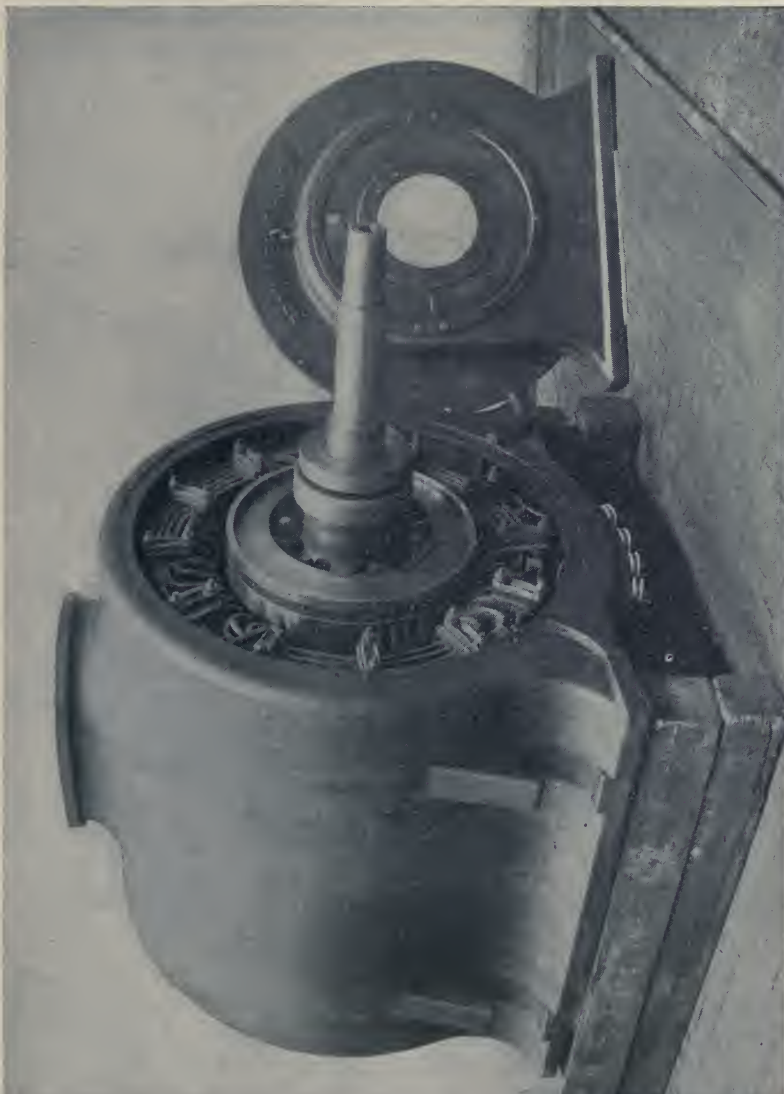


Fig. 528.—Turbo-alternator with end-cover. (Brown, Boveri & Co.)

through a trunk at the end remote from the turbine by means of fans forming part of the rotor core-heads.*

* For description of other turbo-alternators, cp. *Electrical World*, vol. li. p. 64 (Allis-Chalmers, with smooth-rotor); *Electr. Eng.*, vol. xli. p. 406 (Electric Con-

§ 10. **Methods of field excitation.**—The field-magnet system of the alternator must necessarily be excited by a continuous current. Separate excitation by an auxiliary continuous-current dynamo or *exciter* is therefore almost universally employed, its great recommendation being that the exciting current can readily be altered by means of a rheostat R in series with the field-winding so as to maintain the alternating voltage at the required value when the load varies. If a separate

FIG. 529.—Totally enclosed turbo-alternator. (Brown, Boveri & Co.)

small exciter is used for each machine its armature may be driven directly from the alternator shaft (Figs. 492, 494, and 500), or through rope or belt gearing. The method then, however, has the disadvantage that any change of speed of the prime mover also causes a change of speed and of voltage in the exciter. Hence it is better that the exciter

struction Company's 1000 kw. with smooth rotor), vol. xxxix. p. 786 (Dick, Kerr & Co.'s 2000 kw. with salient poles), vol. xxxix. p. 118 and vol. xliii. p. 438 (where ull particulars are given of 970 KVA. 50 ∿ 3-phase 1500 revs. and of 3200 KVA. 49 ∿ 980 revs. turbo-alternators manufactured by Messrs. Kolben of Prague).

should be driven by a separate steam engine or turbine, in which case one machine will serve to excite several alternators in combination, each having its own field-regulating resistance. The alternating current supplied by the alternator itself can also be used to excite the field if *rectified*, or converted into a direct form; the machine can then be

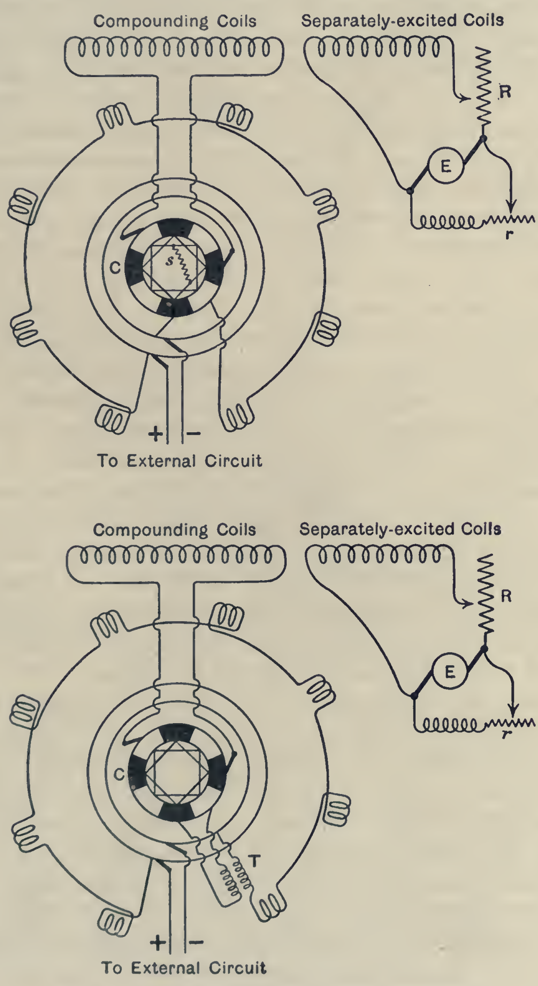


FIG. 530, i. and ii.—Compounding of alternator.

made self-exciting, or if given an initial excitation from a separate source exercises a compounding effect which increases with the load just as in the compound-wound continuous-current dynamo. Two sets of magnet bobbins are then required, the one separately excited and the other self-excited by the armature current (Fig. 530, i.). The rectifying commutator C has as many sectors as there are poles, and alternate sectors are connected together to form two separate sets into and out of which the alternating current is led. A pair of brushes is then arranged to press upon the commutator at two points corresponding to zero value of the current. Thus the circuit between the two sets of sectors is completed through the brushes and the series or compounding coils; the connections of these latter with the sectors are reversed at each reversal of the alternating current, so that the current through them is unidirected. It is not, however, necessary to rectify the whole of the armature current; a portion only as derived from a branch circuit may be used. Thus in Fig. 530, i., if the sets of sectors are connected across by a resistance *s* as shown, only a part of the whole current is shunted through the brushes. Further, the whole of the voltage of the alternator need not be impressed upon the rectifying commutator, but only that of one or two coils. Even then the voltage may be too high to be convenient, so that the alternating current may be passed through a transformer (Fig. 530, ii.), and the secondary current thus obtained be rectified. The commutator must be attached to the rotating portion of the alternator, but the transformer T may be independent of the machine if the field-magnet rotates and the armature is stationary. If the width of the brushes is less than the width of the insulation between the sectors, there must be some sparking owing to the discontinuity of the circuit and the inductance of the field-winding. On the other hand, if the width of the brushes be greater than the width of the insulation between the sectors, both the supply circuit and the field-circuit are short-circuited at the moment when the brushes bridge over the insulation. Various devices have been tried to obviate these difficulties, but on the whole have not found much favour in practice.

Automatic compounding can also be applied to polyphase machines. Thus in a 3-phase alternator each of the three currents may be taken through a transformer, the secondary of which feeds the rectifier; the compounding effect is thus dependent upon the combined currents of all the three phases. Or the common junction of the star winding may be replaced by a small delta, and the three terminals of the delta are then connected to the sectors of the rectifier which are united into three sets.* The result is that some small portion of the current is shunted through the rectifier, and as the phases are each represented separately, if the currents in the three lines are not equal or the circuits are not balanced, the regulation is but little affected.

§ 11. **Compounding of alternators.**—Although the self-exciting principle has been used with some success for compounding purposes in the case of small machines, the problem cannot be said to have been satisfactorily solved. While the series turns may be adjusted to suit

* Cp. Atchison, "Some Properties of Alternators under Various Conditions of Load," *Journ. Inst. Electr. Eng.*, vol. xxxiii. p. 1091.

the normal load and normal load-factor so as to maintain the voltage fairly accurately, there remains the difficulty that the compounding effect is really required to vary not merely with the value of the current, but also with the angle of lag of the current according as the external load is inductive or non-inductive. Hence an alternator which is adjusted for a load of lamps is not so well adjusted for a load of motors, and will require an alteration of its separate excitation.

Various attempts have been made to meet this difficulty, among which may be mentioned the self-exciting alternator invented by Mr. E. F. Alexanderson,* which possesses many novel merits, and the "compensated exciter" † devised by Mr. E. W. Rice, in which the alternating armature current is caused to react upon the field of the exciter to an extent depending not only upon its strength, but also upon its phase relatively to the E.M.F. The exciter armature is built upon the same shaft as the rotating field of the alternator, and has the same number of poles so as to give the same frequency. In addition to the collecting rings by which the continuous current is supplied to the field-winding of the alternator, the exciter armature winding is joined to three collecting rings through which the 3-phase alternating current from a transformer in series with the main circuit is passed. The reaction of the latter current upon the field of the exciter causes its voltage to rise not only with increase in the main current, but also with increase of its angle of lag.

Different principles of compounding ‡ are again to be found in the exciter due to M. Leblanc, and exhibited in connection with the Grammont alternator at the Paris Exhibition of 1900, and in the Boucherot alternator exhibited by MM. Breguet et Cie., the latter being of special interest from its being designed for operation as an asynchronous generator.

The asynchronous generator or, as it is more properly described, "induction generator" (since it is an induction motor mechanically driven above the speed of synchronism), may eventually find a useful field in connection with steam or water turbines, as the simple and strong construction of its squirrel-cage rotor renders it specially suitable for high speeds. Since it also requires a large synchronous machine such as a converter to serve as an alternating exciter, and the latter must itself be excited by a direct current, the expense of the whole plant has up to the present prevented its commercial adoption.§

* Described in *Trans. Amer. Inst. Electr. Eng.*, vol. xxv. p. 61.

† Fully described in *Electrical World*, vol. xxxvii. p. 676; cp. also Atchison, *Journ. Inst. Electr. Eng.*, vol. xxxiii. p. 1092; and Miles Walker, *Journ. Inst. Electr. Eng.*, vol. xxxiv. p. 424.

‡ Both forms are described by M. Guilbert in the *Electrical World*, vol. xxxvii. pp. 302 and 352.

§ But cp. W. L. Waters, "The Non-synchronous Generator in Central Station

If the stator coils of an induction motor are connected across mains between which an alternating voltage is maintained, and if further an engine is coupled to the rotor of the motor, by which its speed may be raised, the first result of so raising its speed is to reduce the current taken by the stator and to cause the driving torque from the motor to decrease until, when it is running at the exact speed of synchronism, no torque is exerted by the rotor and the engine simply supplies the losses from friction and windage, while the stator coils take only a magnetising current from the mains. But when the speed is still further raised by the prime mover above that of synchronism, so that there is a negative slip and the rotor runs faster than the rotating magnetic field, currents are generated in the rotor at this slip frequency, and these in turn will induce watt currents in the stator, in phase with the voltage of the mains, and supplying part or the whole of the load on the mains. It is evident that for the operation of the system there must be some synchronous machine (either generator, motor, or converter) already running on the mains to supply the wattless magnetising current that the machine is itself unable to supply, and to set the voltage and frequency of the system.*

After devising a method of making the asynchronous generator self-exciting and of compounding it, M. Heyland developed a compounded synchronous machine, a description of which is given in Mr. Atchison's paper (pp. 1104–1109), and also in Mr. Miles Walker's paper on "Compensated Alternate Current Generators," *Journ. Inst. Electr. Eng.*, vol. xxxiv. pp. 417–423. The whole subject of compounding alternators is discussed in the latter paper, together with a description of Mr. Walker's own method, which has been adopted in many cases by the British Westinghouse Electric and Manufacturing Company. It will therefore suffice to refer the reader to the above two papers and also to M. Heyland's paper, "Recent Developments in Compounded Alternators" (*Trans. Intern. Elect. Congress*, *St. Louis*, 1904, vol. i. p. 762), for further information on many of the points which arise in connection with the question of compounding alternators.

More recently M. Heyland has proposed a method by which the variation in the amount of the primary leakage under different conditions of load is caused to react directly upon the flux of the exciter, so that its voltage and exciting current vary in the required manner.†

and other Work," *Trans. Amer. Inst. Electr. Eng.*, vol. xxvii., February 1908, together with the discussion ; W. Stanley, "Induction Alternators," *Trans. Amer. Inst. Electr. Eng.*, vol. xxiv. p. 851.

* The action of the inductor generator, together with M. Heyland's method of rendering it self-exciting and of compounding it, is more fully described by A. F. T. Atchison, "Some Properties of Alternators under various Conditions of Load," *ourn. Inst. Electr. Eng.*, vol. xxxiii. pp. 1093–1104.

† *Electr. Eng.*, vol. xxxviii. p. 656 ; *Electrician*, vol. lviii. p. 42 ; cp. W. Cramp, *Journ. Inst. Electr. Eng.*, vol xxxviii. p. 569 ff.

§ 12. **The choice of frequency.**—The frequencies for which alternators are commercially designed range from 100 to 25, and even lower values are beginning to be considered in special cases. Indeed, it may be said that, while in the early days of electric lighting frequencies of 125 or 133 which suited the transformers well were not uncommon, these are no longer now in use, and the general tendency is towards moderate frequencies, by which may be understood any value below 60. When the alternators are driven by belt at a considerable speed a high frequency can be obtained without the necessity for a large number of poles; hence the machines are economical in first cost, and so also are the transformers for use in connection with them on lighting circuits. On the other hand, the drop of voltage over the transmitting lines is increased owing to their greater reactance with the higher frequencies, and when the generators are of large size and are directly connected to the shaft of the steam engine or water turbine wheel, the large number of poles that is required make the machines somewhat expensive in labour and in copper or exciting energy; further, they do not regulate so well for constant potential. An additional disadvantage in the case of engine-driven sets is that at high frequencies they do not work so well in parallel; the maximum angle by which it is permissible for the rotor to be mechanically displaced from the position corresponding to perfectly uniform speed of rotation varies inversely as the number of poles, and if the machines for high frequencies are to be run in parallel the engines must generally either be better balanced as regards their moving parts or must have larger and more expensive fly-wheels. If the alternators are to be used for the supply of energy to polyphase induction motors, still another reason is found for the adoption of a low frequency, since with high frequencies the power-factor and efficiency of the motors are less, at least if their speed is to be moderate. Lastly, if rotary converters form a large part of the load, uniformity of the angular velocity becomes of special importance in order to prevent hunting.

In the other direction there are, however, again certain limits. Owing to the periodic heating and cooling of the carbons in the alternate-current arc the eye can detect and is fatigued by the variation in the light if the frequency is less than 40, and even in the case of incandescent lamps the same effect becomes noticeable with frequencies below 30. Thus for systems combining the supply of both light and of energy to motors, and in which the former is an appreciable part of the whole, a frequency of 60 is often adopted. Except on very long lines the reactance is then moderate, and the regulation of voltage is good, while polyphase induction motors can be built to run well and at reasonable speeds with such a standard. Rotary converters of even large size can also be operated successfully at 60 cycles unless the alternators are directly coupled to very low-speed engines. Both induction motors and converters are, however, better suited by lower frequencies of 40 and 25 cycles respectively, so that, if they form the principal part of the load and complete steadiness of light is not of first importance, as in factory lighting, a frequency of 50 or 40 affords a better compromise for polyphase plants, and the former is in general use on the Continent. Finally, if the whole of the energy is to be supplied to motors of several hundred horse-powers and rotary converters of very large size, or if the transmitting lines are so long as to have considerable capacity, a frequency of 25 is to be recommended. Such a value was adopted by the engineers for the transmission of energy at Niagara, where rotary converters of large size are extensively used, and the results have been such as to amply justify the choice of what was then considered an abnormally low frequency. In consequence a frequency of 25 has now become a standard for similar cases.

For heavy railroad work, single-phase alternating motors have a better power-factor and are more easily designed for sparkless running at 15 cycles per second, but on the other hand this frequency does not suit the steam turbo-alternator, and it remains at present an open question how far the advantage on the motor side will warrant the adoption as a standard of $16\frac{2}{3}$, 15, $12\frac{1}{2}$, or 10 cycles.*

* A. H. Armstrong, "The Choice of Frequency for Single-Phase Alternating-Current Railway Motor"; and N. W. Storer, "25 v. 15 Cycles for Heavy Railways," *Trans. Amer. Inst. Electr. Eng.*, vol. xxvi. part ii. pp. 1377 and 1385, with the discussions thereon. For the opposite extreme of very high frequencies, cp. the descriptions of alternators by B. G. Lamme, *Trans. Amer. I.E.E.*, vol. xxiii. p. 417 (frequency 10,000), and E. F. W. Alexanderson, *Proc. Amer. I.E.E.*, June 1909 (frequency 100,000 for wireless telegraphy).

CHAPTER XXV

ALTERNATOR DESIGN

§ 1. **Preliminary calculation of dimensions of alternator.**—The starting-points in the design of an alternator for a given output are usually our knowledge of the desired frequency and of the speed of the driving engine or turbine, whence the number of pairs of poles is at once fixed from the equation $f=\frac{pN}{60}$. The peripheral speed in the case of a rotating magnet may vary between 4000 and 6000 ft. per min. (20 to 30 metres per sec.), and even when no portion of the copper winding revolves, seldom exceeds 8000 ft. per min. (40 metres per sec.) except in the case of steam-turbo-alternators. If then V be the peripheral speed in ft. per min. which is assumed as permissible, the diameter in ft. is given by the equation $D'=\frac{V}{\pi N}$, and the pole-pitch can be thence approximately determined. The power-factor of the circuit on which the alternator is to work is either given or is known from previous experience, and the permissible drop or rise in volts is usually specified, so that $E_e=\eta E_o$, where η may be, say, 0·9 or 0·8.

From the approximate equation (178) applicable to the intermediate case of a partially inductive load, or

$$\eta E_o = \sqrt{E_o^2 - e^2 \cos^2 \phi_e} - e \sin \phi_e$$

we have

$$e \sin \phi_e + \eta E_o = \sqrt{E_o^2 - e^2 \cos^2 \phi_e}$$

Squaring and solving the quadratic equation,

$$\frac{e}{E_o} + \eta \sqrt{1 - \cos^2 \phi_e} = \sqrt{1 - \eta^2 \cos^2 \phi_e}$$

$$\frac{e}{E_o} = \sqrt{1 - \eta^2 \cos^2 \phi_e} - \eta \sqrt{1 - \cos^2 \phi_e} = Q$$

where $\frac{e}{E_o}$ is the ratio of the synchronous impedance voltage at full-load to the open-circuit voltage for the same excitation.

The numerical ratio Q can be worked out and tabulated once for all as below.* As η decreases, *i.e.* as the permissible drop increases, Q increases; as the power-factor decreases, Q decreases.

* Fischer Hinnen, *Electr. Eng.*, vol. xx. p. 629.

TABLE OF VALUES OF Q.

$\cos\phi_e$	Values of $\eta=$				
	0·95	0·9	0·85	0·8	0·75
1·00	0·31	0·436	0·525	0·600	0·66
0·95	0·134	0·237	0·325	0·400	0·467
0·9	0·107	0·195	0·274	0·347	0·41
0·85	0·09	0·171	0·245	0·311	0·376
0·8	0·08	0·155	0·221	0·287	0·35
0·75	0·073	0·142	0·210	0·272	0·33
0·7		0·131	0·199	0·254	0·314
0·65		0·124	0·188	0·246	0·304
0·6		0·12	0·180	0·235	0·291
0·55			0·172	0·228	0·280
0·5			0·167	0·222	0·274

The necessary value of Q having been thus determined from the data of the specification, it remains to secure a corresponding result in the actual construction of the machine. Now, the essential basis of the synchronous-impedance method is that, approximately speaking, as the short-circuit current at full excitation stands to the open-circuit voltage, so does the full-load current stand to the full-load synchronous impedance voltage, *i.e.*

$$I_o : E_o :: I : e$$

or

$$\frac{I}{I_o}=\frac{e}{E_o}=Q$$

Thus the reciprocal of Q is the multiple which the short-circuit current must be of the known full-load current. Finally, since $I_o=\frac{X}{\lambda t}$ and $t=\frac{\tau}{2mp}$

$$\frac{e}{E_o}=Q=\frac{I\,.\,\lambda t}{X}=\frac{I\cdot\frac{\lambda}{m}\cdot\frac{\tau}{2}}{pX}$$

where X is the full-load excitation which on open circuit is assumed to give $E_o=\frac{E_e}{\eta}$. Therefore $\frac{I\cdot\frac{\tau}{2}}{pX}$ or the ratio $\frac{\text{effective AT on armature}}{\text{total AT on field}}$ must be $Q\,.\,\frac{m}{\lambda}$, or the *ampere-wires per pole* must be

$$\frac{I\tau}{2p}=X\,.\,\frac{Q\,.\,m}{\lambda} \qquad (184)$$

Thus from eq. (184), if we assume from previous machines a normal value for λ or for $k \cdot k_d \cdot \sqrt{2}$, we can obtain the necessary relation between $\frac{I_T}{2p}$ and X. As a general rule $\frac{Q \cdot m}{\lambda}$, or the ratio $\frac{AT_a}{AT_f}$, averages about 0·25, and under favourable conditions with good design may be taken as high as 0·3.

§ 2. **The value of λ.**—Reverting to the considerations of Chapter XXIII. § 22, we have to add further numerical data as to the values which $\lambda = k \cdot k_d \cdot \sqrt{2}$ may be expected to have in different cases.

It is found in practice that the value of k ranges from 1·08 to 1·8 in 3-phase heteropolar machines, and on the average may be taken as 1·36. In inductor alternators of the homopolar type, owing to the greater effect of the primary leakage in saturating the field-magnet, it may be as high as 2. In single-phase alternators when the average value of X_b or X_a, as reckoned in Chapter XXIII. § 8, is taken, k varies from 2 to 3, and in general may be reckoned as about 2·25.

As the normal case in 3-phase alternators, the value of k_d may be taken for two slots per pole and per phase, namely—

$$k_d = 1{\cdot}176 \frac{B}{b} \cdot \sin \frac{\pi}{2} \cdot \frac{b}{B}$$

and if we assume a pole-width $b = 0{\cdot}66$ B, $k_d = 1{\cdot}53$.

Taking the average value of 1·36 for k as above, we thus have

$$\lambda = 1{\cdot}36 \times 1{\cdot}53 \times \sqrt{2} = 2{\cdot}95$$

In practice it ranges from 2·55 to 3·8, and 2·95 may be taken as a fair average value. Machines with short non-overlapping coils show a higher value for λ than those with long coils, owing to the fact that the former do not embrace the main magnetic circuit so completely. Bar-wound armatures with closed slots also show large values for λ.

In the single-phase alternator the pole ratio is usually rather larger, and with the coil-side divided between two slots, $k_d = 0{\cdot}5$, whence

$$\lambda = 2{\cdot}25 \times 0{\cdot}5 \times \sqrt{2} = 1{\cdot}6$$

the limits of its variation being approximately from 1·4 to 2·1.

In general the value of λ is influenced by the number of slots per pole and per phase, and is less the greater the number of such slots, since the secondary leakage is then less and the back ampere-turns are not so concentrated on the main magnetic circuit. Thus in a single-phase alternator* with one slot per pole, λ' was 1·56; when the same type and size of machine was wound as a 3-phase generator with three slots per pole, the subdivision of the winding between the three slots partly compensated for the increased back ampere-turns which would

* Behrend, "Factors which Determine the Design of Monophase and Polyphase Generators," *Electr. Eng.*, vol. xxvi. p. 727.

otherwise have resulted, so that the new value of $\lambda'''=2{\cdot}9$ was considerably less than three times the λ' of the single-phase case.

The absolute value of the pole-pitch, and so indirectly the frequency, has an appreciable influence upon λ. The secondary leakage is largely a constant whatever the pole-pitch, and therefore its ratio to the main flux varies inversely as the pole-pitch. Further, the value of λ depends not only upon the actual number of the demagnetising turns of the armature, but also upon their effect in increasing the primary leakage. The greater the permeance $\mathcal{P}_l$ of the leakage paths, the higher the density of the resultant flux in the field-magnet; its reluctivity rises and $f'(B_m)$ in the third term of the right-hand side of eq. (181) increases. But with reduced value of the pole-pitch, $\mathcal{P}_l$ rather tends to increase, and in consequence with an increase of frequency unless the pole-pitch can be maintained constant, X_m increases and the value of λ is higher.

§ 3. **The armature ampere-wires per pole.**—The ampere-turns on one magnetic circuit may be expressed as a multiple of the ampere-turns required over the double air-gap, or $X=0{\cdot}8\ B_g\,.\,2l_g \times c$; the value of the multiplier c or the ratio of the total ampere-turns of the circuit to those of the air-gaps varies from $1{\cdot}2$ to 2, but may approximately be regarded as constant for the same type and proportions of machine, say at $1{\cdot}28$ within the range of the working part of the open-circuit curve. Therefore

$$\frac{I\tau}{2p}=\frac{Qm}{\lambda}\,.\,X$$

$$=\frac{Qm}{\lambda}\,.\,0{\cdot}8\ c\,.\,B_g\,.\,2l_g \qquad \qquad (185)$$

and this equation shows that the ampere-wires per pole must bear a certain relation to the air induction and length of gap, so that if the latter are high the former may also be large. Since B_g has a certain maximum value which it is not advisable to exceed with due regard to economy in the excitation, to hysteresis and eddy losses, and to the magnetic pull on the armature if there is any want of symmetry in the several fields, it may be regarded as more or less constant. The remaining factor is therefore the length of air-gap, and the above equation may be used to determine the permissible number of ampere-wires per pole for a given gap, or conversely the necessary gap for a given value of $\frac{I\tau}{2p}$.

Although the ratio of the armature to the field ampere-turns, or, to use the new form of expression given above, the number of armature ampere-wires per pole is a most important factor in the design of alternators, it is evident from the wide variations of λ with different types of machines and different arrangements of the winding that the reactance voltage is not definitely determined by the armature ampere-wires per pole. In different cases, therefore, much may be done to vary the one or the other so as to suit the exact nature of the load. Thus if the load-factor varies through a wide range, it is especially important that the armature ampere-wires per pole should be low, although the true armature reactance voltage may be comparatively high; such a case is met by the employment of a very small number of turns per coil, which may then be entirely embedded in the iron of the armature in closed slots or tunnels. On the other hand, if the power-factor is high and does not vary very much, and especially on non-inductive loads, it would be better to employ a larger number of armature ampere-wires per pole, arranged in shallow open slots, so that the actual reactance voltage is not so high; in other words, it becomes then of more importance to minimise the electric part of the reaction due to the secondary leakage rather than the magnetic part due to the back ampere-turns.

§ 4. **The armature ampere-wires per unit length of circumference.**—But as in the case of continuous-current dynamos, a knowledge of the permissible number of ampere-wires per pole is but little else than a warning to the designer, and must be further extended to give some indication of the dimensions of the armature. In other words, the normal relation of the ampere-wires per unit length of the circumference as given by practice must be introduced in order to make a preliminary calculation as to the diameter of the armature.

The value that may be assigned to the effective ampere-wires per unit length of circumference, or $a_w = \frac{I \cdot \tau}{\pi D}$, depends in the first place upon the permissible armature reaction as affecting the drop of volts that may be allowed between no-load and full-load; and secondly, but to a lesser degree, upon the permissible rise of temperature. Owing to the first consideration, it is affected by the nature of the load, whether non-inductive or highly inductive, upon which the machine is to be run, and also to some extent by the frequency. Experience shows that in 3-phase generators of usual construction, if cos ϕ_e lies between 1 and 0·8, and if the regulation is to be from 7 per cent. in small to 4 per cent. in large machines on unity power-factor, and from 22 per cent. in small to 15 per cent. in large machines on a power-factor of 0·8, the ampere-wires per inch length of circumference may gradually rise as the size of machine is increased through ranges of value, as in the following table:—

	50∿	25∿
High voltage	250 – 400	300 – 450
Low voltage	300 – 450	375 – 550

An average value is therefore about 400, and if the above values are much exceeded an undue expenditure of copper in the exciting coils is necessitated. With power-factors less than 0·8, a_w must be reduced in proportion to $\cos\phi_e$ if the drop of volts is to be maintained at the same percentage value, but since within the limits of practice a greater drop of volts is usually expected if the power-factor be low, a_w remains practically constant at the values given above. From this approximate result the preliminary forecast of the diameter must now be checked.

If the drop on inductive loads is to be the same in a single-phase alternator as in a 3-phase generator, the value of a_w would have to be reduced to 200–250 per inch, but in practice a_w may be as high as 70 per cent. of its value in the 3-phase generator, or say 300 per inch of circumference, owing to the fact that the single-phase alternator is mostly used on lighting circuits with high-power factors. Indeed, in such cases there is but little difference between the output of a given machine when wound for a single-phase circuit of high power-factor and when wound for 3-phase circuits of only moderate power-factor.

§ 5 **The dimensions as dependent upon the watts per rev. per min.**—The quantity that has been described as the "dimensional torque constant" of a machine, namely, the ratio of the watts per rev. per min. to D^2L, shows less uniformity in the alternator than in the continuous-current dynamo owing to the greater diversity of conditions in the former. There may be one, two, or three phases, and the voltages in use cover a very wide range; the frequency, and considerations of peripheral speed and pole-pitch further combine to influence largely the minimum size of a machine for a given output, while in addition the degree of regulation that is specified may vary greatly, whereas in the continuous-current dynamo practically the same degree of sparklessness is called for in all machines. The variations of power-factor are even more important, so that they are best eliminated by considering only the ratio of the apparent output in volt-amperes per rev. per min. to D^2L, *i.e.* by presupposing unity power-factor.

If VA be the apparent output in volt-amperes, and m be the number of phases, while E_e and I are the effective values of the terminal voltage per phase and of the current per phase, then $\frac{VA}{m} = E_e \cdot I$, and $I = \frac{a_w \cdot \pi D}{\tau}$, where a_w are the effective-ampere-wires per unit length of circumference.

The E.M.F. which must be induced per phase at full-load is not known at the outset, but will exceed the terminal voltage by, say, 5 per cent., so that $E_a = 1{\cdot}05\, E_e = \frac{K}{m} \cdot 2Z_a \cdot \frac{pN}{60} \cdot \tau \times 10^{-8}$ volts.

Combining these equations, the volt-amperes per rev. per min. are

$$\frac{VA}{N} = \frac{mE_e \cdot a_w \cdot \pi D}{N \cdot \tau} = \frac{K}{1{\cdot}05} \times 2Z_a \cdot \frac{p}{60} \cdot a_w \cdot \pi D \times 10^{-8}.$$

The width of the pole-face being $\beta \cdot \frac{\pi D}{2p}$, for Z_a may be substituted $\beta \cdot \frac{\pi D_{\prime\prime}}{2p} \cdot L_{\prime\prime} \times B_g \times 6{\cdot}45$, and

$$\frac{VA}{N} = \frac{\pi^2 D_{\prime\prime}^2 \cdot L_{\prime\prime} \cdot \beta \cdot K \cdot a_w \cdot B_g \times 6{\cdot}45 \times 10^{-8}}{1{\cdot}05 \times 60}$$

$$= \beta \cdot K \cdot a_w \cdot B_g \cdot D_{\prime\prime}^2 L_{\prime\prime} \times 10^{-8}$$

β is approximately 0·66 to 0·75 in heteropolar machines, and on an average with normal designs it may be assumed that $K = 1{\cdot}11$, and $\beta = 0{\cdot}66$, the latter value giving an approximately sinusoidal E.M.F., whence

$$\frac{VA}{N} = 0{\cdot}73 \times 10^{-8} \times a_w \cdot B_g \cdot D_{\prime\prime}^2 L_{\prime\prime} \qquad (186)$$

which is exactly analogous to eq. (152) for the continuous-current dynamo.

The required degree of regulation for constant potential which has such a great influence on the cost is here not regarded, since, apart from the question of the depth of air-gap and excitation copper, it must be taken care of in the figure assigned to a_w.

Finally, if $a_w = 300$ and $B_g = 6000$ are taken for small machines of high frequency, and $a_w = 550$ and $B_g = 9000$ for large machines of low frequency, we have values ranging from

$$\frac{VA}{N} = 0{\cdot}013\ D_{\prime\prime}^2 L_{\prime\prime} \text{ to } 0{\cdot}036\ D_{\prime\prime}^2 L_{\prime\prime}$$

and

$$D_{\prime\prime}^2 \cdot L_{\prime\prime} = 77 \frac{VA}{N} \text{ to } 28 \frac{VA}{N}$$

Any such approximate formulæ * admit of wide deviations according to the circumstances of the design, and it is evident that curves connecting either the dimensional torque constant, or its reciprocal the size constant, with the volt-amperes per rev. per min. have not the same validity in the case of the alternator as in that of the continuous-current dynamo. Fig. 531 can therefore only be regarded as indicating roughly the order of the figures under ordinary conditions of regulations, etc., for frequencies of 50 or 25 in 3-phase machines with internal rotating fields and driven at steam-engine speeds. Certain general conclusions may, however, be drawn therefrom. The apparent useful torque per cubic inch of D^2L naturally tends to be less in the alternator than in the continuous-current machine, since the diameter of the armature in the former must allow room for the internal poles, and

* For alternators of fairly large size Messrs. J. C. Macfarlane and H. Burge (*Journal Inst. Electr. Eng.*, vol. xlii. p. 261) give $a_w \cdot B_{\prime\prime} = 4 \times 10^6$, whence the dimensional torque on the assumptions as to polar arc, etc., which have been adopted above, $= 0{\cdot}03$ volt-amperes per rev. per min.

owing to the number of these to give the usual frequencies of practice an efficient design necessitates a short armature core of large diameter. The square of the diameter has thus a preponderating influence upon the size constant. In many cases, too, a large diameter and narrow

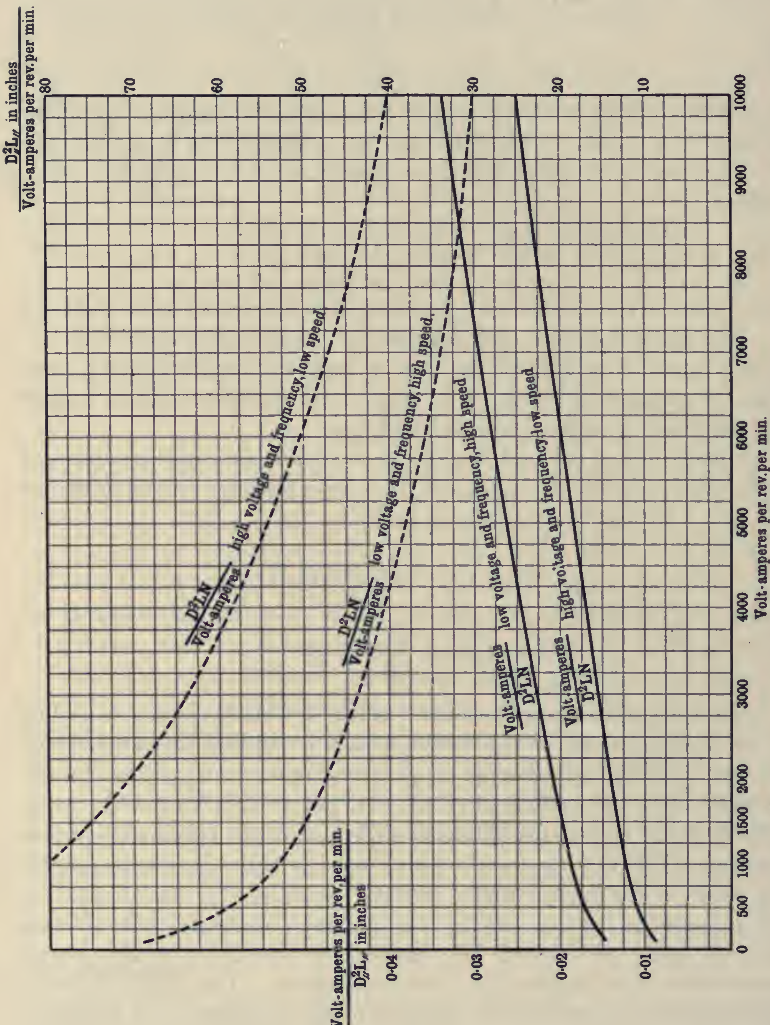


Fig. 531.—Torque constant and size constant of alternators.

width of core is of no disadvantage, since it gives a large fly-wheel effect for a given mass of copper and iron. With short magnet-cores the cost of the magnetically and electrically effective portions of the alternator is also more independent of the diameter than is the case with the continuous-current dynamo.

The dimensional torque constant does not so quickly reach a maximum value in alternators as in continuous-current dynamos, but continues to ascend slowly, owing to the gradually increasing value that may be assigned as mentioned above to a_w with increasing size. Apart from turbo-alternators with artificial ventilation,* the maximum value of $\frac{VA}{D^2LN}$ for very large machines approaches 0·036 to 0·04, yet so low a figure as 0·025 may be regarded as evidencing an economical design for machines even up to 10,000 volt-amperes per rev. per min. On the other hand, under favourable conditions of speed and voltage, 0·02 may be obtained even with so small an output as 1000 volt-amperes per rev. per min.

A low frequency renders possible a high value for the dimensional torque constant, owing to the larger pole-pitch and fewer number of poles.

Lastly, the single-phase alternator does not show such high values as the polyphase alternator, owing to the greater influence of its armature ampere-turns on the regulation and in the production of eddy-current losses.

For the same value of the volt-amperes per rev. per min., *i.e.* for the same torque, there is no great difference between the D^2L of machines whether giving a small output and driven by a low-speed steam-engine, or giving a large output and driven at a high speed by a steam turbine. In either case the combined conditions of voltage, frequency, and speed may be favourable or unfavourable, and although the turbo-alternator is at some disadvantage owing to the small choice in the number of poles for normal frequencies and to the difficulty of dissipating the heat from its small superficial area,† yet on other grounds it often benefits by its small number of poles.

Having determined the approximate value of D^2L, such values for the separate factors must be taken as will give a reasonable value for the pole-pitch as fixed by the frequency and an economical section to the field-magnet cores from the point of view of exciting energy and copper. As a general rule, considerations of peripheral speed limit the diameter as explained above, and a very usual value for the pole-pitch is about 10 inches, although in turbo-alternators this may become 50 inches or more. It is therefore more usually the length of core that remains to be fixed from the D^2L.

* The 3200 KVA turbo-alternator at 980 revs. per min. by Messrs. Kolben & Co. of Prague (*Electr. Eng.*, vol. xliii. p. 438) was artificially ventilated by forced draught and had a dimensional torque of 0·03 volt-amperes per rev. per min. or a size constant of 33·2.

† As an example of a turbo-alternator giving a large output for its size, and showing that the above figures may often be considerably bettered, may be mentioned the 970 KVA 50 ∿ 3-phase 1500 revs. alternator manufactured by Messrs. Kolben, of which the size constant was as low as 39·4 (*Electr. Eng.*, vol. xxxix. p. 118).

§ 6. **Design of 3-phase generator.**—Let it be required to design a 3-phase generator with Y or star-connected armature to give 750 KVA or 2000 volts of interlinked pressure between the lines, and 216 amperes per phase with a frequency of 50 when running at about 95 revs. per min. The power-factor is given as $\cos \phi_e = 0{\cdot}85$, so that while the kilovolt-amperes or kilowatts of output with unity power-factor would be $\sqrt{3} \times 2000 \times 216 = 750$, the true output on the partially inductive external load is only $750 \times 0{\cdot}85 = 637$ kilowatts.

In order to give the frequency required, $f = \frac{pN}{60} = \frac{32 \times 94}{60} = 50$, so that the speed must be 94 revs. per min., and the number of poles $2 \times 32 = 64$. The armature is to be of the stationary external type, and the magnet system is to have poles of alternate sign arranged radially on the rim of a centre wheel. For a cast-steel yoke-ring a suitable peripheral speed will be given if we assume a speed of 5000 ft. per min. at the face of the poles. The external diameter of the rotating portion will therefore be

$$D' = \frac{V}{\pi N} = \frac{5000}{3{\cdot}14 \times 94} = 17 \text{ ft.}$$

A rise of 17·5 per cent. of the terminal volts is to be permitted when the full-load is switched off, so that the open-circuit E.M.F. with constant speed and the full-load excitation may be 2350; hence $\eta = 0{\cdot}85$, and by reference to the calculated table of $\sqrt{1 - \eta^2 \cos^2 \phi_e} - \eta \sqrt{1 - \cos^2 \phi_e}$, the corresponding value of Q is found to be 0·245. The pole-pitch at the face of the pole-shoes is $\frac{204'' \times 3{\cdot}14}{64} = 10''$, an average figure for which λ may be taken as 2·95. The ratio of the effective ampere-turns at full-load on the armature to those on the field or $\frac{AT_a}{AT_f} = \frac{Qm}{\lambda}$ by eq. $(184) = \frac{0{\cdot}245 \times 3}{2{\cdot}95} = 0{\cdot}249$. As a preliminary assumption, let the single air-gap be taken as $l_g = 0{\cdot}375'' = 0{\cdot}95$ cm., and the flux-density therein be, say, 6900, so that on open circuit for the same excitation the flux-density would be about $\frac{6900}{0{\cdot}85} = 8100$. We then have $X_g = 0{\cdot}8 \times 8100 \times 1{\cdot}9 = 12{,}300$, and $X = c \,.\, X_g = 15{,}750$, if $c = 1{\cdot}28$, as in § 3. While the value of X has here been approximately estimated, it is usually more or less known from previous experience with machines of the same type, similarly proportioned, so that the designer thence arrives at the number of ampere-wires per pole, or

$$\frac{I \,.\, \tau}{2p} = X \,.\, \frac{Qm}{\lambda} = 15{,}750 \times 0{\cdot}249 = 3920$$

and

$$I \,.\, \tau = 3920 \times 64 = 251{,}000$$

An average value for the ampere-wires per inch length of the circumference being $a_w = 390$, for the present diameter of a little over 17 ft., $I \cdot \tau = 640 \times 390 = 249{,}600$, a value which agrees nearly, so that in conjunction with $l_g = 0{\cdot}375''$ it may be adopted. The number of active wires will then be $\tau = \frac{249{,}600}{216} = 1156$, but as this must be divisible by the number of phases and by the number of slots, either one or two per pole and per phase, we are limited to either 64 slots per phase with 6 wires in each slot, or 128 slots per phase with 3 wires per slot. In both cases $\tau = 1152$, and in order to minimise the leakage reactance and produce an approximately sinusoidal wave of E.M.F., two slots per pole and per phase will be chosen. The ratio of pole-width to pole-pitch will be taken at $\beta = 0{\cdot}66$.

The value of K for such an arrangement will, however, probably be less than 1·11, say, 1·07 by Table II. Chapter XXII. Inserting, then, the values 1·07, 390, 0·66, and 6900, for K, a_w, β and B_g in formula (186),

$$D_{''}^2 L_{''} = \frac{1}{0{\cdot}019} \cdot \frac{VA}{N} = 53 \cdot \frac{VA}{N}$$

The volt-amperes per rev. per min. are $\frac{750{,}000}{94} = 8000$ nearly, whence $D_{''}^2 L_{''} = 424{,}000$, and $L_{''} = 10{\cdot}2''$. A small increase in B_g will enable the round dimension of 10″ to be employed for the length of the armature core, and this change will also have the advantage of slightly improving the section of the magnet-core.

Allowing a loss of volts over the armature resistance of about $2\frac{1}{2}$ per cent., or of 50 volts on 2000, the loss over one branch of the armature winding will be $\frac{50}{\sqrt{3}} = 29$ volts. The resistance of the one branch of the armature must therefore be $\frac{29}{216} = 0{\cdot}134$ ohm.

If $2\frac{1}{2}''$ are allowed on each side for the projection of the micanite trough beyond the core past the armature end-plates, and a further $2\frac{1}{2}''$ for the projection of the straight ends of a coil so as to give room for the bending upwards of the coils of the other phases, the total length of a straight side of a coil is $10 + 10 = 20''$, while the average length of an end-connection is equal to the pole-pitch. The length corresponding to one active wire will therefore be $20'' + 10'' = 30'' = 0{\cdot}835$ yd., and the total length of wire in one phase of 384 wires $= 384 \times 0{\cdot}835 = 320$ yds. If the heating coefficient reckoned from a temperature of 60°F. be 1·17, the resistance per 100 yds. of the wire must be $\omega' = \frac{0{\cdot}134}{1{\cdot}17 \times 3{\cdot}2} = 0{\cdot}0358$ ohm, and the required area is $\frac{0{\cdot}0024}{0{\cdot}0358} = 0{\cdot}067$ sq. inch. As the round wire corresponding to this area will be somewhat stiff to bend, and when arranged with the three turns one above the other will necessitate too deep a slot, two wires each 0·210″ in diameter may be wound in parallel, giving six wires per slot disposed in three layers of two side by side. The joint area of the two wires will be 0·069 sq. inch, and the current density $\frac{216}{0·069}=3130$ amperes per sq. inch, which is as high as is permissible. The resistance of a phase is then 0·111 ohm cold, or $0·111 \times 1·17 = 0·13$ ohm when hot, giving $0·13 \times 216 = 28·1$, or nearly 29 volts, as estimated at the outset. In order to allow for eddy-currents, this resistance requires to be increased by some 30 per cent. to an equivalent $r_a' = 0·169$ ohm, so that the virtual loss of volts becomes 36·5. The corresponding watts lost in the armature are $36·5 \times 216 \times 3 =$ 23,600, of which $8·4 \times 216 \times 3 = 5440$ can be assigned to eddy-currents.

For 2000 volts a thickness of 0·1″ will be sufficient for the insulating lining of the slot, so that with allowance for the double-cotton-covering or braiding of the wire and for some margin in the winding, the slot may be made 1·25″ deep over-all × 0·75″ wide, with an opening of 0·275″ at the top closed by a hard-wood strip (Fig. 487, ii.).

The number and arrangement of the slots having now been settled (Fig. 532), an approximate estimate of the leakage reactance voltage must be made. From the formulæ of Chapter XXIII. §. 24—

$$\lambda_a = \frac{12·57}{2}\left(\frac{0·875}{3 \times 0·75} + \frac{0·375}{0·5}\right) + \lambda_k$$

the average width of slot above the wires being $w_3 = 0·5$. The equivalent radius of the group of wires in two slots is estimated from a drawing as $r_q = 0·99''$, whence $\lambda_k = 9·2 \log \frac{10}{0·99} + 1 = 10·25$

Therefore $$\lambda_a = 7·15 + 10·25 = 17·4$$
and $$l\lambda_a = 25·4 \times 17·4 = 442$$

Similarly $\lambda_b = \frac{\lambda_k}{2} = 5·125$, and the total length of a straight-sided coil is $l_1 = 20''$, so that $l_1 - l = 10''$, and

$$(l_1 - l)\,\lambda_b = 25·4 \times 5·125 = 130$$

Since all the end-connections follow the same path and have an equivalent radius of $r_T = 0·557''$ with a total distance between them of 20″

$$\lambda' = 4·6 \log \frac{20}{0·557} + 0·5 = 7·65$$

and $$\frac{\pi D}{2p} \cdot \lambda' = 25·4 \times 7·65 = 194$$

The inductance of a phase is therefore by eq. (183)

$$L_a = 2 \times 32 \times 36 \times (442 + 130 + 194) \times 10^{-9}$$

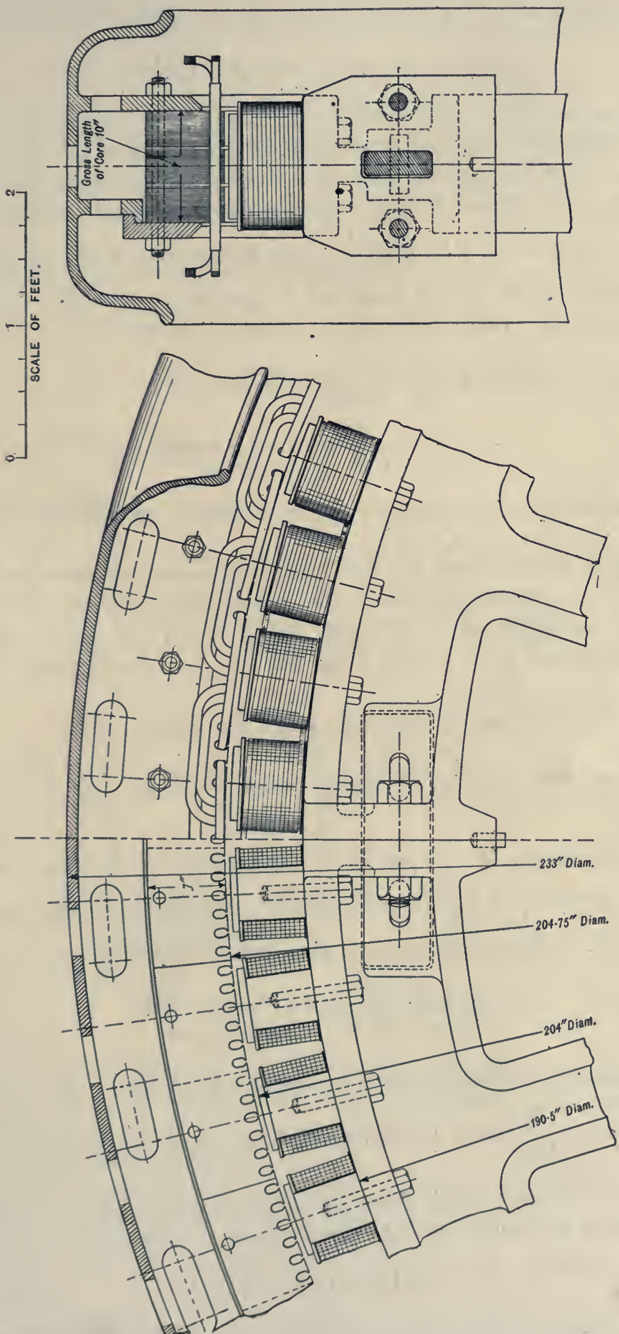


FIG. 532.—Three-phase generator for 750 KVA.

and with an addition of 15 per cent. for the influence of the other phases is

$$1{\cdot}15 \times 1{\cdot}763 \times 10^{-3} = 2{\cdot}03 \times 10^{-3} \text{ henrys}$$

The reactance is then $2\pi f L_a = 6{\cdot}28 \times 50 \times 2{\cdot}03 \times 10^{-3} = 0{\cdot}638$ ohm $= x_a$.

The combined effect of the secondary leakage and of the calculated magnetising armature-turns may now be compared with the value previously assumed for λ from experimentally obtained curves. The interpolar gap being 3·4″, $\frac{c}{l_g} = \frac{1{\cdot}7}{0{\cdot}375} = 4{\cdot}55$, and $K_2 = 2{\cdot}16$ by Fig. 265. The effective width of the polar arc is therefore $6{\cdot}6 + 2{\cdot}16 \times 0{\cdot}375 = 7{\cdot}41$″, and by eq. (169)

$$k_d = 3 \cdot \frac{4}{\pi^2} \cdot \frac{10}{7{\cdot}41} \cdot \sin 66{\cdot}6 \times 0{\cdot}9659$$
$$= 1{\cdot}176 \times \frac{10}{7{\cdot}41} \times 0{\cdot}9178 = 1{\cdot}455$$

The demagnetising ampere-turns of the armature on short-circuit are therefore $X_a = k_d \cdot \sqrt{2}\, t I_o = 12{\cdot}3\, I_o$.

The E.M.F. consumed by the leakage reactance on short-circuit is $e_{sa} = 0{\cdot}638\, I_o$, and combined with the loss of volts over the armature resistance as explained in Chapter XXIV. § 22, the vector sum is approximately say $1{\cdot}015 \times 0{\cdot}638\, I_o$, or say $0{\cdot}65\, I_o$. Analogously to eq. (176)

$$E_a = 1{\cdot}07 \times 2Z_a \times 50 \times 384 \times 10^{-8} = 410\, Z_a \times 10^{-6}$$

so that the flux to give the above value of E_a is $\frac{0{\cdot}65\, I_o}{410} \times 10^6 = 1585\, I_o$.

The axial length of the pole-face may be made ½″ less than that of the core, so that $\frac{a}{l_g} = \frac{0{\cdot}25}{0{\cdot}375} = 0{\cdot}66$, and $K_1 =$ say, 0·88 by Fig. 263 of Chapter XV. The effective area is therefore, by equation (66)

$$s_g = (9{\cdot}5 + 0{\cdot}88 \times 0{\cdot}375 - 0{\cdot}11)\,(7{\cdot}41)$$
$$= 72{\cdot}2 \text{ sq. in., or } 466 \text{ sq. cm.}$$

The excitation over the air-gaps is therefore in general

$$X_g = 0{\cdot}8 \times \frac{1{\cdot}9}{466} \times Z_a = 3260 \times Z_a \times 10^{-6}, \text{ and to give } Z_a = 1585\, I_o$$

is $5{\cdot}16\, I_o$. On short-circuit then, and neglecting any loss of magnetic potential over the field-magnet as small, equation (181) gives

$$X = 12{\cdot}3\, I_o + 5{\cdot}16\, I_o = 17{\cdot}46\, I_o$$

or with a small allowance for reluctance in the field-magnet, say $18\, I_o$.

Thence $\lambda = \frac{X}{tI_o} = \frac{18}{6} = 3$, which is near enough to the assumed value of 2·95. The accuracy of either approximation will again be tested at a later stage, and $\lambda = 3$ will be found practically correct.

If the synchronous impedance method were perfectly valid it would only then be necessary to design the field-magnet so that the excitation X gave the permitted open-circuit voltage $E_o = \frac{2350}{1·73} = 1350$ per phase. The short-circuit current must be on this hypothesis, $I_o = \frac{I}{Q} = \frac{216}{0·245} = 881$, and the synchronous impedance per phase $\frac{e}{I} = \frac{0·245 \times 1350}{216}$ or $\frac{E_o}{I_o} = \frac{1350}{881}$, *i.e.* 1·53 ohm. The normal excitation must correspondingly be

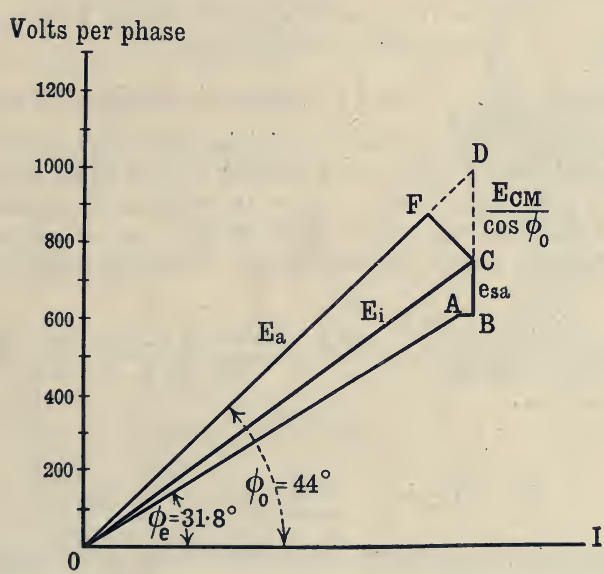


FIG. 533.—Vector diagram of E.M.F.'s.

$X = \lambda t I_o = 3 \times 6 \times 881 = 15{,}900$, and must give 2300 volts of interlinked pressure on open circuit. But in practice it is necessary to proceed by the full method which is applicable to the true case of the saturated magnet as follows :—

The calculated value of the E.M.F. consumed by leakage reactance at full-load is $e_{sa} = 0·638 \times I = 138$ volts.

When the three vectors of $E_e = \frac{2000}{1·73} = 1152$ terminal volts per phase, of $Ir_a' = 36·5$, and of $e_{sa} = 138$ are combined together graphically as in Fig. 533, their vector sum is found to be $E_i = 1263$.

By formula (172) the cross-magnetising ampere-turns of the armature are

$$X_{CM}=3\cdot\frac{4}{\pi^2}\cdot\frac{10}{7\cdot41}\sqrt{2}\times6\times(1-\cos 66\cdot6)\times0\cdot9659\times I\cos\phi_o$$
$$=1\cdot176\times6\cdot9\times I\cos\phi_o=8\cdot12\ I\cos\phi_o$$
$$=1755\cos\phi_o \text{ at full current.}$$

The cross-flux is therefore by equation (175)

$$Z_c=\frac{1\cdot257\,X_{CM}}{1\cdot9/466}=309\ X_{CM}=2510\,I\cos\phi_o$$
$$=542{,}000\cos\phi_o \text{ with full current}$$

and the cross E.M.F. is by equation (176), assuming a sinusoidal curve as an approximation,

$$E_{CM}=1\cdot11\times2Z_c\times50\times384\times10^{-8}=1\cdot07\ I\cos\phi_o$$
$$=232\cos\phi_o \text{ with full current.}$$

We thus find $\frac{E_{CM}}{\cos\phi_o}=232$, and the diagram is completed by producing BC to a length $CD=232$ volts and joining OD. Upon OD let fall the perpendicular CF; then the E.M.F. which each phase must give if distortion were absent is $E_a=1250$ volts, to which the armature flux under full-load at the given power factor must be proportional. The angle ϕ_o can now be measured as 44°. Or analytically by eq. (177)

$$\tan\phi_o=\frac{E_e\,.\,\sin\phi_e+x_aI+\dfrac{E_{CM}}{\cos\phi_o}}{E_e\cos\phi_e+Ir_a'}=\frac{610+138+232}{980+36\cdot5}=\frac{980}{1016}=0\cdot965$$

whence $\phi_o=44°$.

$$OD=\sqrt{980^2+1016^2}=1411$$

and

$$E_a=OD-\sin\ _o\,.\,\frac{E_{CM}}{\cos\phi_o}=1411-161=1250.$$

By equation (167) the demagnetising ampere-turns of the armature are on full-load

$$X_{DM}=k_d\,.\,\sqrt{2}\,.\,tI\sin 44°$$
$$=1\cdot455\times\sqrt{2}\times6\times216\times0\cdot6947$$
$$=12\cdot3\times216\times0\cdot6947$$
$$=1850$$

The flux corresponding to $E_a=1250$ is $1250\times410\times10^{-6}=3\cdot05\times10^6=Z_a$. With a density in the armature of $B_a=5000$, $2ab=610$ sq. cm., or the cross-sectional area of iron in the core below the slots must be half this amount, *i.e.* $=47\cdot3$ sq. ins. If there are two air-ducts in the core each 0·4″ wide, and a deduction of 10 per cent. is made for insulation between the laminations, the net width of iron in the 10″ gross width of core is $9\cdot2\times0\cdot9=8\cdot28″$. The radial depth must therefore be 5·72″, say $5\frac{3}{4}$ in., which with a slot $1\frac{1}{4}''$ deep makes the external diameter of the discs 218·75″. The number of teeth under a pole with allowance for fringe may be taken as 4. The width of a tooth is 0·94″, whence the area of iron in the teeth corresponding to one field is $0{\cdot}94 \times 8{\cdot}28 \times 6{\cdot}45 \times 4{\cdot}4 = 220$ sq. cm., and the maximum uncorrected density $\frac{3{,}050{,}000}{220} = 13{,}900$, a comparatively low figure.

In order to make a preliminary determination of the requisite size of magnet core, a leakage coefficient of, say, $\nu = 1{\cdot}2$ may be assumed,

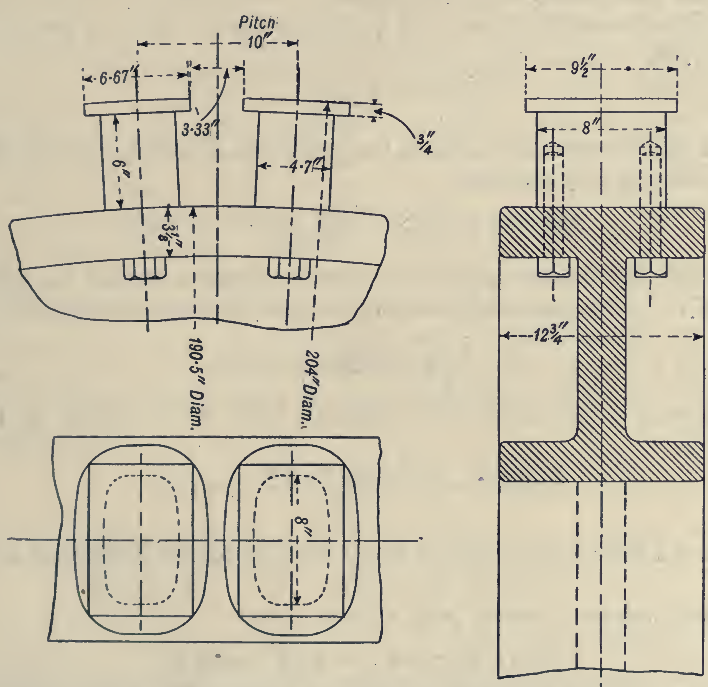


FIG. 534.—Yoke and poles of 750-KVA alternator.

whence $Z_m = 1{\cdot}2 \times 3{\cdot}05 \times 10^6 = 3{\cdot}66 \times 10^6$. With poles of cast-steel carefully tested, the flux-density may be carried as high as $B_m = 17{,}750$, so that the necessary area is $\frac{3{\cdot}66 \times 10^6}{17{,}750} = 206$ sq. cm., or 32 sq. ins. An overhanging edge is required for the pole-shoe to hold the exciting coil in position, and $7\frac{5}{8}''$ length $\times\, 4\frac{1}{2}''$ width with well-rounded corners may, in the first place, be chosen for the two dimensions of the magnet core. An approximate sketch of one magnetic circuit, such as Fig. 534 (which will subsequently have to be corrected if not found suitable) must now be made in order to give the length of path in the pole and yoke, and more especially to enable the designer to calculate as closely as possible the value of $\mathcal{P}_l$, the field leakage permeance. In order to minimise the leakage, the length of the pole will be as short as is compatible with the requirement of room for the exciting coils and a small loss of watts therein, say 6″ between the overhanging edge of the pole-shoe and the yoke-ring. The radial depth of the pole-shoe will also be small, say $\frac{3}{4}$″, in order to keep the exciting coil as near to the top of the pole as possible. The yoke may be given ample proportions so as to increase the fly-wheel effect, say $12\frac{3}{4}$″ wide × $3\frac{1}{8}$″ deep, making a total cross-section $2s_y$, of 510 sq. cm.

The permeance between the pole-tips on either side of a pole is by eq. (70*a*)

$$\frac{0{\cdot}75 \times 9{\cdot}5}{3{\cdot}33} \times 2{\cdot}54 \times 2 = 10{\cdot}85$$

and that between the flanks of the pole-shoes in either direction and on both sides by eq. (72*a*)

$$1{\cdot}86 \times 0{\cdot}75 \times \log \frac{\pi \times 3{\cdot}33 + 3{\cdot}33}{3{\cdot}33} \times 4 = 3{\cdot}44$$

both the above being acted on by the full excitation between the poles, or X_p. The permeance between the sides of the poles is approximately

$$\frac{6 \times 7{\cdot}6}{5{\cdot}2} \times 2{\cdot}54 \times 2 = 44{\cdot}6$$

and from the flanks in either direction and on both sides of the core is

$$1{\cdot}86 \times 6 \times \log \frac{\pi \times 2{\cdot}25 + 5{\cdot}2}{5{\cdot}2} \times 4 = 16{\cdot}6$$

The two latter being acted on by $\frac{X_p}{2}$ must be halved if regarded as in parallel with the armature path, whence

$$\mathcal{P}_l = 10{\cdot}85 + 3{\cdot}44 + 22{\cdot}3 + 8{\cdot}3 = \text{say } 50$$

The full-load excitation can now be calculated as follows, and is conveniently reckoned out for a half-circuit or per pole. For greater accuracy the equivalent length of the air-gap may be calculated from Fig. 273, and m is found to be about 1·02. The slots being half-closed, m was previously not taken into account, but it is evident that to the same extent the true air-gap may be kept on the small side, say to 0·94 cm. Thence by eq. (67)

$$\frac{X_g}{2} = 0{\cdot}8\ Z_a \,.\, \frac{1{\cdot}02 \times 0{\cdot}94}{466} = 0{\cdot}00164\ Z_a$$

For $Z_a = 3{\cdot}05 \times 10^6$, we thus find $\frac{X_g}{2} = 5000$, the average density in the air being $B_g = 6550$.

$B_a = 5000$, and the length of path corresponding to an arc between the pole-tips and at a radius below the slots gives $\frac{l_a}{2} = 4 \cdot 3$ centimetres; $f'(B_a) = 2 \cdot 4$, so that the loss of magnetic potential over the armature core only demands some 10 AT, and is practically negligible. As the density in the teeth is also low, the excitation expended over them may roughly be taken into account by doubling the loss over the armature core. To $\frac{X_g + X_a + X_t}{2} = 5020$ must be added $\frac{X_{DM}}{2} = 925$, and thence $\frac{X_P}{2} = 5945$. The leakage flux is then $1 \cdot 257\ X_P \,.\, \mathcal{P}_l = 1 \cdot 257 \times 11{,}890 \times 50 = 745{,}000$, and $Z_m = Z_a + \zeta = (3 \cdot 05 + 0 \cdot 745) \times 10^6 = 3 \cdot 795 \times 10^6$, whence $\nu = 1 \cdot 242$, which agrees nearly with the assumed value when determining the size of the magnet core. Since, however, it is slightly higher, and the core is more highly saturated than was anticipated, it will be advisable to make the core rather more oval with an extreme length of 8″ and an extreme width of 4·7″, which with allowance for the rounded corners will give an area of 33·4 square in., or 215 square centimetres. The leakage permeance will be hardly affected by this change. The length of path in the magnet-core $= 6 \cdot 75'' \times 2 \cdot 54 = 17 \cdot 1$ centimetres, and in the yoke up to the dividing-line between the poles is, say, $5'' = 12 \cdot 7$ centimetre. In order to allow for the magnetic joint between the core and the yoke at the point of maximum flux, it will be safer to assume an equivalent air-gap rising to about $\frac{1}{4}$ millimetre with the full-load density within it of $B_m = 17{,}650$. Hence

$B_m = 17{,}650$	$f'(B_m) \,.\, \frac{l_m}{2} = 78 \times 17 \cdot 1 =$	1335
Allowance for joint	$= 0 \cdot 8 \times B_m \times 0 \cdot 025$ $=$	353
$B_y = 7{,}450$	$f'(B_y) \times \frac{l_y}{2} = 4 \cdot 1 \times 12 \cdot 7 =$	52
		1740

Adding $\frac{X_m + X_y}{2} = 1740$ to $\frac{X_P}{2} = 5945$, we find the total ampere-turns to be provided on each pole to be $\frac{X}{2} = 7685 = AT_c$.

The open-circuit or no-load characteristic is quickly calculated in tabular form (p. 982), and is plotted in Figs. 473 and 475.

The partial magnetic characteristics are given in Fig. 535, and thence by the principles of Chapter XXIV. § 15 the external characteristic under full-load current and with a power-factor of 0·85 is calculated on p. 983. In equation (177) under the full-load current of 216 amperes, $e_{sa} = 138$, $\frac{E_{CM}}{\cos \phi_o} = 232$, and $I r_a' = 36 \cdot 5$: $\sin \phi_e = 0 \cdot 527$ and $\cos \phi_e = 0 \cdot 85$, so that by giving successive values to E_e per phase $\tan \phi_o$ and E_a are

Z_a in 10^6 C.G.S. lines.	1		1·5		2		2·5		2·75		3		3·25		3·5	
$\frac{X_g}{2} = 1640 Z_a$		1640		2460		3280		4100		4510		4920		5330		5740
$\frac{X_a}{2}$		4		5		6		8		9		10		12		15
$\frac{X_t}{2}$		4		5		6		8		9		10		12		15
$\frac{X_P}{2}$		1643		2470		3292		4116		4528		4940		5354		5770
$\zeta = 1·257 \times X_P \times \mathcal{S}_l$	0·207		0·31		0·414		0·516		0·57		0·62		0·675		0·726	
$Z_m = Z_a + \zeta$	1·207		1·81		2·414		3·016		3·32		3·62		3·925		4·226	
$B_m = \frac{Z_m}{215}$	5620		8420		11,200		14,000		15,400		16,800		18,250		19,650	
$f'(B_m)$	1·6		3·1		4·8		12		20		48		116		250	
$\frac{X_m}{2} = f'(B_m) \times 17·1$		27		53		82		205		342		820		1980		4272
Allowance for joint		40		75		140		210		260		300		365		415
$\frac{X_y}{2}$		5		8		15		24		35		48		75		115
$\frac{X}{2}$		1720		2606		3529		4555		5165		6108		7774		10,572
E.M.F. per arm. phase	410		615		820		1025		1130		1230		1335		1438	
E.M.F. between lines.	710		1065		1420		1775		1960		2130		2310		2490	

E_c between Lines.	E_c per Phase.	E_a	Z'_a	ϕ_o	$\sin\phi_o$	$\times 1330 = \frac{X_{DM}}{2}$	$\frac{X'_g + X'_a + X'_t}{2}$	$\frac{X_P}{2}$	ζ	Z_m	$\frac{X_m}{2}$	$\frac{X}{2}$
0	0	140	0.342	84.4	0.995	1320	565	1885	0.237	0.579	30	1915
260	150	260	0.635	70	0.9397	1247	1048	2295	0.288	0.923	60	2355
520	300	402	0.98	61.15	0.876	1166	1615	2781	0.35	1.33	90	2871
780	450	548	1.33	56	0.829	1100	2190	3290	0.414	1.74	110	3400
1040	600	697	1.7	51.35	0.781	1040	2800	3840	0.484	2.18	195	4035
1300	750	847	2.07	48.75	0.752	1000	3420	4420	0.555	2.62	290	4710
1560	900	996	2.43	46.4	0.726	967	4010	4977	0.626	3.056	450	5427
1820	1050	1146	2.8	45.2	0.71	945	4610	5555	0.699	3.5	890	6445
2080	1200	1295	3.16	43.8	0.692	925	5210	6135	0.77	3.93	2500	8635

found. The demagnetising ampere-turns with the full-load current are $X_{DM} = 2660 \sin \phi_o$.

The result is plotted as the intermediate full-current curve of Fig. 473.

It is evident that on open circuit the full-load excitation will give 2300 volts, or a rise of 300 volts = 15 per cent., which is less than the permitted amount. The excitation for 2000 volts and unity power-factor is from Fig. 473 seen to be 6050 per pole, and under these conditions the rise of volts would be to 2130 or 6 per cent. nearly. On a completely inductive load with $\cos \phi_e = 0$, the necessary excitation would be 8900 per pole, and the rise of volts nearly 19 per cent.

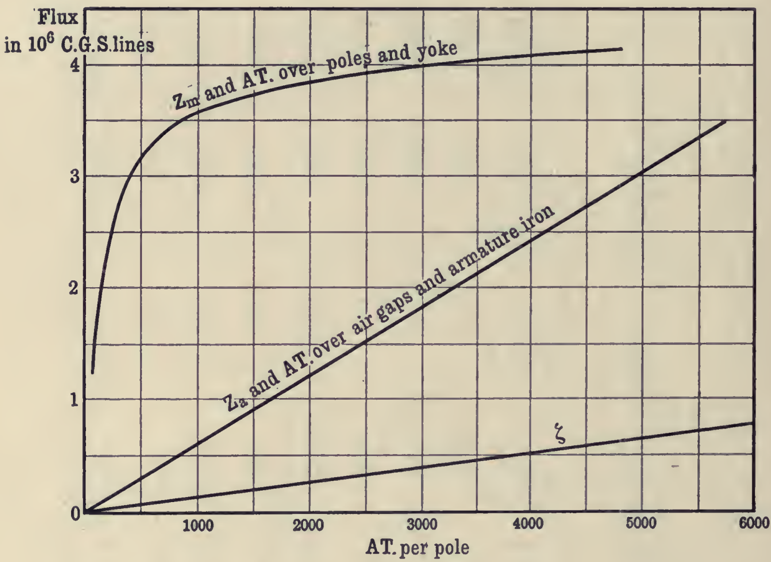


FIG. 535.—Partial magnetic characteristics of 750-KVA alternator.

On short-circuit, $\tan \phi_o = \frac{0{\cdot}638 + 1{\cdot}07}{0{\cdot}169} = 10{\cdot}1$, whence $\phi_o = 84^\circ{\cdot}4$ nearly, and the apparent synchronous impedance $z' = \sqrt{1{\cdot}71^2 + 0{\cdot}169^2} = 1{\cdot}713$. Thence $E_a = OD - \sin \phi_o \,.\, \frac{E_{CM}}{\cos \phi_o} = I_o(1{\cdot}713 - 1{\cdot}063) = 0{\cdot}65\, I_o$, and $Z_a = \frac{E_a}{410} \times 10^6 = \frac{0{\cdot}65 I_o}{410} \times 10^6 = 1585 I_o$. The air-gap and armature excitation per pole is $\frac{X_g + X_a + X_t}{2} = 1648 Z_a \times 10^{-6} = 2{\cdot}61\, I_o$, and $\frac{X_{DM}}{2} = 6{\cdot}14\, I_o$. Hence $\frac{X_P}{2} = 8{\cdot}75 I_o$; $\zeta = 1{\cdot}257 \frac{X_P}{2} \times 2 \times 50 = 1100 I_o$, and $Z_m =$

$2685I_o$; the necessary excitation $\frac{X_m}{2}$ must then be added to $\frac{X_P}{2}$ to obtain $\frac{X}{2}$. Calculation shows that on short-circuit saturation of the field hardly enters into the question, and the short-circuit current is a straight-line function. Or with sufficient accuracy it may at once be said from the no-load curve of Fig. 473 that for low values of the net excitation $\frac{X_R}{2} = 4{\cdot}4E_a$, and to give $E_a = 0{\cdot}65I_o$, $\frac{X_R}{2} = 2{\cdot}86I_o$; whence $\frac{X}{2} = I_o\,(2{\cdot}86 + 6{\cdot}14) = 9I_o$ (Fig. 536), or $X = 18I_o$. Under the full-load excitation of 7685 AT per pole the short-circuit current is $\frac{7685}{9} = 854$ amperes, or 3·95 times the full-load current.

After making allowance for a thick end-plate under the pole-shoe to hold the magnet-coil in place and for insulation, the net winding length is, say, 5·6″. The perimeter of the pole when insulated is 23·6″, and if the radial depth of the turns is about 1·6″, the mean length of the turn is l_x = 29·25″ = 0·814 yard. The exciting voltage is 100, but in order to retain complete control over the alternator voltage and allow a margin so that the excitation can be increased in case of a lower power-factor occurring in practice, it will be advisable to allow 10 volts in the adjustable rheostat, leaving 90 at the terminals of the field winding.

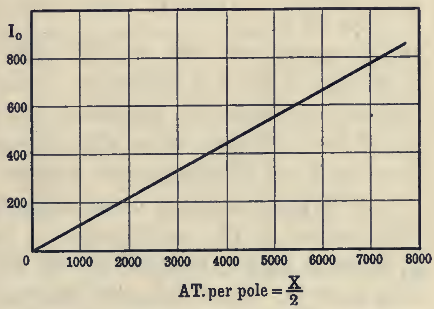


FIG. 536.—Short-circuit current curve of 750-KVA alternator.

Thence by equation (76), $\omega' = \frac{90 \times 100}{7685 \times 64 \times 0{\cdot}814 \times 1{\cdot}15} = 0{\cdot}0196$ ohm per 100 yards, and the necessary area of the wire is $\frac{0{\cdot}0024}{0{\cdot}0196} = 0{\cdot}122$ square inches. Either a thin wide strip wound edgewise in one layer, or a rectangular section wound on the flat in many layers, may be adopted; the section of the former is in itself uneconomical, but the thin insulation which suffices between adjacent turns allows the winding space to be utilised to as great advantage as the stouter section with its double-cotton covering. Thus with bare copper strip $1\frac{1}{2}$″ deep × 0·082″ thick with insulation between the turns 0·015″ thick, $\frac{5{\cdot}6}{0{\cdot}097} = 57\frac{1}{2}$ turns per pole can be obtained. Or deducting one turn from the quotient $\frac{5{\cdot}6}{0{\cdot}45} = 12\frac{1}{2}$ we obtain with rectangular wire 0·435″ × 0·285″ d.c.c. to

$0.45'' \times 0.3''$, $11\frac{1}{2}$ turns per layer and 5 layers, or $57\frac{1}{2}$ turns per pole. In either case there are 3680 turns in all, of length 3000 yards. Their resistance cold is $0.0196 \times 30 = 0.588$ ohm, and when hot 0.676, giving an exciting current of $\frac{90}{0.676} = 133$ amperes. The total loss over the field winding is therefore 11,950 watts = 1.88 per cent. of the net output, or with the additional loss in the rheostat 13,300 watts and 2.09 per cent. of the output. The current density is 1080 amperes per square inch. The outer periphery of the coil being about 33″, its cooling surface with end-flanges is approximately $(33 \times 5.6 + 28 \times 1.5 \times 2) = 270$ square inches, and the ratio $\frac{S_c}{W} = \frac{270 \times 64}{11950} = 1.44$ square inches per watt. The mean peripheral speed of the coil is 4830 feet per minute, whence $1 + 0.35\left(\frac{v}{1000}\right)^{1.3} = 3.71$, and the mean rise of temperature is estimated as $\frac{260}{1.44 \times 3.71} = 48^\circ.6$ F. even with the more heavily insulated wire and in agreement with the low figure assumed for the mean heating coefficient of the coil throughout its entire depth. Either winding can therefore be used; the peripheral speed is not so great as to forbid the use of the rectangular wire, while the section of the edge-wise strip is so thin that it will be troublesome to wind.

§ 7. **Experimental measurement of armature inductance.**—By means of the oscillograph or ondograph the true angle of lag ϕ_o between E_a and the current can be measured * when the alternator is at work on a non-inductive resistance for which $\phi_e = 0$. For under these conditions it follows from equation (177) that

$$2\pi f(L_a + L_c) = \left(r_a' + \frac{E_e}{I}\right) \tan \phi_o \quad . \quad . \quad . \quad . \quad . \quad (177a)$$

The inductance of one phase of the armature varying according to its position relatively to the poles has, when plotted, the characteristic shape of Fig. 537. The electrical degrees are here reckoned from a zero position when the axes of poles and coils coincide; although not always the case, the smallest figure is then usually obtained, the largest value occurring when the axes of coils and poles are crossed.† The temporal value may be expressed as $L = A + B \cos 2\pi f . 2t + L_a$, where $A + B = L_c$ and $A - B = L_d$. If, therefore, an alternating current of the normal frequency is passed through one phase, and the voltage across the ends with the corresponding current be noted, the quotient $\frac{E}{I}$ first

* Blondel, "Tests of Alternators," *Trans. Intern. Electr. Congress, St. Louis*, 1904, vol. i. p. 631.

† Cp. especially Rushmore, *Trans. Intern. Electr. Congress, St. Louis*, 1904, vol. i. p. 742 ff.; and Hobart and Punga, *Elektrische Kraftbetriebe u. Bahnen*, vol. v. p. 611 ff.; cp. also Chap. XXIII. §§ 23 and 24, for other references.

for the coil-sides of the phase exactly over the centre of the field-poles, and secondly with the coil-sides midway between the poles, gives the impedance for the two positions, from which can be determined the two values of the reactance or of the inductance, namely, $L_c + L_a$, and $L_d + L_a$. As the field-system becomes saturated the inductances diminish, especially when the axes of coils and poles are crossed, so that it is very necessary to excite the field-magnet at the same time with its normal current. The value of the alternating current through the phase is of lesser importance, as the armature current has but little effect in saturating the magnetic circuit. From the values of $A + B + L_a$, and $A - B + L_a$, B can be determined, and also $A + L_a$, but it is still necessary to calculate A in order to separate out L_a. This is best done by means of an approximate estimate of L_c, *i.e.* of $A + B$. In calculating L_c from the data of Chap. XXIII. § 11, it must be borne in mind that only a single-phase current is used for the test, and

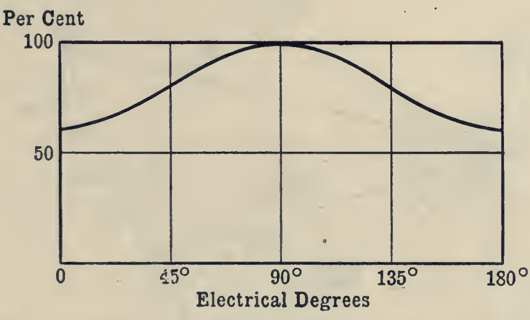


FIG. 537.—Armature inductance as dependent upon position relatively to field-poles.

that k_c must correspond thereto. The calculation of L_d is still more liable to error through the indefiniteness of the amount of damping. The pulsating character of either the cross or direct flux (cp. Chapter XXIII. § 12) introduces, in fact, considerable doubt as to the validity of the results when applied to 3-phase alternators. On the other hand, if a 3-phase current were used for the same experiment with the rotor stationary in its two crucial positions, the current passing through its alternations in each position does not truly represent the facts when the field-magnet rotates at the speed of synchronism. To supplement the above test, therefore, even better information is obtained, *e.g.* in the case of a star-wound 3-phase armature by sending the alternating current through one phase A, and collecting it again from the two other phases B and C in parallel, and again by passing the alternating current through two of the phases, say A and B, in series, the rotor in each experiment being set relatively to the coils of phase A in its correct position corresponding to these momentary cases. The phase A is thus at least placed under the conditions which would actually hold at the moments when the working current was passing through its maximum or through 0·866 of its maximum value, and the inductance includes, as it should, any mutual inductance in these positions from the other phases.

§ 8. **The testing of alternators.**—If a pair of similar alternators are available, the best method of determining their efficiency is by a modification of the Hopkinson test, which renders it suitable for alternating currents. The two machines are coupled rigidly together, but with a slight difference in the positions of their armature coils relatively to the poles in the two cases, so that a difference of phase

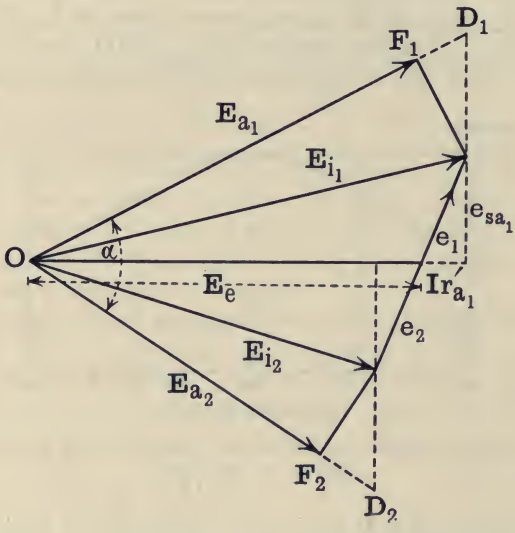


FIG. 538.—Two alternators mechanically coupled together, with external current in phase with terminal voltage.

of the induced E.M.F.'s results. When the combination is now driven through a transmission dynamometer or through a continuous-current motor of which the efficiency and losses at different speeds are accurately known, the one alternator acts as a motor and the other as a generator; by altering the difference of phase of the two machines the input of the one and output of the other are regulated until they reach the required amounts.* Then, as in Chapter XX. § 10, if W^1 be the output of the generator as measured by a wattmeter, and L be the total power supplied from the external source less the loss of energy over the two

* A complete account of such a test is to be found in *Proc. C. E.*, vol. cxxx. p. 247, "Alternating-Current Dynamo Tests" (H. F. Parshall). Cp. *Electr. Eng.*, vol. xxxvii. p. 375.

armature resistances, *i.e.* $L = W - I^2 (R_{aD} + R_{aM})$, the efficiency of the generator is similarly to equation (161)

$$\eta = \frac{W^1}{W^1 + I^2R_{aD} + \frac{L}{2} + A_x^2R_x}$$

With particular values of the two excitations the current at the terminals will be in phase with the terminal voltage E_e; the vector diagram of the two alternators in parallel will then be as in Fig. 538, the generator acting as if working on a non-inductive resistance. Hence if a comparison of the readings of a wattmeter with those of an ammeter and voltmeter across the terminals shows that the current is in phase with E_e, by equation (177*a*) the value of $L_a + L_c$ can be determined as

$$2\pi f (L_a + L_c) = \left(r_a' + \frac{E_e}{I}\right) \tan \frac{\alpha}{2}$$

where α is the total phase difference between the two machines.

Since Ir_a' is small as compared with E_e, it is evident that in Fig. 538 D_1D_2 will almost become a straight vertical line, and the armature E.M.F.'s and fluxes of the two alternators are then very nearly identical. The terminal E.M.F. vector falls midway between E_{a1} and E_{a2}, and since ϕ_o has the same value in both cases, namely, $\pm\frac{\alpha}{2}$, it results that the excitations are equal, the direct magnetising ampere-turns of the armatures being in each case demagnetising; for though the current is oppositely directed in the motor, it has also the opposite phase relation to the induced E.M.F.

If the two alternators are coupled together without any difference of phase, and they are unequally excited (Fig. 539), the vector difference $\overline{E_{i1} \,.\, E_{i2}}$ between the induced E.M.F.'s, or C_1 C_2, is the fall of potential over the impedances of the two armatures in series, corresponding

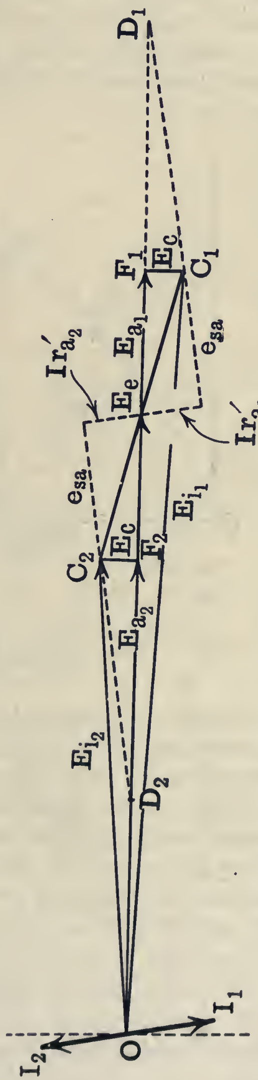


FIG. 539.—Two alternators mechanically coupled together without difference of phase.

to the resultant current which is displaced by nearly 90° from the E.M.F.'s. Since one of the components of the vector C_1C_2, namely, $I(r_a{}'_1 + r_a{}'_2)$ is small, C_1 C_2 is nearly equal to F_1 F_2, *i.e.* to the difference between the armature E.M.F.'s $E_{a1} - E_{a2} = 2e_{sa}$. Whatever occurs in the generator armature is reversed in the motor armature, so that E_{CM} has the same numerical value in the two machines, and so also has X_{DM}, but in the generator it is demagnetising and in the motor forward-magnetising. On the apparent synchronous impedance assumption, therefore, the open circuit E.M.F. of the generator would be $E_{o1} = OD_1$, to which would correspond X_1, while the open circuit E.M.F. of the motor would be $E_{o2} = OD_2$, to which would correspond X_2. Knowing the two excitations X_1 and X_2, and adopting a single total characteristic

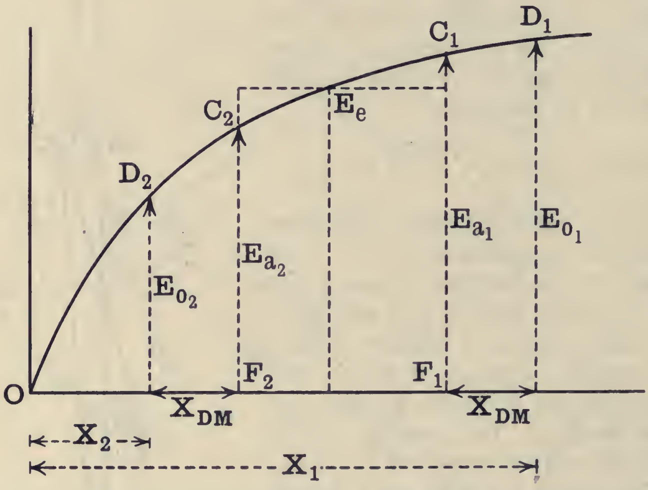


FIG. 540.—Determination of X_{DM} and e_{sa} from excitations of two alternators mechanically coupled together.

of excitation with approximate allowance for the full back turns (the transverse reaction being practically negligible), we can thence check the assumed values of e_{sa} and X_{DM} by the construction of Fig. 540. On either side of the point on the characteristic corresponding to the terminal voltage, mark off two points C_1 and C_2, the one as much higher than E_e as the other is lower, *i.e.* by the amount e_{sa} in each case; the differences $X_1 - OF_1$ and $OF_2 - X_2$, *i.e.* the amount by which X_1 exceeds OF_1 and X_2 falls short of OF_2 must then each be equal to X_{DM}, demagnetising in the one case and magnetising in the other, while $E_{a1} - E_{a2}$ must yield the observed current.

The three phases of two similar 3-phase machines can be similarly coupled, care being taken that the measuring ammeter and wattmeter do not interfere with the balance of the phases. Analogous methods of determining the various factors when only a single machine is available by running it as a synchronous motor have been described by Prof. André Blondel (*Trans. Intern. Elect. Congress, St. Louis*, 1904, vol. i. p. 625).

An extension of the Hopkinson method of testing to the case of a single alternator has been given by Mr. Mordey.* By it the armature coils were divided into two unequal groups and connected in opposition, one group acting as a motor, and the larger group as a generator, so that only the difference of their E.M.F.'s was effective in circulating an almost wattless current round the armature, exactly as in Fig. 474. With the heteropolar type of field-magnet and numerous poles it is equally easy to divide the poles into two unequal groups and connect these in opposition, two adjacent poles at each end of the groups having the same polarity. But the objection to both variations is that the motor fields are powerfully strengthened by the armature current and the generator fields similarly weakened. The inductions are therefore very different, and the motor group must be reduced in number to allow for its much greater density of flux. Unbalanced magnetic pulls are then set up between the rotating and stationary parts, and vibration ensues.

To overcome this objection, Mr. B. A. Behrend has suggested dividing the field coils into two equal sets and exciting them with different strengths of field-current, so that when the armature reaction is superposed the inductions of the resultant fields again become much more nearly equal in the motor and generator groups.† The same difference in the total fluxes of generator and motor poles, necessary to circulate the armature current, is in fact spread over the pair of groups instead of being obtained by many weak poles overbalancing a few strong poles, and the current is not so much out of phase. When the number of poles is large the method is found to yield satisfactory results. Other modifications have also been suggested by Mr. S. P. Smith ‡ which reach the same end, namely, the elimination of vibration, even more perfectly.

The object of all such methods is to do away with the necessity for a full-load prolonged run at the makers' works in order to determine the heating and regulation. With the large alternators of modern times it may not be possible to supply the necessary driving power at full-load at the makers' factory; hence the development of such compromise tests which only call for driving power to make up the losses. In practice, too, the regulation can be calculated from the open-circuit characteristic and low power-factor or short-circuit tests quite as accurately as it can

* "On Testing and Working Alternators," *Journ. Inst. Electr. Eng.*, vol. xxii. p. 116.

† *Trans. Intern. Electr. Congress, St. Louis*, 1904, vol. i. p. 528.

‡ "The Testing of Alternators," *Electrician*, vol. lv. p. 508, and especially *Journal Inst. Electr. Eng.*, vol. xlii. p. 190, with discussion.

be taken by direct observation under the disadvantageous conditions of testing at the factory, when it may be difficult to maintain perfectly constant steam pressure and speed.*

The retardation method of testing the various separate losses under different conditions is especially suitable to the case of alternators, owing to the large moment of inertia of the rotor which allows of the readings being taken very accurately. The results of such a test on a 150 kilowatt alternator are given by M. Routin in *L'Éclairage Électrique* (vol. ix. p. 170), and an electrical method of measuring the change of speed as the alternator slows down has been suggested by Dr. Sumpner.†

A truly non-reactive circuit for absorbing the output of an alternator on a full-load test is not easily obtained. A water-bath is non-inductive and is the most convenient form, but it may possess some capacity, so that there remains some doubt ‡ whether cos ϕ_e is 1. With wire coils or transformers lightly loaded, a wattmeter must be used to determine the true power. Special choking coils for use in combination with a water bath to give a definite power-factor have been devised and are of great convenience.§

In addition to other tests, the insulation resistance of an alternator armature should be measured, and its ability to withstand puncture under the application for one minute of an alternating potential of sine form and of R.M.S. value twice or thrice that of the machine. With high potentials the pressure should be measured with a static voltmeter or spark-gap, as the multiplying ratio of the testing transformer is affected by the capacity current which flows into the machine as a condenser. In certain cases it becomes advisable to test the effect of a short-circuit on the alternator, in order to ensure that the coils are sufficiently stiff and well supported to withstand the mechanical stress from the abnormal current which temporarily passes.

§ 9. **Efficiency of alternators.**—The efficiency of polyphase alternators varies but little from that of continuous-current dynamos of the same output and speed, so that it is equally well represented by the curves of Figs. 414 and 415, the losses in the exciter not being included. ‖ The exciting energy varies from about 3 per cent. of the output in a 30-

* For suggestions as to testing of alternators in the factory, see R. Goldschmidt, "Artificial Loading of Alternating-Current Machinery," translated from *E.T.Z.* in *Electr. Eng.*, vol. xxviii. p. 848 ; S. Senstius, *Trans. Amer. I.E.E.* "Heat Tests on Alternators," vol. xxv. p. 311, with discussion.

† *Journ. Inst. Electr. Eng.*, vol. xxxi. p. 634.

‡ But cp. Dr. W. M. Thornton, *Journ. Inst. Electr. Eng.*, vol. xxxv. p. 157, and M. B. Field, *Journ. Inst. Electr. Eng.*, vol. xxxii. p. 651, and K. Wallin, *E.T.Z.*, vol. xxvii. pp. 739–40.

§ "Artificial Load for Testing Electrical Generators," by R. K. Morcom and D. K. Morris, *Journ. Inst. Electr. Eng.*, vol. xli. p. 137.

‖ Cp. R. Goldschmidt, *Journ. Inst. Electr. Eng.*, vol. xl. pp. 466–468.

kilowatt alternator to 2 per cent. in a 500-kilowatt machine and 1 per cent. in machines of 800 kilowatts or over; but great differences exist between different types, and even in the same machine the amount varies greatly with the nature of the load if the regulation is much affected by the presence or absence of external inductance. Indeed, the increased field excitation required for a power-factor of 0·8 as compared with unity will as a general rule lower the efficiency of the whole machine some 1 or $1\frac{1}{2}$ per cent. With inductor generators for a power-factor = 1, the loss in excitation is less than in heteropolar machines, and may be brought as low as $\frac{1}{2}$ per cent., but this advantage becomes less marked as the power factor is decreased. The copper loss in the armature varies in machines of 500 kilowatts and upwards from 1 to 2·5 per cent. of the output at full-load, and the loss by eddy currents and hysteresis varies from 3 to 4 per cent., so that in general the latter is the greater of the two. The one may, in fact, be reduced at the expense of the other, and if a high efficiency is required at light loads, the copper loss in the armature should be given a higher ratio to the constant loss in the armature iron.

The estimate of the losses from hysteresis and eddy-currents is a matter of some uncertainty. Messrs. Hobart and Punga give as an approximate formula for the core loss at no-load,

$$\text{watts} = k\left\{\left(\frac{B_a}{1000}\right)^2 . \text{ kilogrammes of core} + \left(\frac{B_t}{1000}\right)^2 . \text{ kilogr. of teeth}\right\}$$

where k for 50 periods and ordinary discs 0·5 mm. thick ranges from 0·085 to 0·1. But with open slots the eddy currents in the conductors near the openings may increase k 10 per cent. or more.*

Owing to the smaller number of slots and their comparatively large size in the alternator, the eddy current effects in the pole-pieces are rather different in character from those in continuous-current machines; usually with one slot per pole per phase the frequency is lower, and even when the slots are half-closed the ratio of the width of opening to the air-gap is large. The shape of each half-wave of E.M.F. is dissimilar, which may be traced to the effects of hysteresis in the teeth; and the shapes of the complete waves differ considerably in different parts of the pole-face, while further they vary greatly according to the magnitude and nature of the armature load, whether inductive or having capacity.† Experiments emphasise the great difference in the magnitude of the effects from entirely open as compared with closed slots.

* *Elektrische Kraftbetriebe und Bahnen*, vol. v. p. 568.

† Cp. "An Investigation into the Periodic Variations in the Magnetic Field of a Three-phase Generator by means of the Oscillograph," by G. W. Worrall and T. F. Wall, *Journ. Inst. Electr. Eng.*, vol. xxxvii. p. 148, and "Magnetic Oscillations in Alternators," by G. W. Worrall, *Journ. Inst. Electr. Eng.*, vol. xxxix. p. 206, and vol. xl. p. 413.

The eddy-currents in the solid conductors of an alternator which are set up by the armature current itself, or, more strictly speaking, the loss by non-uniform distribution of the current in the bars and its calculation have been fully discussed in papers by Messrs. M. B. Field * and A. B. Field.† Using the curves of Mr. Field, Messrs. Hobart and Punga ‡ have calculated, in the case of the 5000-kw. Siemens-Schuckert alternator described in Chap. XXVII. § 7, that the ratio $\frac{\text{actual copper loss}}{\text{ohmic loss}}$ was 1·48 in the case of the solid conductors at the bottom of the slots and 5 in the case of the laminated conductors in the halves of the slots nearer the openings. Each of these increases only applies to the portions of the conductors within the slots, *i.e.* to $\frac{51 \text{ cm.}}{284 \text{ cm.}} = 0{\cdot}18$ of the whole, therefore the ratio for the whole armature phase works out to $0{\cdot}64 + 0{\cdot}18(1{\cdot}48 + 5) = 1{\cdot}8$, which shows that the additional loss is as much as 80 per cent. of the ohmic loss. Actually, too, the increase would be somewhat more owing to a similar effect in the end-connections from their own secondary leakage or inductance.

Even in the 3-phase alternator the armature reaction is not strictly constant, but, as previously mentioned, varies periodically; experiment shows that an harmonic of triple or sextuple frequency according to the nature of the winding § results therefrom, and this oscillation of the field sets up eddy-currents and hysteresis in the pole-shoes and fluctuation in the exciting current.||

All causes combine to render the loss by eddy-currents and hysteresis greater in the single-phase than in the 3-phase alternator, and on this account the efficiency of the former is appreciably lower than that of the latter. The chief reason is found in the fluctuation of the armature reaction in the single-phase alternator, owing to which a current in the armature of frequency f sets up eddy-currents in the field-magnet of twice that frequency as described in Chapter XXIII. § 12. Thus in the case of the 385-kilowatt machines of frequency 100 when running at 300 revs. per min., of which the full tests are given in the paper of Mr. Parshall cited in § 8, the eddies in the armatures of the two machines amounted to about 1400 watts on no-load, but when the

* *Journ. Inst. Electr. Eng.*, vol. xxxvii. p. 83, "Idle Currents."

† *Trans. Amer. Inst. Electr. Eng.*, vol. xxiv. p. 761, "Eddy-Currents in Large Slot-Wound Conductors."

‡ *Elektrische Kraftbetriebe und Bahnen*, vol. v. p. 569.

§ Cp. W. M. Thorton, *Journ. Inst. Electr. Eng.*, vol. xxxv., p. 156, and G. W. Worrall, *Journ. Inst. Electr. Eng.*, vol. xxxix. p. 215, with discussion thereon.

|| Upon the fluctuation of the inductance of the armature appears also to depend the fact that an alternator can be run with some load even when its field excitation is zero; cp. H. Herman, "Parallel Operation of Alternators," *Electr. World*, vol. li. p. 1100, and Dr. C. P. Steinmetz in *Trans. Amer. Inst. Electr. Eng.*, vol. xxvi. part II. p. 1047 (discussion on "Interaction of Synchronous Machines").

armature current was given values corresponding to about half, three-quarters, and full-load, the additional eddy-loss in the field-magnets of the two machines was as follows :—

Current in System.	Additional Eddy-loss in Generator and Motor.
84·2 amperes	5,820
126·4 "	10,560
177·8 "	21,000

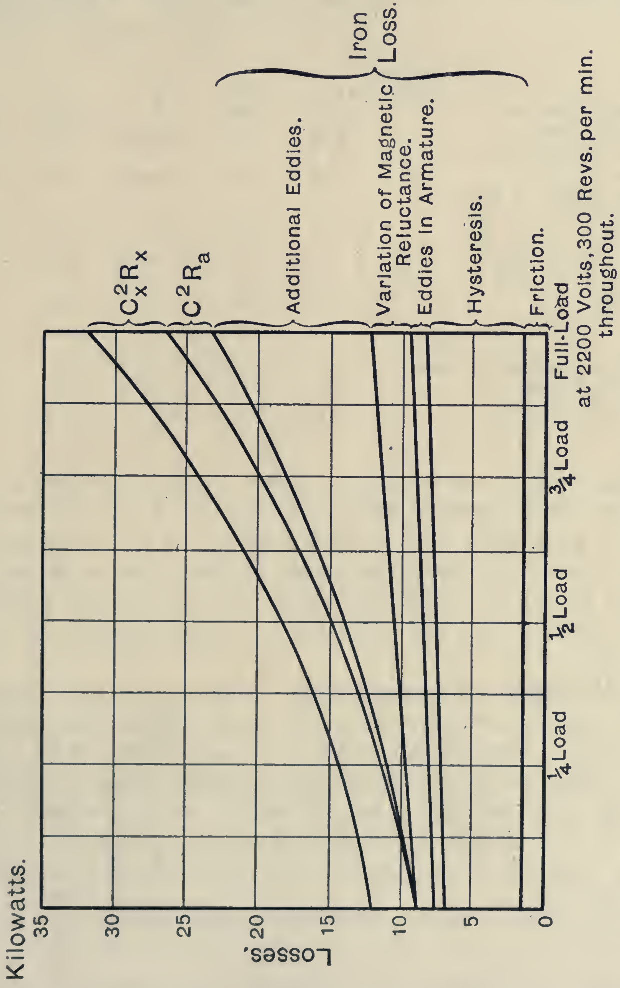


FIG. 541.—Losses in 385-KW. single-phase alternator.

It thus increased approximately as the square of the current, and the total iron loss is shown (Fig. 541) to be by no means a constant quantity in the single-phase alternator. Owing to the completeness of the tests the following figures may be added as an accurate analysis of the various losses in a single-phase alternator of high frequency for electric lightning, when working on a non-inductive load:—

		No Load.	¼ Load.	½ Load.	¾ Load.	Full Load.
External power in kilowatts		0	96·25	192·5	288·75	385
Current in ampères		0	43·75	87·5	131·25	175
Terminal volts		2200	2200	2200	2200	2200
Friction	per cent. of input	12·5	1·36	0·71	0·48	0·36
Hysteresis in armature	,,	44	4·98	2·8	2·015	1·62
Eddies in armature	,,	5·85	0·905	0·544	0·415	0·288
Variation of magnetic reluctance	,,	11·75	1·45	0·946	0·767	0·696
Additional eddies in magnet due to armature current	,,	0	1·175	1·66	2·11	2·645
Copper loss in armature	,,	0	0·226	0·426	0·608	0·816
Copper loss in field-winding	,,	25·9	2·98	1·66	1·375	1·272
Total loss as percentage of input		100	13·076	8·746	7·71	7·697
Percentage efficiency		0	86·93	91·25	92·29	92·31
Total input in kilowatts		12	110·5	211	313	417

In large single-phase alternators driven by steam-turbines at high speeds and low frequencies such as 25 cycles per sec., the armature reaction is great owing to the large pole-pitch, and in consequence the pulsation of the flux is also considerable, so that it may be necessary to reduce the amount of the eddy-currents by the use of copper dampers * or bars dovetailed into the pole-faces and short-circuited by copper rings or discs.

§ 10. **Weights of alternators.**—According to the frequency, degree of regulation, the fly-wheel effect introduced into the rotor, etc. the weight of an alternator per watt per rev. per min. just as the specific torque constant shows very great variations under different conditions. As a rough approximation it may be said to be proportional to the square root rather than to the ⅔rds power as in continuous-current dynamos, and has such values as are given by the relation $W = 1260\left(\frac{VA}{N}\right)^{\frac{1}{2}}$ lbs. For large slow-speed machines with $\frac{VA}{N} > 10{,}000$

* For figures as to the serious nature of the loss in the single-phase as compared with the 3-phase alternator and as to the effect of dampers, see W. L. Waters, "Modern Development in Single-phase Generators," *Trans. Amer. Inst. Electr. Eng.*, vol. xxvii. July 1908. Also Hobart and Punga, *Elektr. Kraftbetriebe und Bahnen*, vol. v. p. 570, for tests on the 5000-kw. alternator described in Chapter xxvii. § 7.

and $D^2_{\prime\prime}L_{\prime\prime} > 300{,}000$ and a high dimensional constant of say 27·7, this becomes $W = 240(D^2_{\prime\prime}L_{\prime\prime})^{\frac{1}{2}}$ lbs. Owing to the different relation of the diameter and length in such machines as compared with continuous-current machines, the increase in weight with increasing $D^2_{\prime\prime}L_{\prime\prime}$ is not so rapid. But with smaller alternators with $\frac{VA}{N}$ between 1000 and 10,000 the two types are more similar, and the weight of the alternator becomes approximately $W = 28(D^2_{\prime\prime}L_{\prime\prime})^{\frac{2}{3}}$, except in the case of turbo-alternators, which form a class by themselves.

CHAPTER XXVI

THE WORKING OF ALTERNATORS IN PARALLEL

§ 1. **Parallel working of alternators.**—The series operation of alternators is not possible unless they are connected together by a rigid mechanical coupling. If only connected together electrically, they exert no mutual control on one another; if for any reason one machine falls slightly behind another in phase, the lagging machine has the greater part of the load thrown upon it, which causes it to lag still more, until finally the two settle down into exact opposition of phase, their E.M.F.'s neutralise each other, and no current flows.

Alternators can, however, be run in parallel, the mutual control which they exert electrically upon one another being in most cases sufficient to keep them "in step," as it is termed. If two alternators be running at the same frequency, and further with equal E.M.F.'s rising and falling together, that is to say, if they *synchronise* in phase, their armature circuits may be coupled together, and they will continue to run in parallel, dividing the load between them in proportion to the power supplied to each. If a generator is to be thrown into parallel with another, or is to be joined to a network on which other generators are already working, then at any instant there should be no difference of potential between any pair of corresponding terminals which are to be united together, so that no cross current will flow between the machines due to such difference of potential. Thus there are required for parallel operation (1) *equality of frequency*, (2) *synchronism of phase*, and of less importance, (3) *equality of voltage*. The general principle of the connections for working two low-tension 3-phase alternators in parallel is shown in Fig. 542. Machine A is at work, its triple-pole switch S being closed on the three leads, *a*, *b*, *c* of the distributing network. The current in one phase is read on the ammeter I, while the wattmeter W measures the power of one phase, its main terminals being in series with one main lead and its shunt coil being connected to the ends of one arm of the Y winding, *i.e.* between a' and the centre of the Y winding. The shunt-wound exciter E supplies the exciting current to the field-magnet M with a regulating rheostat R in series. The machine B is now run up to its full speed, and its excitation regulated by means of the rheostat R′ in series with its field-winding until the voltmeter V′ gives the same reading as the voltmeter V. The effective voltages of the two machines are now alike, and assuming their E.M.F. curves to be similar, so also are the maxima value of the E.M.F.'s. A further and finer regulation of the speed and phase must now be made by means of the phase lamps PP belonging to machine B. It will be seen that in virtue of their connection they are in series between two arms a' and c' of the armature winding on the one side and two wires a and c of the network on the other side, so long as the main switch of B is open; they therefore indirectly measure by their light the difference of potential between these points, and when this is zero their light goes out, and the triple-pole switch of B may be closed. When

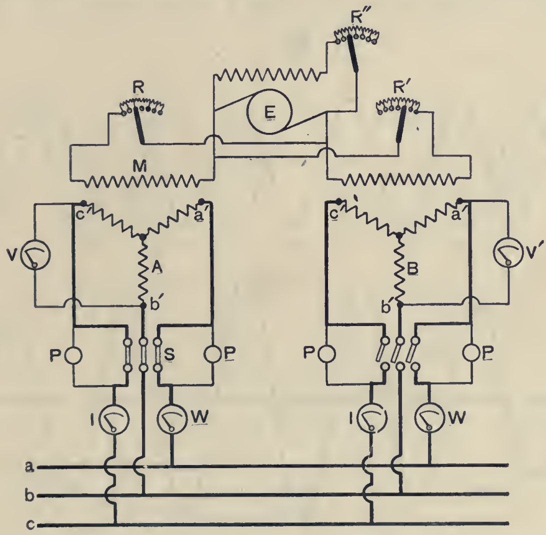


FIG. 542.—Connections for two 3-phase machines to be worked in parallel.

the machines are completely out of step, in the arrangement shown, the voltage on each pair of lamps may rise to the sum of the terminal volts of two machines.

Let us now suppose that the periodicity of machine B is not quite the same as that of the network or of machine A. Starting from a point of time when the instantaneous voltage of both machines is zero, let the full-line curve E_1 (Fig. 543) represent the instantaneous E.M.F. between the wires a and c, while the dotted curve E_2 represents the instantaneous E.M.F. between the terminals $a'c'$ of machine B. The algebraic difference of the two shown below by the heavy full line e_c gives the difference of potential on the lamps, proportional to which is the current through them. From the nature of the resultant curve it will be seen that the lamps are traversed by an alternating current of which the amplitude waxes and wanes in accordance with the effective value of the current as determined by its amplitude at the time; in other words, periods of brightness and darkness alternate. The number of complete waves of the alternating current through the lamps itself coincides with the number of periods of the more rapid of the two periodic voltages, and one "beat" or pulsation of the light from complete darkness to maximum brightness and back to darkness occurs in the time in which the number of complete periods passed through by the voltage E_1 and the voltage E_2 differs by 1 (in our figure 8 and 7 respectively), *e.g.* if the periodicity of the machine A be $f = 50$, and that of machine B is only 49, at the end of one second the difference of their numbers of periods

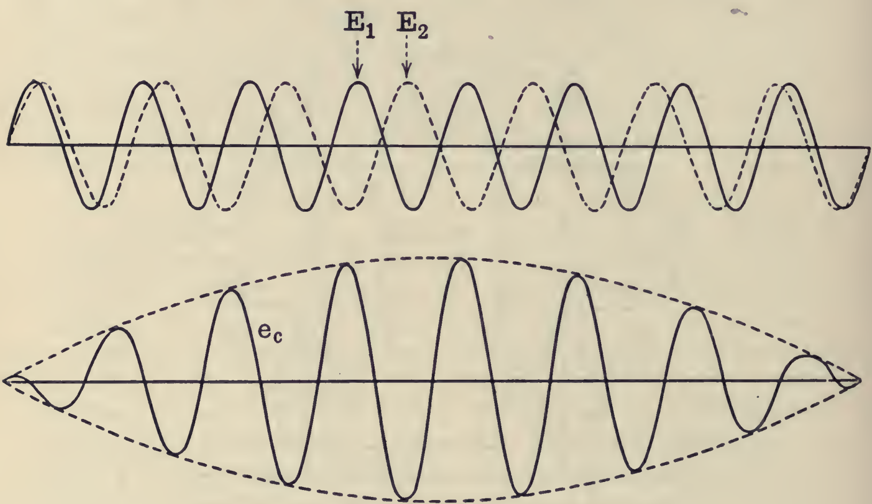


FIG. 543.—Pulsating voltage on phase-lamp.

will be 1, and the lamps will have passed through one pulsation. One beat per second of the light will therefore indicate a difference in the periodicities of $\frac{1}{50} = 2$ per cent. But by regulation of the driving power each pulsation of the light may easily be extended to, say, 5 seconds, when the difference of the periodicities will be only $\frac{2}{5}$ per cent. or to 10 seconds with large machines. Complete synchronism may not be exactly obtained, yet if the triple-pole switch is now closed *in the middle* of an interval of darkness, but little cross current will flow, and the two machines will fall into step.

The two instruments of Fig. 542 enable a machine to be conveniently taken out of parallel as follows. The steam stop-valve or inlet gate of the turbine is closed until the wattmeter reads zero. If the ammeter still shows some cross-current passing, the field rheostat of the machine is altered until it is reduced to zero, when the main switch can be opened without disturbance to the machines still remaining at work and without spark.

§ 2. **Phase indicator.**—The arrangement of phase lamps shown in Fig. 542 is the simplest form of *synchroniser* or *phase indicator*, to ensure that the difference of phase between the two machines may not be too great at the moment when they are thrown into parallel. It is, however, only suitable for low-tension machines, and many other devices are employed based on the same or similar principles. Thus the method may be immediately adapted to high-tension machines, if, instead of the phase lamps being directly connected between the

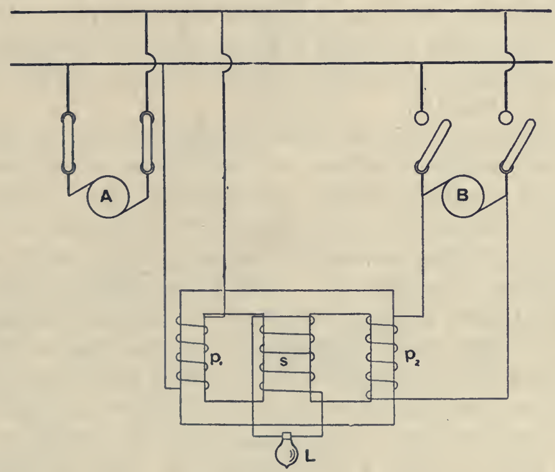


FIG. 544.—Phase indicator for parallel working.

corresponding terminals, the primaries of two small transformers be inserted, to the secondaries of which the lamps or a voltmeter are applied as before. Or the secondaries of two small synchronising transformers are connected in series with one or more lamps in circuit. A still more common variation is to employ a single transformer with two primaries wound on it; one of these p_1 is connected to the bus bars, and the other p_2 to the terminals of the alternator which is to be thrown into parallel as in Fig. 544, which shows the arrangement for single-phase machines. A common secondary s has its terminals connected to an incandescent lamp, and the advantage is gained that the phase is indicated without any direct connection of the machine to the network; when the phases are in synchronism both poles of the incoming alternator B are for the first time connected to the bus bars by the closing of the double-pole switch. The primaries may be arranged to oppose one another when the machines are in step, so that the lamp is dark at the proper moment for closing the switch (synchronising "dark"), or they may be arranged so that when the E.M.F.'s synchronise the lamp burns brightly (synchronising "bright"). The latter is perhaps the best, since the moment of maximum brightness is better distinguished than the middle of intervals of total darkness, and, when synchronising "dark," rupture of the filament of the lamp might happen to coincide with the coming of the machines into step. In either case the lamps should not be fully incandesced, but should be run below their true voltage in order that the eye may not become fatigued by their brilliancy. Small candle-power high-voltage lamps are best, since their thin filaments heat and cool quickly with less time-lag behind the variations of current. A preferable arrangement is obtained with a hot-wire parallelling voltmeter in place of the lamps, the machines being switched into parallel when the voltmeter gives its maximum reading. The determination of this permits of greater accuracy, and the instrument has little or no time-lag.

The above methods do not indicate whether the want of synchronism is due to the incoming machine running too fast or too slow, and to effect a good parallel may take a considerable time. Hence various forms of synchroniser have been devised which possess the additional advantage of signalling whether the incoming prime mover requires to be slowed down or speeded up. One of the earliest of these was the Siemens and Halske 3-phase synchroniser, in which three lamps arranged in a triangle brighten in a clockwise or counter-clockwise order of succession according as the speed of the incoming machine is too high or low. Various purely optical methods have also been devised, but more recently "synchroscopes" depending upon the action of a small induction motor, as the Everett-Edgecumbe rotary synchroniser or again the Lincoln synchroniser, have come into extensive use. In these the pointer moves in one or the other direction according as the incoming machine is too fast or too slow. The Westinghouse Company have also introduced an automatic synchroniser for use in large stations with machines of great power, and electrically or electro-pneumatically controlled switches.*

While in normal working it is sufficient to synchronise one phase of a polyphase machine, when a new machine is first set to work it is necessary to verify the connections of each phase, or at least of two out of the three phases.

* Cp. A. Hay, *Alternating Currents*, §§ 77, 78; F. H. Davies, "Synchronising Devices," *Electr. Eng.*, vol. xxxii. p. 513; *Electr. Eng.*, vol. xxxiv. p. 876, vol. xxxix. p. 466A; P. MacGahan and H. W. Young, "Synchronising," *The Electric Journal*, vol. iv. p. 485; Franklin and Williamson, *Alternating Currents*, 2nd ed. p. 288; and A. R. Dennington, *Electr. World*, vol. li. p. 771.

§ 3. **Alternator vector diagrams.**—The case of two synchronous alternating-current machines running in conjunction, whether both as generators on an external network or one as generator and the other as motor, the former also feeding an external network, may be represented diagrammatically in two ways, either (1) by a clock diagram of vectors whose directions and signs have reference to each of the two branch circuits ABEFG and CDEFG, which the two machines respectively form with the external network, or (2) by a diagram of vectors whose directions and signs have reference to the series circuit ABEDCG existing between the two machines (Fig. 545).

On the first method of representation the vectors of the terminal E.M.F.'s which are necessarily equal in the two machines must also be shown as coincident; so also must the vectors of the currents I_{e1} and

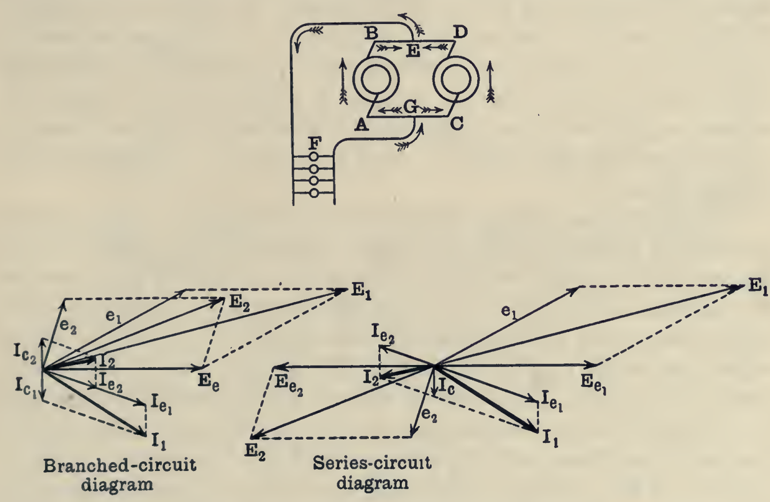


FIG. 545.—Two methods of representing alternators in parallel.

I_{e2}, supplied by each to the network when both are acting as generators, coincide in direction, although not necessarily equal in magnitude. But these currents may be only part of the total currents passing through the machines; there may be also a permanent "corrective" component I_c flowing round the circuit ABEDCG between the two machines in series, if their respective excitations and steam supplies are not so balanced that their terminal E.M.F.'s are naturally equal. Or, if the phases of the armature E.M.F.'s are not at any moment correctly related to the excitations and steam supplies, there may be a temporary or fluctuating "synchronising" component I_s tending to supply the necessary correction of phase; or, finally, one machine may be driving the other as a motor, so that the only current through the latter is a series current. The vectors of any such "series" current or component, whether corrective or synchronising, must in the two machines necessarily have opposite directions in relation to the two branch circuits which are the basis of the present method of representation, and must be equal when only two machines are assumed, since the one supplies it to the other.

On the second method, the terminal E.M.F. of one machine at EG, or the bus-bar voltage, appears as diametrically opposite to the terminal E M.F. of the other machine. The shares of the total external current which are supplied by each to the external network now appear as opposite in direction, while the vector of any corrective or series current has the same direction for both.

Method (1), which may be shortly described as the "branched-circuit" diagram, is the more appropriate to the case of two alternators working in parallel, and is thus to be adopted for our present purpose. Method (2), or the "series-circuit" diagram, is the more appropriate to the case of a synchronous motor driven by a source of alternating voltage, since, if the external network is suppressed, the diagram is reduced to the form most suitable to a transmission of power between a single generator and motor in series with a pair of intervening transmitting lines.

By the branched-circuit method the E.M.F. actually induced in the armature of each machine or E_i is to be regarded as the vector sum of the terminal voltage E_e and the voltage consumed by the impedance of the machine $e = z_a I$, but since the angle between the two components may be more than 90° when they are transferred to the common centre, either component may be larger than the induced E.M.F. which they combine to equal. In the series method of representation, especially when used for the synchronous motor case, the bus-bar voltage may be treated as combining with the induced E.M.F. of the motor to give a resultant E.M.F. which is the voltage consumed by the impedance of the armature of the motor. The working of the synchronous motor and of the alternator in parallel is, in fact, intrinsically the same. *E.g.* an over-excited synchronous motor run at the far end of a transmission line, and absorbing a leading current so as to raise the power-factor, may from many points of view also be considered as a generator run in parallel with the station generators and supplying the necessary lagging current at the far end, so that the transmitting line is relieved from the necessity of transmitting anything but the working current.*

Thus the whole problem of the representation of the working of alternators in diagram form may be said to consist in finding the possible value which the third vector may take with given values of the other two vectors in order to form a closed triangle, and it is of the greatest importance to adopt a definite method of representation of the

* Cp. M. B. Field, "Idle Currents," *Journ. Inst. Electr. Eng.*, vol. xxxvii. p. 87.

three vectors which may be applied to different problems without fear of confusion.

§ 4. **The power as dependent upon the phase angle between E_i and E_e.**—The possible values of the impedance-voltage vector $e = z_a I$ are with constant values of E_i and E_e entirely dependent upon the phase angle γ between the induced and terminal E.M.F.'s; further, for any one possible value of the current (except the minimum or maximum) there are always two possible phase relations between E_i and E_e, in one of which E_i leads before E and in the other lags behind it by the same angle. From the position of e is also fixed the phase of the current which lags behind e by the angle ϕ_a, so that what

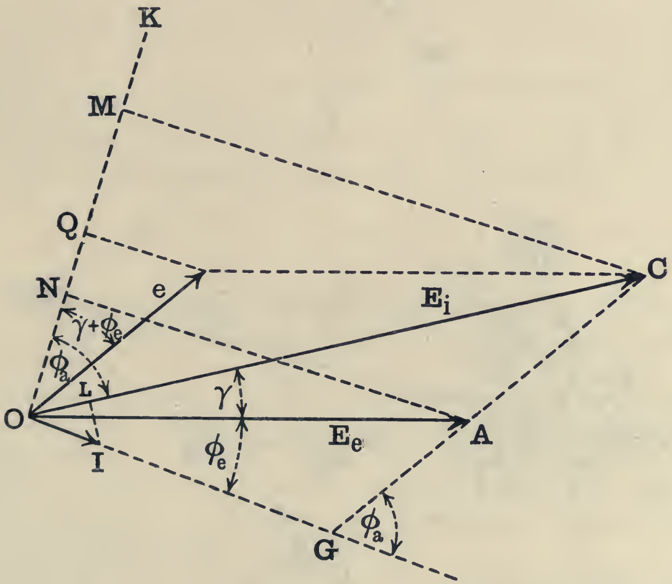


FIG. 546.—Total power of alternator as dependent upon variable angle γ.

is obtained by an analysis of the possible values of e for different values of the angle γ is a knowledge of the different values which the *power* of the machine can assume, and whether this power is positive, *i.e.* the machine acting as a generator, or negative, when the machine is acting as a motor.

The total rate at which energy is developed by the armature of the alternator is $E_i I \cos(\gamma + \phi_e)$, and is therefore given by the product of the induced E.M.F. OC with the projection upon it of the current or OL (Fig. 546). At the angle ϕ_a ahead of the current vector OI let a line be drawn on which falls the E.M.F. consumed by the impedance z_a of the armature, *i.e.* AC, which is also shown as e when transferred to the centre. Draw OK at the angle ϕ_a ahead of the induced E.M.F.; this line is then also as much ahead of or behind e as E_i is ahead of or behind I; *i.e.* the angle between OK and e is $\gamma+\phi_e$. The projection of e upon OK is therefore $OQ = e\cos(\gamma+\phi_e) = z_a I\cos(\gamma+\phi_e)$. The component OQ divided by z_a thus gives the working component of the current or OL. This current can then be expressed in terms of the assumed constant quantities E_i and E_e and of the variable γ if OQ can be so expressed, and this is effected when it is seen that OQ is the difference of the projections of E_i and E_e upon OK. The angle between E_i and OK is ϕ_a, and between E_e and OK is $\gamma+\phi_a$; therefore

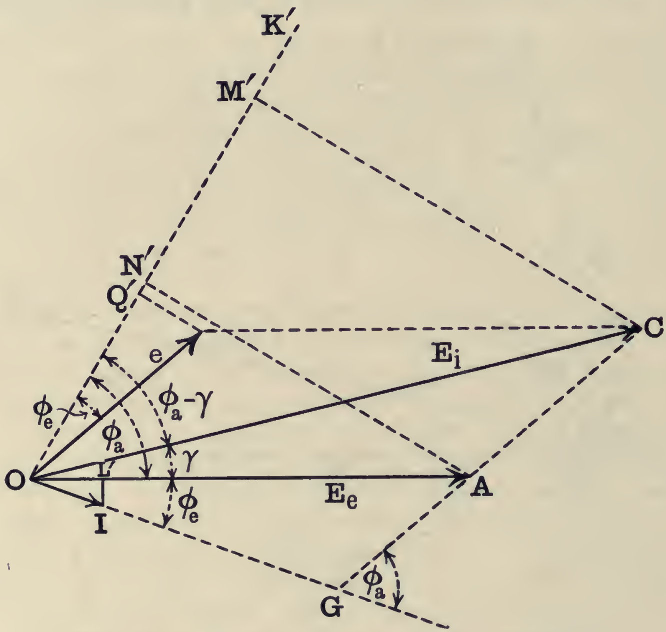


FIG. 547.—Useful power of alternator in terms of variable angle γ.

$OQ = OM - ON = E_i\cos\phi_a - E_e\cos(\gamma+\phi_a)$. The total rate at which energy is developed by the machine electrically as a generator or mechanically as a motor is thus

$$P = \frac{E_i}{z_a}\{E_i \cdot \cos\phi_a - E_e\cos(\phi_a+\gamma)\} \quad . \quad . \quad . \quad . \quad . \quad (187)$$

In precisely the same way is found the expression for the rate at which energy is delivered to the network, or is developed from it in the armature of the machine. Draw OK′ ahead of the vector e by the angle ϕ_e, so that it is as much ahead of or behind e as E_e is ahead of or behind I (Fig. 547). The rate at which energy is received by the network from the machine, or is received by the alternator from the network, is $E_e I \cos \phi_e$, and is given by the product of the terminal E.M.F. with the projection upon it of the current vector or OL′. The projection of e upon OK′ is $OQ' = e \cos \phi_e = z_a I \cos \phi_e$; so that when this is divided by z_a it gives the current component OL′ which is in phase with the terminal E.M.F. But OQ′ can again be expressed in terms of E_i and E_e and of the variable γ when it is seen that it is the difference between the projections on OK′ of E_i and E_e, *i.e.* $OQ' = OM' - ON' = E_i \cos(\phi_a - \gamma) - E_e \cos \phi_a$.

Therefore the rate at which energy is delivered to or received from the network is

$$P' = \frac{E_e}{z_a}\{E_i \cos(\phi_a - \gamma) - E_e . \cos \phi_a\} . \quad . \quad . \quad . \quad . \quad (188)$$

According as the energy is delivered from the alternator to the network, or *vice versa*, P′ is positive or negative.

The algebraic difference between the above expressions must necessarily be the loss over the resistance of the armature, or $P - P' = r_a' . I^2$

$$= \frac{(E_i^2 + E_e^2) \cos \phi_a - 2E_iE_e . \cos \phi_a . \cos \gamma}{z_a}$$

The current is always a minimum when E_i and E_e coincide, or $\gamma = 0$, and is always a maximum when E_i and E_e are diametrically opposite, and $\gamma = 180°$.

By giving all values to γ in the above expressions, and plotting the results either by rectangular co-ordinates or by polar co-ordinates, a complete analysis is obtained of the possible combinations of the three E.M.F.'s in a given alternator with constant induced and terminal E.M.F.'s, apart of course from the question whether all these combinations are equally practicable.

Unless $E_i = E_e$, and the machine is either a generator on open circuit or an ideal synchronous motor running light without any constant losses from friction, etc., so that it is entirely neutral, there must always be some current flowing. The result of progressively increasing the angle γ though a complete cycle is then in general divisible into four stages, according as—

(1) P is positive, P′ negative, the former increasing and the latter decreasing.

(2) Both P and P′ are positive.

(3) P is again positive and P′ negative, but the former is finally decreasing and the latter is increasing.

(4) Both P and P′ are negative.

Thus, if $E_i \lesseqgtr E_e$ there is some value of γ, say γ_1, for which $E_e . \cos(\phi_a + \gamma_1) = E_i . \cos \phi_a$, this value being a negative angle of lag of

E_i behind E_e when $E_i > E_e$, and a positive angle of lead of E_i before E_e when $E_i < E_e$. For this value of γ, $P = 0$, and P' is negative; γ_1 will therefore form our starting-point in the cycle. There must also be some other value γ_2 similar to γ_1, but for which $E_i . \cos(\phi_a - \gamma_2) = E_e . \cos \phi_a$, and P' in its turn becomes zero. Between these two limits we have the first stage (1); as γ is made larger than γ_1, *i.e.* as the angle of lag is made smaller or the angle of lead is made larger, P becomes positive and increasing, while P' is negative and decreasing. Thus, during this comparatively short stage both the mechanical energy absorbed by the armature of the alternator from its prime mover and converted into electrical energy, and also the electrical energy delivered to it directly from the bus bars or network, combine to heat the armature.

(2) From this point between $\gamma = \gamma_2$ and $\gamma = 2\phi_a - \gamma_2$ the machine is acting as a generator in the normal manner, supplying not only the whole of its own copper loss, but also a surplus of useful electrical energy to the network. E_i then precedes E_e, and the useful electrical output reaches its maximum when $\gamma = \phi_a$, and thence declines to zero when $\gamma = 2\phi_a - \gamma_2$ and $E_i . \cos(\phi_a - \gamma)$ again becomes $= E_e . \cos \phi_a$.

(3) The third stage between $\gamma = 2\phi_a - \gamma_2$ and $\gamma = 360° - (2\phi_a + \gamma_2)$ resembles the first, in so far that both the absorbed mechanical energy of the generator and the energy received electrically from the network combine to heat the armature of the generator. But the former declines, while the latter increases, so that finally, when $\gamma = 360° - (2\phi_a + \gamma_2)$, P is again zero.

(4) In the final stage $E_e . \cos(\phi_a + \gamma)$ exceeds $E_i - \cos \phi_a$, so that P is negative, and so also is P'. The network is therefore not only supplying the copper loss of the alternator, but is also driving it as a motor.

The total rate of development of energy by the alternator is a maximum when $\gamma = 180° - \phi_a$, and this value is positive, so that the machine is then acting as a generator.

If $E_i = E_e$, it is evident that both γ_1 and $\gamma_2 = 0$, and the first of the above four stages entirely disappears, so that in this case we have

(1) $\gamma = 0°$ to $\gamma = 2\phi_a$; generator supplying useful electrical output and its own ohmic loss.

(2) $\gamma = 2\phi_a$ to $\gamma = 360° - 2\phi_a$; both the absorbed mechanical energy and the electrical energy received from the network are expended in heating the alternator's armature.

(3) $\gamma = 360° - 2\phi_a$ to $360°$; *i.e.* for negative angles of lag from 0° to $2\phi_a$ the network not only supplies the ohmic loss, but also power to drive it as a motor.

Finally, if the armature reactance is high as compared with its resistance, so that, as is usually the case, ϕ_a approaches 90°, again both

γ_1 and γ_2 become zero; and further, the intermediate stage also disappears, so that we simply have

(1) $\gamma = 0^\circ$ to 180°; generator.

(2) $\gamma = 180^\circ$ to 360°; motor.

§ 5. **Inequality of voltage when switching into parallel.**—Let an alternator be run up on open circuit until its frequency is the same as that of the network to which it is to be connected, and let the phase of its open circuit E.M.F. E_o be synchronised with that of the terminal voltage E_e of the bus bars, the throttle valve of the engine giving just enough steam to supply the losses from friction, windage, hysteresis, and eddy-currents. In the first place, let the network be fed by such powerful machines that the additional alternator when switched into parallel cannot sensibly affect their running, so that any current which flows through its armature produces a negligibly small effect upon the rest of the system; further, let E_o differ in magnitude from E_e. If the ohmic resistance of the armature is negligible in comparison with its reactance, the E.M.F. acting at the terminals of the machine is at the first moment simply the difference $E_o - E_e$, and in phase therewith; to this corresponds a corrective current $\frac{E_o - E_e}{x_a}$, of which the phase is at right angles to either the induced or terminal E.M.F., and which is positive or negative according as E_o or E_e is the larger. The corrective current accordingly either lags behind the armature E.M.F. by 90° and so partially demagnetises the machine, or leads before the armature E.M.F. and assists in strengthening its magnetism. In either case it so alters the magnetisation that the vector sum of the induced E.M.F. and reactance voltage becomes equal to the terminal E.M.F. of the network. But now, if the machine also has ohmic resistance in its armature, the cross current between the machine and the other generators, *i.e.* $\frac{e}{z_a}$ lagging behind the cross E.M.F. e (or the vector joining E_i and E_e) by some angle ϕ_a less than 90°, will, as it grows, cause the machine to act either as a generator (if $E_o > E_e$) or as a motor (if $E_o < E_e$), slightly retarding or accelerating its rotor; the driving torque from the steam supply is assumed to remain constant, so that the additional power is in the first case in reality obtained from the kinetic energy of the fly-wheel, and in the second case electrically from the network. Any such current or component of the total armature current which tends to alter the position of the rotor relatively to the poles so as to bring the machine into such phase-relationships as satisfy the conditions of excitation or steam supply, may be defined as a *synchronising* current; it does not necessarily tend to actual coincidence of the phase of the alternator E.M.F. with that of the terminal voltage, or with that of the other alternators connected to the network, but it causes such a relative movement of the rotor as would at that moment bring it more nearly into a position of stability for continuous running. Thus in the present case the synchronising current does not make the armature E.M.F. of the additional machine to coincide in phase with the bus bar voltage, but to lag or lead according as E_o is greater or less than E_e. Further, the synchronising current is from its nature either temporary or always varying in magnitude. As, *e.g.*, it grows up in our case, it is accompanied *pari passu* by a shifting of the phase of E_o or of the centre of the poles as the rotor is retarded or accelerated, and by a consequent shifting of the phase of the cross E.M.F. e which joins the extremities of E_i and E_e. The instantaneous velocity of the vectors is not therefore constant, and the true facts cannot be strictly represented by a simple clock diagram of instantaneous or effective values. The change of velocity of the rotor is, however, but slow as compared with the periodic time of the current, and the approximate assumption may be made that the case can at any moment be represented by a vector diagram. The synchronising power in watts per phase may thus at the initial moment be taken as $\frac{E_o - E_e}{z_a} . E_o . \cos \phi_a$, and is a maximum; at any subsequent moment, according to the value which the angle between the terminal and induced E.M.F.'s has reached, it is as in § 4,

$$\frac{E_i\{E_i . \cos \phi_a - E_e \cos (\phi_a + \gamma)\}}{z_a}$$

The cumulative effect of this synchronising power causes the rotor to overshoot the correct position, whereupon it reverses and causes the rotor to return and again pass the correct position; oscillation or "phase-swinging" is thus set up. If it were not for the presence of damping forces these oscillations would continue indefinitely, but usually they are gradually annulled after a brief time and a final steady state is reached in which (on our assumption that there is no surplus of steam supply available) any heat loss in the armature is derived from the network, and yet the terminal voltage of the alternator is equalised with that of the network. The final condition is that shown in Fig. 548, from which it will be seen that the cross-current I, which persists in this particular case, is at right angles to the induced E.M.F. of the alternator; it no longer, therefore, has any synchronising power, and this would only again arise if further oscillatory displacement continued. The final angle between E_i and E_e is then the value γ_1 at which $E_e \cos (\phi_a + \gamma_1) = E_i . \cos \phi_a$ as explained in § 4, and the armature ampere-turns are purely direct magnetising, since ϕ_o is 90°. In Fig. 548 E_o is $> E_e$, and therefore the current lags behind E_i, which is also equal to E_a by 90°, and reduces the magnetisation, the demagnetising effect being indicated by ED, which is at right angles to I. If E_o had been $< E_e$, I would have been diametrically opposite, leading 90° ahead of E_i.

From this point, if the throttle valve of the engine be opened so as to admit more steam, the driving torque is increased and the rotor is slightly accelerated. The angle γ between the induced and terminal E.M.F.'s gradually changes from negative to positive, and thence increases, so that we pass successively through all the stages described in § 4. The power of the alternator increases as it becomes a generator, all the successive values given in § 4 under stage (2) being passed through.

In any finished diagram of E.M.F.'s, such as Fig. 548, the same relations are obtained, the synchronous-impedance method being

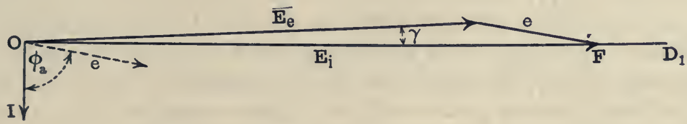


FIG. 548.—Alternator in parallel with $E_o > E_e$, and supplying no power.

assumed as correct, if instead of the true armature reactance x_a is used the value $x_a' = x_a + \frac{E_{CM}}{I \cos \phi_o}$, or the apparent synchronous reactance which is a fixed quantity for any given machine. The apparent synchronous impedance z_a' then replaces z_a, and correspondingly the angle ϕ_a' replaces ϕ_a, while the open-circuit E.M.F. E_o is used throughout. Thus Fig. 549 reproduces Fig. 548, and the line AD_1 is now $e' = z_a' I_1$ which precedes the current I_1 by the angle ϕ_a' instead of by ϕ_a. In all cases when the apparent synchronous impedance may

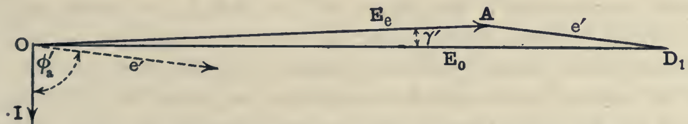


FIG. 549.—Preceding diagram on synchronous-impedance basis.

safely be taken as a constant, OD_1 is then equal to the open-circuit voltage E_o produced in the alternator by its total excitation, and the angle γ' between E_o and E_e replaces the angle γ between E_i and E_e. The drawing of the diagrams is thereby much simplified. But it must be pointed out that the true power of each machine is not obtained by the projection of the current vector upon E_o, but upon the induced E_i which no longer appears. The error is however but small, and the convenience of the method warrants its adoption for all approximate calculations, or to illustrate the actions which take place in alternators working in parallel. The results of § 4 can then be re-stated in terms of the open-circuit E.M.F. and of the apparent synchronous impedance, if for E_i and z'_a are substituted E_o and z_a'. We thus have

$$P = \frac{E_o}{z_a'}\{E_o \cdot \cos\phi_a' - E_e \cdot \cos(\phi'_a + \gamma')\} \quad . \quad . \quad . \quad . \quad . \quad (189)$$

and

$$P' = \frac{E_e}{z_a'}\{E_o \cdot \cos(\phi_a' - \gamma') - E_e \cdot \cos\phi_a'\} \quad . \quad . \quad . \quad . \quad . \quad (190)$$

while for particular relations γ' assumes new values which may be marked as γ_1', γ_2', etc.

In Fig. 548 the assumed quantities are $E_i = 15$, $E_e = 12$, and for the alternator $r_a' = 0{\cdot}25$, $x_a = 1{\cdot}4178$, so that $z_a = 1{\cdot}44$ and $\phi_a = 80°$. The final value of γ_1 is then given by the relation $12 \cdot \cos(80° + \gamma) = 15 \times 0{\cdot}1736$, whence $\gamma_1 = -2{\cdot}55$, or a negative angle of lag.

On the synchronous-impedance method $\frac{E_{CM}}{I\cos\phi_o} = 0{\cdot}9$, whence $x_a' = x_a + 0{\cdot}9 = 2{\cdot}3178$, and $z_a' = 2{\cdot}33$ instead of $1{\cdot}44$, and $\phi_a' = 83{\cdot}85°$.

But the case must also be considered of a machine A synchronised and connected to another machine B which is comparable in size and which is already feeding the network. The effect of inequality of voltage is then similar, but in addition machine B is oppositely affected. If the machines are of equal size and similar, and further, have negligibly small ohmic resistance, then on no-load the corrective current simply rises to a value $I_c = \frac{E_{o1} - E_{o2}}{2\,x_a'}$ such that when acting upon the series circuit of pure inductance it produces a terminal E.M.F. equal to the arithmetical mean of the two no-load voltages; being correctly synchronised, the phases of the E.M.F.'s coincide and the effect of the wattless current is seen to be purely magnetic. But if the armatures have ohmic resistance, when the alternator A giving E_{o1} volts on open-circuit is synchronised with the terminal E.M.F. of the second machine E_e, then as before the difference $E_{o1} - E_e$ causes a synchronising current through machine A which at the initial moment may be taken as $\frac{E_{o1} - E_e}{z_a}$ lagging ϕ_a behind the terminal E.M.F. It is therefore a generator current, forming an additional load on machine A if E_{o1} is $> E_e$, slightly retarding it when the steam supply is assumed to remain constant. But the same synchronising current also flows through the armature of machine B, and in relation to this machine is a motor current; or, more correctly speaking, it enters as a component into the total armature current of machine B, and from its direction has the effect of reducing its magnitude. It thus allows the rotor of machine B to be accelerated, and the two machines oscillate on either side of their final steady position. After a few oscillations equilibrium is again established when the terminal E.M.F.'s are equalised, and also the power developed by each machine is in accordance with their respective steam supplies.

If at the moment of switching into parallel, E_e is greater than E_{o1}, the reverse holds good; the corrective current is at first a motor current through machine A, accelerating it, while machine B is retarded by more work being thrown on to it.

Since the reactance of ordinary machines is many times greater than their resistance, the final position of the vector of the corrective current is in both cases very nearly in quadrature with the E.M.F. of either machine, so that it is almost wattless. The component current which at the moment of switching into parallel begins by transferring energy from one machine to the other, after the phase angles of the two machine pole-centres have settled down into their correct relative positions, ends by having a nearly pure magnetic effect. The general result is that the final corrective current which flows through the armatures, in series, on the assumption that the steam supplies of both machines are fixed and that their governors do not come into action, affects to some slight extent the watts lost over the resistance of the armature of machine B, and in consequence its output into the external network is indirectly affected.

§ 6. **Effect of want of synchronism of phase when switching into parallel.**—If γ' be the initial divergence of phase between the open-circuit E.M.F. E_o of the incoming machine A and the terminal E.M.F. E_e from a number of powerful alternators which may be credited with a vector of terminal pressure revolving with perfectly uniform velocity, the mechanical power which the armature of machine A should absorb and convert into electrical energy, or, *vice versa*, is given by eq. (189). This is positive when γ' is positive and machine A leads, or negative when γ' is negative and machine A lags. But now if the steam supply is limited to the amount required to make up the constant losses, the driving torque remains constant; hence, if γ' is positive, it is evident that as a synchronising current grows up it causes an additional electrical resisting torque which can only be balanced by a retardation of the rotor. The rotor and fly-wheel then part with some of their stored kinetic energy, which appears as electrical energy. Or, if γ' is negative, the rotor has work done on it by the network, and is accelerated so that the divergence of phase is reduced.

If the network is fed by a single similar machine B of equal size, the synchronising current reacts on machine B in exactly the contrary sense. Thus the temporary changes in the speeds of the rotating parts supply a certain amount of positive or negative watts, and, after some free oscillations, having a periodic time to be afterwards defined, the machines by their mutual control pull into step and a state of equilibrium is reached. If machine A is not supplied with any more steam, and E_o originally $= E_e$, its rotor will have fallen into synchronism with

E_e, and it will be carrying no load or current, the synchronising current having itself disappeared.

§ 7. **Inequality of frequency when switching in.**—The action when the machines have not quite the same frequency at the moment of parallel connection, which was already considered in § 1, is very similar to the last case. We have in effect a progressive succession of cases of want of synchronism of phase, and as before, but to an even greater degree, the case cannot be truly represented by a simple clock diagram of effective values with fixed phase relations, since the instantaneous values of the quantities are not altering after a sine law, and their vectors have not a strictly constant velocity.

The action depends, too, upon the exact instantaneous phase of the two E.M.F.'s at the moment of switching in. Even if these initially coincide, and so are in direct opposition with regard to the series circuit between the armatures, a resultant difference of potential at once begins to arise, as explained in § 1. A cross current grows up which is, however, no longer determined almost solely by the resistance of the phase lamps, as in § 1, but by the resistance and inductance of the two armatures in series. The phase of the cross current is thereby shifted more nearly into agreement with the E.M.F. of the machine which is at the moment leading and into opposition to the E.M.F. of the lagging machine. The synchronising current corresponds to the changes in the kinetic energy of the rotors of the machines, the lagging machine being accelerated and the leading machine retarded, while if the inequality of frequency is considerable, the mechanical governors of the prime movers may further be called into action.

§ 8. **The synchronising watts of a single machine coupled to a large network.**—When an alternator is switched into parallel it is usually running on open circuit, and is not initially provided with any surplus driving torque from the steam above that required to run it unloaded. Under these conditions the synchronising watts per phase in relation to a large network upon which the machine's reaction is negligible are given by the same expression as eq. (189), and since all phases have an equal effect, the total synchronising watts of the machine are $= \dfrac{m E_o \{E_o \cdot \cos \phi_a' - E_e \cdot \cos (\phi_a' + \gamma')\}}{z_a'}$

whether the angle γ' is due to an initial divergence of phase between E_o and E_e, or occurs during the process of equalising the terminal voltage when unequally excited. These synchronising watts are in reality the effect of change in the stored energy of the rotor, and with the synchronising current they vanish when γ' reaches such a value γ_1' that $E_e \cdot \cos (\phi_a + \gamma_1') = E_o \cdot \cos \phi_a'$. But in the above case of a machine coupled to a network of which the vector of terminal E.M.F. may be regarded as unalterable in speed, it is equally simple to consider the general case of any divergence from the true correct position γ', either positive or negative, which the E.M.F. vector will finally take up, whether there be any surplus driving torque or not, and whatever may be the cause of the divergence. Let the phase angle between E_o and E_e be at any moment $\gamma' + \theta_e$ where γ' is the true position corresponding to the actual steam supply and excitation, and θ_e is the momentary electrical displacement from this position. The total normal watts per phase in steady work should be

$$W_n = \frac{mE_o}{z_a'}\{E_o \,.\, \cos\phi_a' - E_e \,.\, \cos(\phi_a' + \gamma')\}$$

while the actual watts are

$$W_1 = \frac{mE_o}{z_a'}\{E_o \,.\, \cos\phi_a' - E_e \,.\, \cos(\phi_a' + \gamma' + \theta_e)\}$$

The unbalanced excess or deficit as compared with the normal is

$$W_s = W_1 - W_n = \frac{mE_o \,.\, E_e}{z_a'}\{\cos(\phi_a' + \gamma') - \cos(\phi_a' + \gamma' + \theta_e)\}$$

When expanded this becomes

$$W_s = \frac{m \,.\, E_o \,.\, E_e}{z_a'} 2 \sin\left(\phi_a' + \gamma' + \frac{\theta_e}{2}\right) \sin\frac{\theta_e}{2}$$

and when $\frac{\theta_e}{2}$ is small, so that it may be neglected in comparison with $(\phi_a' + \gamma')$ and $\frac{\theta_e}{2}$ in radians may be written for $\sin\frac{\theta_e}{2}$

$$W_s = \frac{mE_o \,.\, E_e}{z_a'} \,.\, \sin(\phi_a' + \gamma') \,.\, \theta_e$$

The synchronising watts are thus simply proportional to the electrical-displacement, and

$$\frac{W_s}{\theta_e} = \frac{mE_o \,.\, E_e}{z_a'} \,.\, \sin(\phi_a' + \gamma') \quad . \quad . \quad . \quad . \quad (191)$$

The above assumes that the small displacement falls within the second stage described in § 4, when the running as a generator is stable, *i.e.* γ' must fall within the angles γ_2' when $E_o \,.\, E_e \cos(\phi_a' - \gamma_2') = E_e^2 \cos\phi_a$ and $\pi - \phi_a'$. Within these limits an increase of the angle γ' by any small amount θ_e calls for more power, so that W_s is really synchronising.

§ 9. **The synchronising torque.**—When the synchronising watts are divided by the actual angular velocity, the synchronising torque is obtained $T_s = \frac{W}{\omega}$, corresponding units being employed as will be explained in § 12. For small oscillations the angular velocity may be considered as constant at its mean normal value of ω radians per second $= \frac{2\pi N}{60}$, so that in the above case

$$T_s = \frac{mE_o \,.\, E_e}{\omega \,.\, z_a'} \,.\, \sin(\phi_a' + \gamma') \,.\, \theta_e \quad . \quad . \quad . \quad . \quad (192)$$

It is evident that the unbalanced portion of the torque or T_s really turns upon the value of $\theta_e . \frac{dT}{d\gamma}$, and that for all approximate purposes $\frac{dT}{d\gamma}$ may be regarded as a constant, so that the synchronising torque becomes proportional to the electrical displacement θ_e or $T_s = \theta_e . T_{eu}$, where T_{eu} is the synchronising torque per radian of electrical displacement. The mechanical angle θ to which an electrical displacement of θ_e corresponds will vary inversely as the number of pairs of poles or $\theta = \frac{\theta_e}{p}$; each pair of poles represents 360° electrical degrees, and θ and θ_e only become identical when the machine is bipolar. Finally, therefore

$$T_s = T_u . \frac{\theta_e}{p} = T_u . \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (193)$$

where T_u is the synchronising torque per radian of mechanical displacement, or p times that which corresponds to one radian of electrical displacement.

Now, the acceleration or retardation of the rotor by the synchronising torque causes the rotor to catch up or to fall back upon the true position of steady running, but upon arriving there the rotor has more or less than its true steady velocity. In consequence, it does not then exert the steady-motion torque corresponding to the constant losses and to the steady electrical load, if any, but there is still a balance of torque accelerating or retarding it due to the speed being incorrect. If the difference in the angular velocity is small, this unbalanced torque is proportional to the difference in velocity, and is equal, say, to $b . \frac{d\theta}{dt}$ where b is the couple due to this term per unit angular velocity, and which may be damping or otherwise, and θ is the displacement. The moment of inertia Mk^2 of the rotor multiplied by its angular acceleration $\frac{d^2\theta}{dt^2}$ is equal to the sum of the moments of all forces acting on the rotor about its axis of rotation; the steady driving and resisting torques cancel out, but it follows that $Mk^2 . \frac{d^2\theta}{dt^2}$, instead of being simply equal to the synchronising torque as depending upon the relative position of the vectors E_o and E_e, is equal to this *plus* the term depending upon the difference in the angular velocity, or $b . \frac{d\theta}{dt}$. The general equation of the motion is therefore approximately of the form

$$Mk^2 . \frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + T_u . \theta = 0 \quad . \quad . \quad . \quad . \quad (194)$$

And it is evident that the synchronising current or torque which under sudden variations of load or of driving torque or on switching into parallel tends to produce the correct position of rotor for steady running never in itself attains the desired end, owing to the necessary presence of a certain amount of inertia in the rotor.

§ 10. **The oscillations set up by the synchronising torque.**— The nature of the oscillations set up is thus dependent upon the value of b. If $b=0$, $Mk^2 . \frac{d^2\theta}{dt^2} = -T_u . \theta$, or the angular acceleration is proportional simply to the synchronising torque, and, as in the case of an undamped pendulum, the amplitude of the first oscillation is continually repeated unchanged; there is nothing to increase it, and no damping to diminish it. Usually $4T_u . Mk^2 > b^2$, and in this case the solution of the differential equation is of the form

$$\theta = \epsilon^{-\frac{b}{2Mk^2} . t}(A . \cos qt + B \sin qt) \quad . \quad . \quad . \quad . \quad (195)$$

$$\text{where } q = \frac{\sqrt{4T_u . Mk^2 - b^2}}{2Mk^2}$$

It follows that if b is positive, the amplitude of the oscillations continually diminishes, until they are finally damped out. But if b is negative, however small its value, the amplitude of the oscillations continually increases until the machine falls out of step. In the assumed case that the exciting current is maintained absolutely constant, and that the field-magnets, armature core, and conductors are perfectly laminated, so that there is no damping from eddy-currents in any part of the machine, the value of b has been shown by Professor B. Hopkinson* to be $-\frac{r_a}{4\pi^2 f^2} . E_o^2 \frac{x_a^2 - r_a^2}{(x_a^2 + r_a^2)^2}$.

Hence, if the resistance of the armature is negligible, the tendency is for the first case to be reproduced, *i.e.* oscillations of constant amplitude indefinitely repeated. If the reactance is negligible as compared with the resistance, b is positive and the running is stable; but since in practice x_a is almost invariably greater than r_a, the running is inherently unstable with any variation of load or of driving torque.

There is, however, in addition a true damping or viscous effect due to eddy-currents set up in the iron of solid pole-pieces, and also (except in special cases) to variation produced in the current flowing round the poles in the exciting coils. When phase-swinging or free oscillations are set up the combined fundamental M.M.F. of the armature ampere-turns is no longer stationary in respect to the poles even in the polyphase

* Prof. B. Hopkinson, *Proc. Royal Soc.*, vol. lxxii. p. 235. "The 'Hunting' of Alternating-Current Machines," and paper on "The Parallel Working of Alternators," British Assoc.. 1903, reprinted in *Electr. Eng.*, vol. xxxii. p. 467. Compare lso, A. Russell, *The Theory of Alternating Currents*, vol. ii. pp. 180–190.

machine; as it moves, both the direct magnetising wave and the cross wave vary, so that there results a change both in the total flux of the field and also in the amount of its distortion. The general effect of the eddy-currents thereby set up is to diminish the amplitude of the changes of induction in the pole-pieces, and to cause them to lag a little behind the changes in the M.M.F. waves to which they are due. Whether this will render b positive and assist the damping will depend on their effect upon the torque at the moment when the rotor passes its position of steady motion with more than its steady-motion velocity. Professor B. Hopkinson * has shown that the direct magnetisation due to eddy-currents in short-circuited coils which merely surround the poles or variation in the exciting current may under certain conditions of load $\left(\text{if the watt current in phase with the armature E.M.F. is} < \frac{\sqrt{2}E_o \cdot r_a}{x_a^2 + r_a^2}\right)$ increase the instability, but on the other hand the effect of eddy-currents crossing the face of solid poles always causes a true damping effect. Chiefly, therefore, from the additional damping given in general by eddy-currents in the iron and copper of both armature and field, it results that b is usually positive, and the free oscillations are quickly damped out, so that alternators can be run with success in parallel.

§ 11. **Influence of the reactance and expression in terms of the short-circuit current.**—Apart from the indirect effect of the reactance upon b or upon the damping out of the oscillations, and returning to the simple question of the synchronising watts, it is evident that there must be some amount of reactance present to equalise divergence of voltage without great loss of efficiency, or to give the initial impulse towards synchronism of phase by transference of the power from one machine to another. It is therefore of interest to examine how the synchronising watts are affected by varying amounts of reactance with a given value for r_a' and a given normal phase difference γ'. Since $z_a' = \frac{r_a'}{\cos \phi_a'}$, $W_s = \frac{mE_o \cdot E_e}{z_a'} \sin(\phi_a' + \gamma') \cdot \theta_e$

$$= \frac{mE_o \cdot E_e}{r_a'} \cdot \sin(\phi_a' + \gamma') \cdot \cos \phi_a' \cdot \theta_e$$

In the case of switching into parallel when $E_o = E_e$, since normal $\gamma' = 0$, a particular solution is at once reached; for W_s simply becomes $\frac{mE_o^2}{r_a'} \sin \phi_a' \cdot \cos \phi_a' \cdot \theta_e$, which is a maximum when $\sin \phi_a' \cdot \cos \phi_a'$ is a maximum, *i.e.* when $\phi_a' = 45°$, or the apparent reactance x_a' is equal to the resistance r_a'. Such a low value of the reactance would, however, subject the machine to great mechanical stresses due to the racking action of the synchronising current, so that the higher ratios of the reactance to the resistance which alone occur in practice with iron-

* Cp. *Elec. Eng.*, vol. xxxii. p. 470.

cored and especially with toothed armatures are, in fact, better from the point of view of parallel working. The sudden acceleration or retardation of many tons of metal may evidently impose very severe stresses on the windings and mechanical construction.

As will be seen later, the above relation is also easily traced from eq. (208) in the case of two similar alternators coupled in parallel on open circuit, and subjected to fluctuations of speed.

Under all circumstances the synchronising watts may be expressed in terms of the current that would flow through the armature if short-circuited under the given excitation; for $\frac{E_o}{z_a'} = I_o$, the short-circuit current. Hence from eq. (191) for the single machine coupled to a network with constant pressure vector or fed by other much larger generators

$$\frac{W_s}{\theta_e} = m \cdot \frac{E_o \cdot E_e}{z_a'} \cdot \sin(\phi_a' + \gamma') = mE_e \cdot I_o \sin(\phi_a' + \gamma')$$

On switching into parallel when there is no surplus driving torque from the steam supply, if $E_o = E_e$, so that $\gamma' = 0$, this becomes

$$\frac{W_s}{\theta_e} = mE_o \cdot I_o \cdot \sin \phi_a' \quad . \quad . \quad . \quad . \quad . \quad . \quad (196)$$

And since ϕ_a' in practice usually approaches 90°,

$$\frac{W_s}{\theta_e} = mE_o \cdot I_o, \text{ approximately} \quad . \quad . \quad . \quad . \quad . \quad (196a)$$

§ 12. **Periodic time of a free oscillation.**—It has been shown that for approximate calculations the synchronising power may with sufficient accuracy be assumed to be proportional to the angle of displacement, at least for small values of θ_e. As already stated, the actual variation of the speed is but small, so that the synchronising torque is also practically proportional to the angular displacement. When any body, having a central position of rest from which if it is displaced a controlling force tends to bring it back to its central position, is set vibrating, then, provided the value of the controlling force is itself proportional to the displacement, its motion, as is well known, is a simple harmonic function of the time such as would be given by the motion of a point rotating at uniform velocity in a circle when projected upon a diameter of the circle. The natural periodic time in which each complete to-and-fro swing is effected, then follows a law which is easily expressible by a simple formula.

Thus, if a mass M after displacement from a central position of rest oscillates backwards and forwards on either side of the centre solely under the action of a force towards the centre which is proportional to the displacement, so that there is no damping

$$s = s_{max} \cdot \sin\left(2\pi \cdot \frac{t}{T_p}\right)$$

where $s_{max.}$ is the amplitude or half the total range of swing, t the instant of time is reckoned from the moment when the body passes the central position, and T_p is the periodic time of a complete swing. Let the force $F = f_u \cdot s$, where s is the linear displacement from the mid position, and f_u is the constant proportionality factor between the force and the displacement, or the controlling force per unit of displacement. The velocity of the body as it passes the central position being a maximum $V_{max.}$, let the speed at distance s from the centre be V. The difference in the kinetic energy at point s from the kinetic energy when it passes the centre being equal to the potential energy at that point is also equal to the integral effect of the force acting through the displacement s.

Therefore
$$\tfrac{1}{2}M(V^2_{max.} - V^2) = \int_0^s f_u \cdot s \cdot ds = \frac{f_u \cdot s^2}{2}$$

whence
$$V = \sqrt{V^2_{max.} - \frac{f_u \cdot s^2}{M}}$$

When the body has reached its maximum displacement, and $s = s_{max.}$, $V = 0$, so that
$$V_{max.} = s_{max.}\sqrt{\frac{f_u}{M}}$$

Further, the maximum velocity of the simple harmonic motion is equal to the uniform velocity of the rotating body from which it is derived by projection, *i.e.* to the circumference of the circle whose radius is the amplitude divided by the time of a complete oscillation; hence
$$V_{max.} = \frac{2\pi \cdot s_{max}}{T_p}$$

Equating the two expressions for $V_{max.}$, namely, $s_{max.}\sqrt{\dfrac{f_u}{M}} = 2\pi \cdot \dfrac{s_{max.}}{T_p}$

we find
$$T_p = 2\pi\sqrt{\frac{M}{f_u}}$$

This is often expressed as $T_p = \dfrac{2\pi}{\sqrt{\mu}}$, where μ is the constant ratio which the acceleration towards the centre bears to the displacement.

Thus in words,
$$T_p = 2\pi\sqrt{\frac{\text{mass}}{\text{force per unit displacement}}},$$

or
$$= 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}},$$

since the actual force F = mass × acceleration. Such is the case of the simple pendulum, provided its swings are small. The tangential force acting upon its bob is $Mg \cdot \sin\theta$, where θ is the angle of displacement from the vertical; the actual displacement is the arc moved through. So long, then, as the chord in $\sin\theta = \dfrac{\text{chord}}{\text{radius}}$ may be regarded as equal to the arc, the force is proportional to the arc or to the displacement, $\mu = \frac{g}{l}$ and its natural periodic time becomes $2\pi\sqrt{\frac{l}{g}}$.

In the case of an alternator the displacement of the rotor is again the arc of a circle, but takes place under the action of a true tangential force; and, since the former is assumed proportional to the latter, the same principle applies as in the case of a rectilinear vibration. It is only necessary to replace the simple mass by the moment of inertia Mk^2, the force by a torque T, and the velocity by the angular velocity ω. Our case of rotary motion is therefore represented by the analogy of a body of moment of inertia Mk^2 hung from a wire, or again of a heavy ring fastened by a clock-spring to a central axis, to which a torque T is applied so as to twist it through an angle of θ radians. If the angular displacement is proportional to the torque throughout the range considered, or $T = T_u \,.\, \theta$ where T_u is the controlling torque per unit angle of displacement, from the equation $\frac{1}{2} Mk^2 (\omega^2_{max.} - \omega^2) \int_0^\theta T_u \,.\, \theta \,.\, d\theta$,

it follows that

$$\omega = \sqrt{\omega^2_{max} - \frac{T_u \,.\, \theta^2}{Mk^2}}$$

and when $\omega = 0$, θ reaches its maximum value $\theta_{max.}$,

or

$$\omega_{max.} = \theta_{max.} \sqrt{\frac{T_u}{Mk^2}}$$

When developed on the flat the linear displacement at any moment is equal to the arc of the angular displacement, or $s = \theta r$, and the radius of the circle round which a point moving with uniform velocity will give the equivalent harmonic motion is $s_{max.} = \theta_{max.}\, r$. The maximum velocity as the body passes the centre being $\omega_{max.}\, r$, this is also equal to the uniform velocity $\frac{2\pi \,.\, \theta_{max.}\, r}{T_p}$, whence $\omega_{max.} = \frac{2\pi \,.\, \theta_{max.}}{T_p}$.

Equating the two expressions for $\omega_{max.}$ namely $\theta_{max.} \sqrt{\frac{T_u}{Mk^2}} = \frac{2\pi \,.\, \theta_{max.}}{T_p}$

we find

$$T_p = 2\pi \sqrt{\frac{Mk^2}{T_u}} \quad . \quad . \quad . \quad . \quad . \quad (197)$$

or in words,

$$T_p = 2\pi \sqrt{\frac{\text{moment of inertia}}{\text{torque per mechanical radian}}}$$

$$= 2\pi \sqrt{\frac{\text{angular displacement}}{\text{angular acceleration}}}$$

since the actual torque T = moment of inertia × angular acceleration.

In the case of an alternator the periodic time of a free oscillation is usually expressed in terms of $T_{eu} = \frac{T_s}{\theta_e}$, or the synchronising torque which gives one radian of electrical displacement, rather than in terms of T_u, the synchronising torque which gives one radian of mechanical displacement. Since the latter is p times the former, or $T_u = p \cdot T_{eu} = p \cdot \frac{T_s}{\theta}$.

$$T_p = 2\pi \sqrt{\frac{Mk^2}{p \cdot \frac{T_s}{\theta_e}}} \quad . \quad . \quad . \quad . \quad (198)$$

It only remains to secure the expression of the synchronising torque and of the moment of inertia in corresponding units.

On the C.G.S. system, $W_s \times 10^7$ = ergs per second, and $\frac{W_s}{\omega} \times 10^7 = T_s$ in dyne-centimetres. If Mk^2 be given in kilogrammes of mass $\times$ square of radius of gyration k in metres, $Mk^2 \times 10^7$ will give the moment of inertia in the C.G.S. unit of grammes-(centimetres)^2. The numerical factors then cancel out, and

$$T_p = 2\pi \sqrt{\frac{Mk^2}{\frac{p}{\omega} \cdot \frac{W_s}{\theta_e}}} \text{ seconds} \quad . \quad . \quad . \quad (199)$$

Since $\omega = 2\pi \frac{f}{p}$ or $\frac{2\pi N}{60}$, where f is the frequency, this becomes

$$T_p = \frac{2\pi}{p}\sqrt{\frac{2\pi f \cdot Mk^2}{W_s/\theta_e}} = \frac{15 \cdot 7}{p}\sqrt{\frac{f \cdot Mk^2}{W_s/\theta_e}} = 0.262\,N\sqrt{\frac{Mk^2}{f \cdot W_s/\theta_e}} \quad . \quad . \quad . \quad (200)$$

In British units, $\frac{W_s \times 550}{746}$ = ft.-lbs., per second, and $\frac{W_s \times 550}{\omega \times 746} = T_s$ in lb.-feet. If W = the weight in lbs., and r is the radius of gyration in feet, the moment of inertia in engineers' mass-units $\times$ (feet)^2 is $\frac{W}{g} \cdot r^2$, corresponding to the torque in lb.-feet; hence

$$T_p = 2\pi \sqrt{\frac{1 \cdot 36 \frac{W}{g} \cdot r^2}{\frac{p}{\omega} \cdot \frac{W_s}{\theta_e}}} = \frac{2\pi}{p}\sqrt{\frac{2\pi f \times 1 \cdot 36 \frac{W}{g} \cdot r^2}{W_s/\theta_e}}$$

$$= \frac{18 \cdot 3}{p}\sqrt{\frac{f \cdot \frac{W}{g} \cdot r^2}{W_s/\upsilon_e}} = 0 \cdot 305 N \sqrt{\frac{\frac{W}{g} \cdot r^2}{f \cdot W_s/\theta_e}} \text{ seconds} \quad . \quad . \quad (201)$$

Since $r^2 = \frac{D^2}{4}$, where D is the diameter of gyration in feet and WD^2 in tons-ft.^{2} is conveniently used to compare moments of inertia of fly-wheels, we also have

$$T_p = \frac{76 \cdot 1}{p}\sqrt{\frac{f \cdot WD^2}{W_s/\theta_e}} \text{ and finally, } = 1 \cdot 27 N \sqrt{\frac{WD^2}{f \cdot W_s/\theta_e}} \quad . \quad . \quad (201a)$$

where W is now the weight in tons.

The several expressions for T_p are given as it is variously expressed by different writers. It must be borne in mind that W or M in lbs. or tons or kilogrammes must include all rotating parts (magnet, fly-wheel, crank discs, etc.).

The expressions for the periodic time of a free oscillation have been confirmed very closely by actual experiment;* it varies as the square root of the moment of inertia, and inversely as the square root of E_o^2, *i.e.* inversely as E_o, which is roughly proportional to the excitation.

In the above the effect of damping has been left out, but when $4\ T_u \,.\, Mk^2 > b$, it follows from eq. (195) that the full expression for the periodic time of a free oscillation with damping is

$$T_p = \frac{2\pi}{q} = \frac{4\,\pi\, Mk^2}{\sqrt{4T_u \,.\, Mk^2 - b^2}} \quad . \quad . \quad . \quad . \quad (202)$$

§ 13. **Vector diagrams for steady conditions of working.**—After the preliminary remarks of §§ 5-10 upon the switching into parallel connection we pass to a consideration of permanent conditions as affecting the parallel working of alternators.

It is evident that such a diagram as Fig. 548 only represents a part of the facts. The phase and value of each actual armature current in a number of alternators in parallel is always fixed by the vector $\overline{E_i \,.\, E_e}$, but no distinction was there required between that portion of the total armature current, I, which passes into or out of the external network and the portion which passes only through the armatures of the other generators connected to the network.

The vector of an actual alternating current may be resolved into a pair of components in any number of ways, and the lines upon which it is so resolved should be chosen so as best to bring out the information which is required. In the present connection, when a steady state has been reached, it is best to resolve each armature current along the external-current line and along a line at right angles to the terminal E.M.F., *i.e.* into I_e and I_c; the latter component has no effect upon the external output, and may be strictly defined as a "corrective" current required to equalise some divergence of excitation or phase under permanent conditions of steady working. Such a method of resolution of the current is chosen so that the actual current supplied by each machine to or received from the network is correctly obtained. Thus when the complete diagram is drawn for two or more alternators working in parallel, the vector of the external current must be shown separately, and inclined to the terminal E.M.F. at the angle of lag ϕ_e. This angle for a given network under fixed conditions is a constant dependent upon the relative values of its resistance and its reactance, *i.e.* upon its power-factor. *E.g.*, in Fig. 549, for no steam supply beyond that required by the constant losses, the current I would be resolved into a corrective component I_c at right angles to the terminal E.M.F., and

* Cp. M. Paul Boucherot, "Coupling Fly-wheel Alternators in Parallel," *Trans. Intern. Electr. Congress St. Louis*, 1904, vol. i. p. 693.

a small negative component, in phase with the terminal E.M.F. and by its product therewith giving the loss of watts over the armature resistance which is supplied from the electrical energy of the network.

Since, in Fig. 470, $BG = Ix_e$ where x_e is the reactance of the external circuit, and $OG = IR_e + Ir_a'$, the angle between the vectors of E^i and of the current is such that its tangent is $\frac{x_e + x_a}{R_e + r_a'}$. Taking first the case of two precisely similar alternators working in parallel under ideal conditions of perfect equality in all respects of excitation and steam supply, a similar diagram (Fig. 550) is true for both. But now each may be regarded as working separately on half the external network of

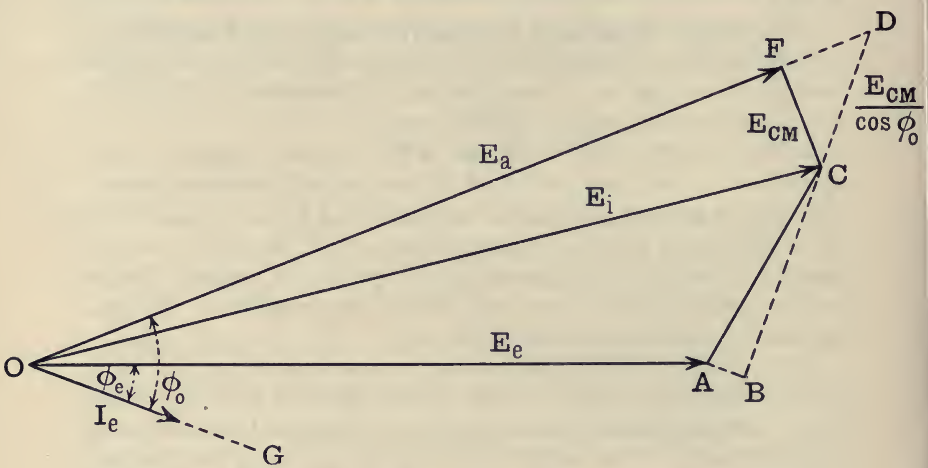


FIG. 550.—Single alternator, or two alternators in perfect parallel.

which the resistance will be $2R_e$ and its reactance $2x_e$; the angle COG between E_i and I is therefore such that its tangent is $\frac{2x_e + x_a}{2R_e + r_a}$, and this angle is independent of what may afterwards be found to be the value of the E.M.F.'s, since the three sides of the triangle all vary together. The angle COG will later be seen to lend important assistance in the process of constructing diagrams to represent the effect of other altered conditions. So long as the alternators are working perfectly in parallel, the extremity of the vector of E_i of either machine must fall on this line at some point C.

§ 14. **Effect of unequal excitation.**—But now let the conditions be altered, and in the first place, with the steam supply and frequency remaining equal, let the excitation of one be raised and of the other be lowered. The complete diagram of the two machines is shown in Fig. 551, in which we return to the full representation in terms of E_i, z_a, and ϕ_a instead of in terms of synchronous impedance. The vectors of

E_{i1} and E_{i2}, and also of E_{a1} and E_{a2}, are now no longer coincident, and from the divergence of the latter it follows that in the steady state

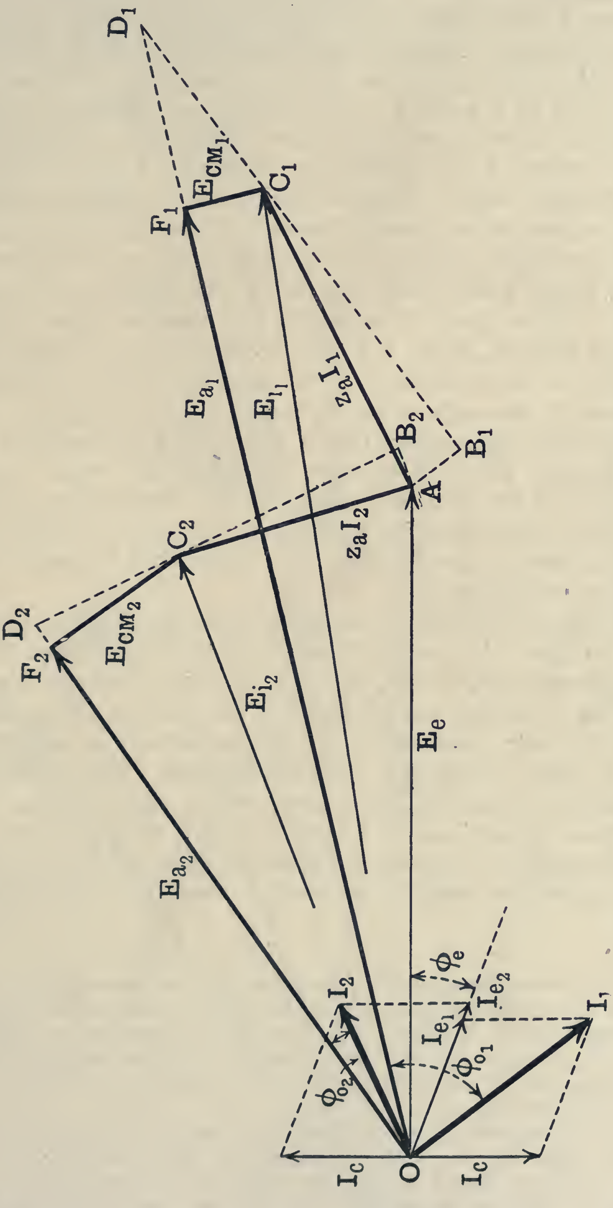


FIG. 551.—Two similar alternators, with equal steam supply and unequal excitation.

the pole-centres of one machine are advanced relatively to the vectors of the terminal E.M.F. or external current, while the pole-centres of the other machine have conversely fallen back. At the same time, the vectors of the total currents in each machine have diverged. The current I_1 of the machine of larger E.M.F. falls behind the external current line, and its negative angle of lag behind the centre of the pole or ϕ_{o1} is the larger. Conversely, the total current I_2 of the machine of smaller E.M.F. draws nearer to E_{a2}, or to the centre of the pole, and its angle of lag ϕ_{o2} is reduced or may become converted into a positive angle of lead, so that it precedes the centre of the pole. Resolving the currents as shown in the diagram, the component I_c lags in respect to E_{a1}, and therefore helps to demagnetise the machine of higher excitation; on the other hand, since it leads in respect to E_{a2}, it assists its excitation, so that in the end E_{t1} and E_{t2} are not so far different as their unequal excitation would by itself cause, and their terminal E.M.F.'s are actually identical. Further, since the component I_c is displaced widely from the positions of either E_{t1} or E_{t2}, it is very largely wattless and has no great effect upon the internal losses in the two machines or upon their output. The increase in the angle of lag ϕ_{o1} causes the total current I_1 to be more powerfully demagnetising than it would be if the machines were equally excited, while the converse change in ϕ_{o2} lessens the demagnetisation of the machine of smaller E.M.F., or positively produces forward ampere-turns upon its magnetic circuit. Thus the effect of an unequal excitation in altering the phases of the E.M.F.'s and currents is almost purely magnetic.

All the vectors of Fig. 551 have the same signification as in Fig. 470; *e.g.*, the lines AC_1 and AC_2 are the E.M.F.'s consumed by the armature impedances, or z_aI_1 and z_aI_2, and the armature currents I_1 and I_2 lag $\phi_a°$ behind these E.M.F.'s. The products of E_{t1} and E_{t2} with the projections upon them of I_1 and I_2 respectively, are equal, since equality of steam supply has been presupposed. The ampere-turns corresponding to E_{a1} and E_{a2}, after taking into account the direct magnetising turns proportional to $I_1 \sin \phi_{o1}$ and $I_2 \sin \phi_{o2}$, are the two unequal excitations. The machine of greater excitation has the larger armature current, but supplies the smaller share of the total external current.

While the finished diagram for two unequally excited alternators in parallel has been at once presented in Fig. 551, the general method of obtaining this from given numerical values may be described. It is best done by considering the E.M.F.'s consumed by the armature impedances not in relation to the total armature currents, but in relation to their two components, *i.e.* the external and the corrective, separately and in succession.

If a line be drawn through the extremity of the vector of the terminal voltage and inclined at the angle ϕ_a to the external current vector OG, the point C at which it intersects the line OCK gives the length AC representing the E.M.F. consumed by the armature impedance in relation to the *mean* external current, or $z_a \cdot \frac{I_{e1}+I_{e2}}{2}$. About this point C the diagram can be drawn in a definite fashion

(Fig. 552). The mean external current $I_e = \frac{OC}{\text{joint impedance}}$. Cos COG $= \frac{r_a' + 2R_e}{\sqrt{(r_a' + 2R_e)^2 + (x_a + 2x_e)^2}} = \frac{\text{joint resistance}}{\text{joint impedance}}$ is the power-factor of the joint system of armature and half the external net work.

Except in the case of unity power-factor, the corrective component I_c is from its

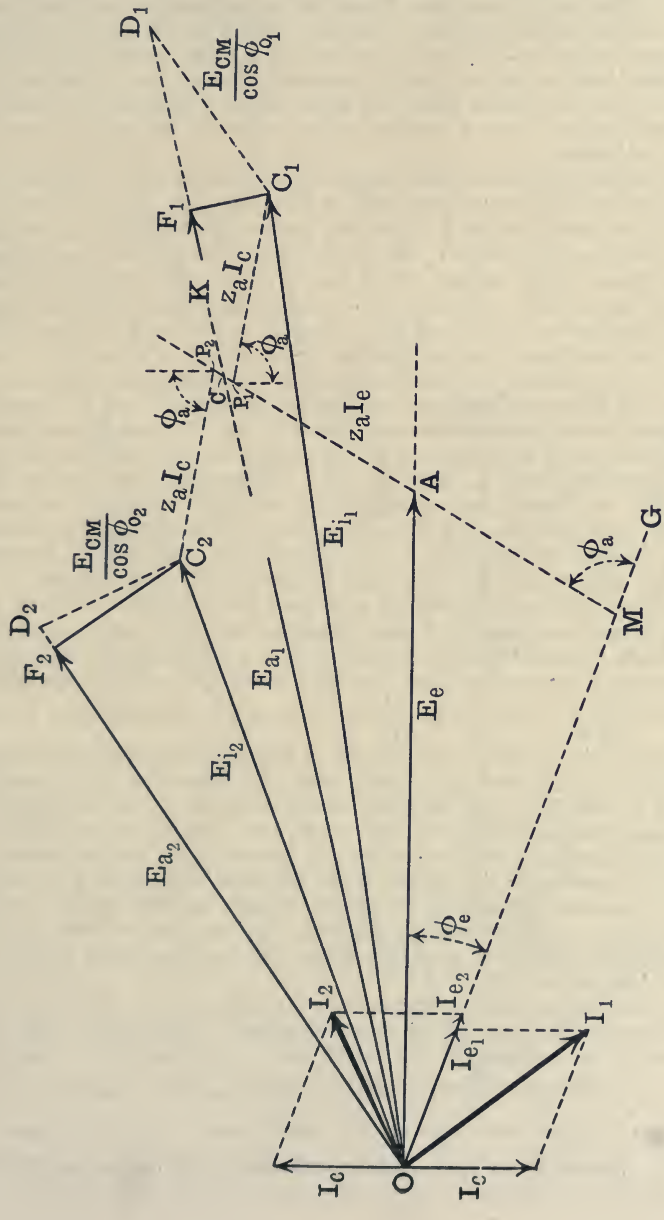


FIG. 552.—Derivation of preceding diagram.

definition inclined at a smaller angle to I_{e1} than to I_{e2}; in consequence, if the external component of the two machines remained equal, the total armature current of machine 1 would be greater than that of machine 2, and the loss of watts over the resistance of the armature would be greater. But the steam supply and the total electrical energy developed in the two machines are ex-hypothesi equal, so that the external output of machine 1 must be smaller than that of machine 2. Thus the presence of the corrective component destroys the perfect equality of the two external outputs, and the external component of machine 1 which has the stronger excitation must be made slightly smaller than that of machine 2, which has the weaker excitation. Since, however, the loss over the armature in either case is but small in comparison with the external output, no great difference is required in order to again reach equality of the total electrical energy developed. The diagram can then be drawn as follows :—

After drawing OCK at the correct angle to the current line OG to represent the given value of the network resistance, draw MAC at the angle ϕ_a to OG, intersecting OCK at the point C which is a little less than the mean value of E_{o1} and E_{o2}. Upon AC take two other points P_1 and P_2, the former as much below C as the latter is above it, and neither differing widely from C; then AP_1 and AP_2 represent respectively the E.M.F.'s consumed by the armature impedances in relation to the external components, or $z_a I_{e1}$ and $z_a I_{e2}$. From P_1 and P_2 draw two lines P_1C_1 and P_2C_2 of equal length, and each inclined to the vertical at an angle ϕ_a; then P_1C_1 and P_2C_2 represent the E.M.F.'s consumed by the armature impedances in relation to the corrective component which has the same value I_c, but is in opposite directions in the two machines. The two lines OC_1 and OC_2 then give the phases and magnitudes of the impressed E.M.F.'s E_{i1} and E_{i2}, and the former must fall well within the no-load voltage E_{o1}, while the latter is more nearly equal to the no-load voltage E_{o2}. By dividing AP_1 and P_1C_1 by z_a, the values of the two component currents I_{e1} and I_c of the machine of larger excitation are found; when compounded, they give the phase and magnitude of the total current I_1. Similarly the quotients of AP_2 and P_2C_2 when divided by z_a give I_{e2} and I_c, and, when compounded, I_2. If the products of E_{i1} and E_{i2} with the projections upon them of I_1 and I_2 are not found to be equal when equality of steam supply has been presupposed, the lengths AP_1 and AP_2, P_1C_1 and P_2C_2 must be corrected until a possible diagram is obtained. At right angles to the currents I_1 and I_2 are then drawn C_1D_1 and C_2D_2 representing $\frac{E_{CM1}}{\cos\phi_{o1}}$ and $\frac{E_{CM2}}{\cos\phi_{o2}}$, which are proportional to I_1 and I_2, and the diagram is completed in the same manner for each machine by letting fall the perpendiculars of the cross E.M.F.'s, C_1F_1 and C_2F_2, which mark the extremities of the armature E.M.F.'s E_{a1} and E_{a2}. The excitations corresponding to E_{a1} and E_{a2}, when the direct magnetising turns proportional to $I_1 \sin\phi_{o1}$ and $I_2 \sin\phi_{o2}$ are duly taken into account, must then tally with the two unequal excitations. If the disagreement is found to be considerable, the position of the line MAC may require to be shifted inwards or outwards, thus altering the terminal voltage, and after one or two trials a correct solution of the problem is obtained. That the diagram fulfils all the required conditions is evident, since it reproduces the same final result as Fig. 551 for equal steam supply.

If the apparent synchronous impedance of the machines may be used for approximate calculation, the diagram is still further simplified. The line MAC′ (Fig. 553) must now be drawn inclined to the current vector at the angle ϕ_a' of which the tangent is $\frac{x_a'}{r_a'}$, where x_a' is the apparent synchronous reactance; similarly, the line C′OG must be drawn at an angle to the current vector of which the tangent is $\frac{2x_e+x_a'}{2R_e+r_a'}$. The lines $P_1'D_1$, $P_2'D_2$ are drawn at the angle ϕ_a' to the vertical, and now represent the E.M.F.'s consumed by the apparent synchronous impedance z_a' in relation to the corrective components, while AP_1' and AP_2' represent the E.M.F.'s

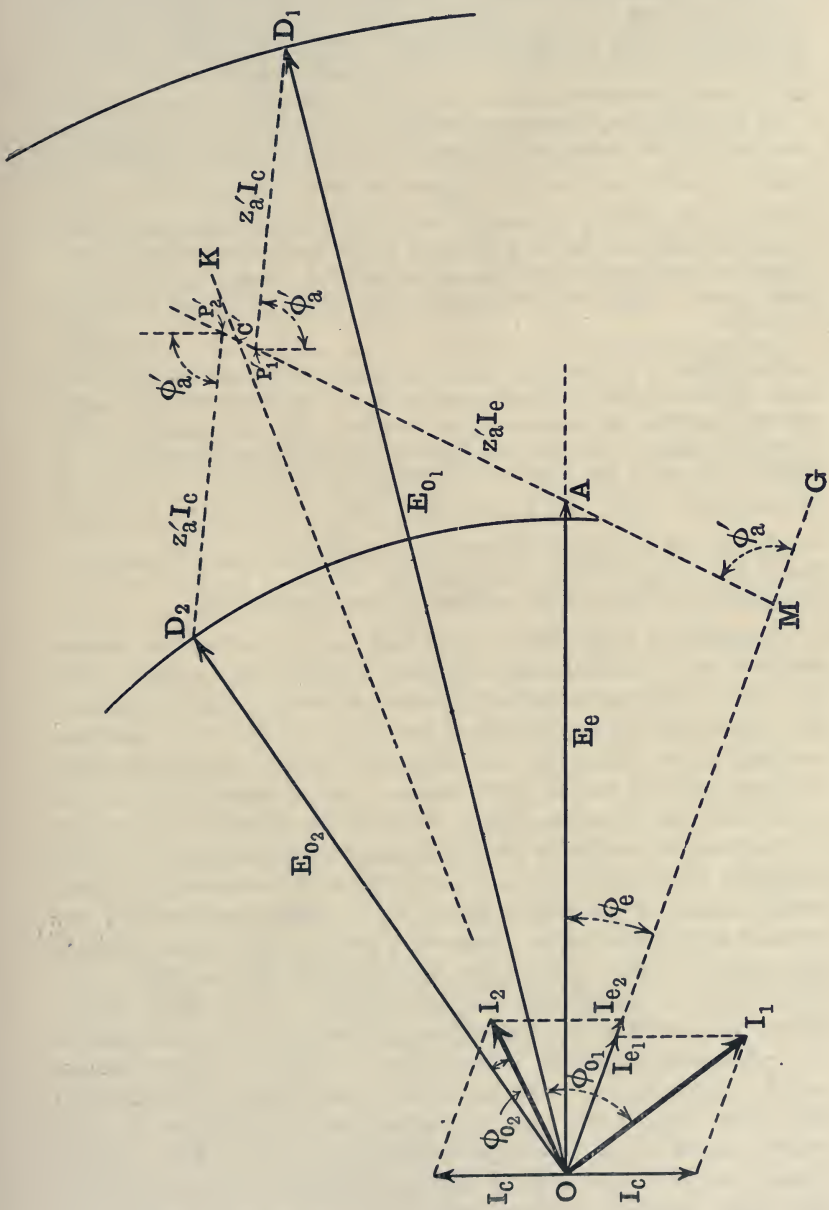


FIG. 553.—Derivation of Fig. 551 on synchronous impedance basis.

consumed by z_a' in relation to the external currents. Such points must then be found that $P_1'D_1$ and $P_2'D_2$ are equal, and in the two cases respectively terminate upon the circles representing the values of the no-load voltages E_{o1} and E_{o2}, and give the required ratio of steam supply.

In Fig. 552 the external network is assumed to have a resistance $2R_e=4$, and a reactance $2x_e=1\cdot456$ or $\cos\phi_e=0\cdot94$ and $\phi_e=20°$, while the alternators have the same quantities as in § 5. Thence $\cos COG=\dfrac{0\cdot25+4}{\sqrt{4\cdot25^2+2\cdot874^2}}=0\cdot829$, and $COG=34°$; the joint impedance is $5\cdot13$.

In Fig. 553, on the synchronous-impedance method, when OD_1 and OD_2 are identified with the open-circuit voltages, $E_{o1}=24$ and $E_{o2}=11\cdot4$. The joint impedance is then $5\cdot68$, and $\cos COG=\dfrac{4\cdot25}{5\cdot68}=0\cdot748$, whence $COG=41\cdot6°$.

In the case of unity power-factor, since the corrective component as before determined is in each case at right angles to the external current, the armature current I_2 is as much ahead of the terminal voltage as I_1 is behind, and both remain equal in value. Hence with equal steam supplies the external outputs remain equal throughout.

The effect of unequal steam supply with unequal excitation can also be traced by similar diagrams. The maximum value of the output and efficiency for a given total steam supply is reached when the phases of the two induced E.M.F.'s E_{i1} and E_{i2} coincide, and when the reactance of the armatures is high as compared with their resistances, which is the usual case of practice, *i.e.* when ϕ_a approaches 90°, the required ratio of the two steam supplies reduces to the same ratio as that of the induced E.M.F.'s $\frac{E_{i1}}{E_{i2}}$. This is again not far different from $\frac{E_{a1}}{E_{a2}}$, but the ratio of $\frac{E_{o1}}{E_{o2}}$, or of the two excitations, does not afford any direct clue, since the demagnetisation affects the machine of larger excitation to a preponderating extent.

Enough has been said to show that two alternators can be satisfactorily run in parallel when unequally excited, but that even under the most favourable conditions there is a certain loss of efficiency in the system due to the additional watts expended over the armature resistances by reason of the corrective current-component, which equalises their E.M.F.'s to a common value. Beyond the general conclusion that equal excitation has not the paramount importance that equality of frequency and synchronism of phase have, the above diagrams have chiefly a theoretical interest as assisting to a clear understanding of the phenomena. The relative dimensions of the vectors are chosen not as representing practical cases, but merely to show how the action of machines in parallel depends essentially upon their respective steam, water, or gas supplies to the prime movers. When once thrown into parallel, equal division of the load between two alternators cannot be secured as in continuous-current dynamos simply by altering the excitation of one. Any such change only alters the phase angle of the current relatively to the E.M.F., or, in other words, causes a cross current magnetising one machine and demagnetising the other. It is, however, desirable that all generators which are worked in parallel should have similar load characteristics, in which case when in parallel their separate field rheostats might be mechanically coupled together, or the exciting voltage of all may be simultaneously altered for any change of load by altering the rheostat in the shunt circuit of the exciter, without causing any cross current or loss of efficiency therefrom.

On a network fed by such large alternators that the bus-bar E.M.F. may be regarded as fixed, the effect of decreasing the field excitation and lowering the induced E.M.F. of a single small alternator with a constant steam supply, is to increase the angle γ until the point of the lowest induced E.M.F. at which the machine can take the power of the steam supply is passed, and it falls out of step. A V-curve of current analogous to that of a synchronous motor can be traced, but the best excitation in this supposed case is not that which gives the minimum loss in the single machine, but that which makes its vector diagram of E.M.F.'s coincide with those of the other machines.

The greater the short-circuit current for a given excitation, or, in other words, the lower the armature reaction, the greater the cross current which must flow to counterbalance a given difference in the field strengths of the machines. Hence at least a moderate amount of armature reaction is on this score to be desired, so as to prevent the cross current from becoming unduly high.

§ 15. **Unequal steam supply with equal excitation.**—In the case of equal excitation but unequal steam supply to two similar alternators the line MAC inclined at the angle ϕ_a to the external current vector must intersect OC at some point C falling within the circle corresponding to the open-circuit E.M.F. for the given excitation (Fig. 554). The product of the mean external current (as deduced from $AC = z_a \times \text{mean } I_e$) when projected upon OC and multiplied by the value of OC will then give the mean horsepower absorbed in developing the mean external output. On either side of C must be taken the two points P_1 and P_2 such that AP_1 and AP_2 are nearly in proportion to the unequal steam supplies. By drawing P_1C_1 and P_2C_2 inclined at the angle ϕ_a to the vertical, and thence calculating the corrective current I_c and combining it with I_{e1} and I_{e2}, the diagram can be completed; and the values must be adjusted until the lines C_1D_1 and C_2D_2 representing $\dfrac{E_{CM}}{\cos\phi_{o1}}$ and $\dfrac{E_{CM}}{\cos\phi_{o2}}$ drawn at right angles to the currents I_1 and I_2, and proportional to them, terminate upon the circle of equal E.M.F.'s. The phases of the armature E.M.F.'s and of the centres of the poles are then given by OD_1 and OD_2. It will be seen that, since the two machines cannot give equal powers, a corrective current arises which causes the greater current of machine 2 to lag behind E_{a2} by a lesser angle than the current of machine 1 lags behind E_{a1}; their respective demagnetising turns are therefore more nearly equalised than would follow from the simple values of the currents, and in consequence their terminal

E.M.F.'s are equalised. The two induced E.M.F.'s E_{i1} and E_{i2} diverge equally on either side of the mean induced E.M.F. OC.

When expressed in terms of the apparent synchronous impedance of the armature z_a' and of E_o, the diagram of Fig. 554 is redrawn as Fig. 555. The calculations of relative power thence deduced, although

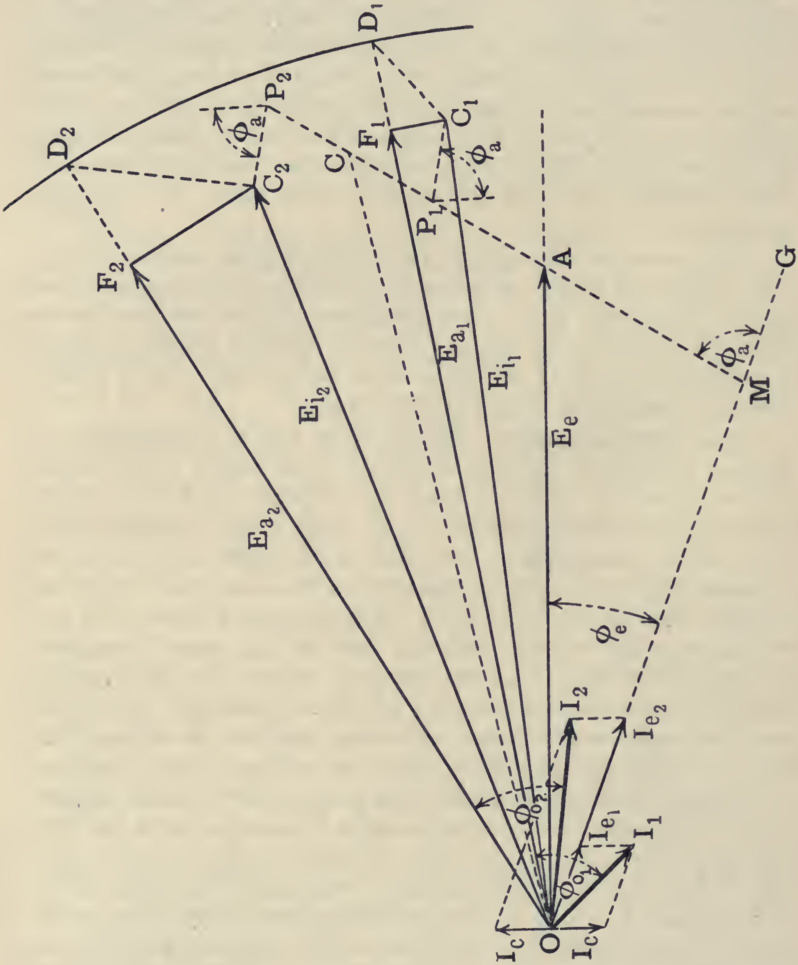


FIG. 554.—Two similar alternators equally excited, with unequal steam supply.

not strictly correct, are not far from the true values obtained when the induced E.M.F.'s are used instead of the open-circuit E.M.F.'s.

So far in Fig. 555 the vector $\overline{E_i\ E_e}$, or AC_1 has been treated as made up of the combination of AP_1 and PC_1, and in Fig. 556 the vector $\overline{E_{o1}\ E_e}$, or AD_1, as made up of AP_1 and PD_1. But it is equally valid to consider them as made up of the combinations respectively of AC and CC_1 and AC′ and $C'D_1$. And similarly for the second machine. This amounts to assigning to each machine an *equal* share of the external current $I_e = \frac{I_{e1} + I_{e2}}{2}$ proportional to which is the mean length AC or AC′, and combining this equal current with a corrective component I_c' which is inclined to the vertical, and which is therefore no longer purely at right angles to the terminal

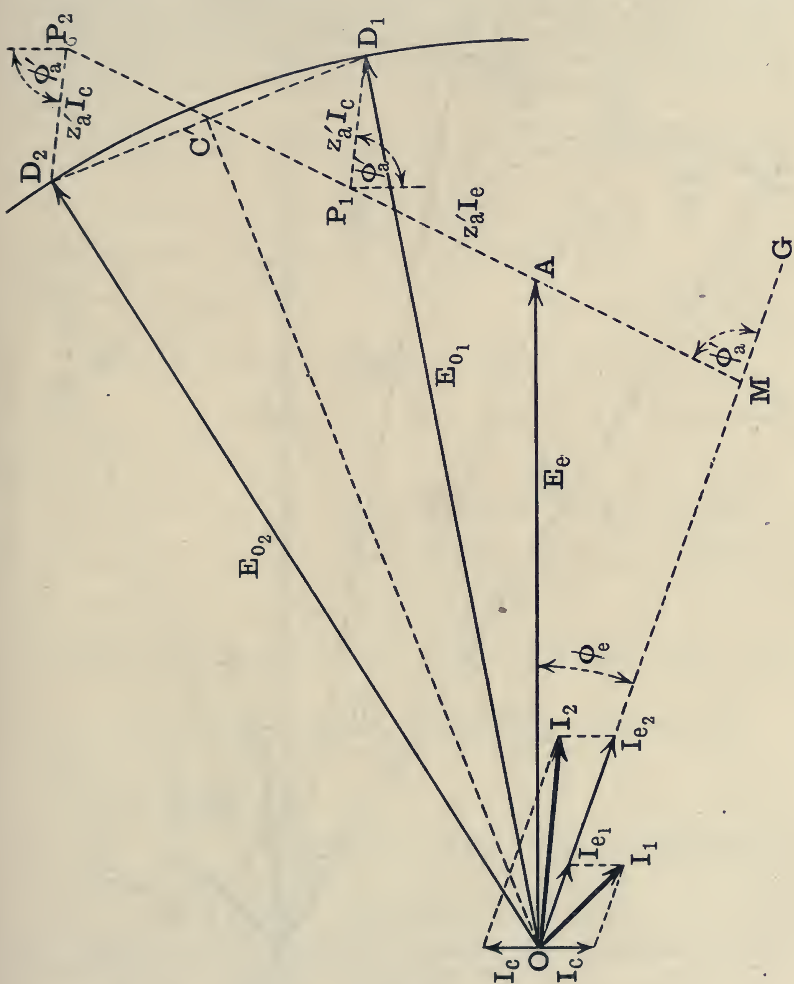


FIG. 555.—Preceding diagram on synchronous-impedance basis.

voltage E_e. The new corrective component which has been symbolised by I_c', and which is obtained by dividing the vector $\overline{E_{t1}\ E_{t2}}$ or $\overline{E_{o1}\ E_{o2}}$ by $2z_a$ or $2z_a'$, must now be plotted at an angle ϕ_a or ϕ_a' to the line C_1C_2 or D_1D_2. In Fig. 556 this has been done for the approximate diagram on the synchronous-impedance assumption; the vector $\overline{E_{o1}\ E_{o2}}$ or $D_1C'D_2$ is there shown transferred to the centre, and lagging behind it by the angle ϕ_a' is $I_c' = \frac{D_1C'D_2}{2z_a'}$. The resultant diagram is the same as before, but is differently expressed. While the particular relation for the corrective current which was first made use of, namely, at right angles to the terminal voltage, was chosen to show at once what proportion of the total external current was given by each machine, the present resolution of the currents will be seen later to be especially suited to the case where there is a rhythmical fluctuation of the speed, and each machine does give as its normal load half the total external current.

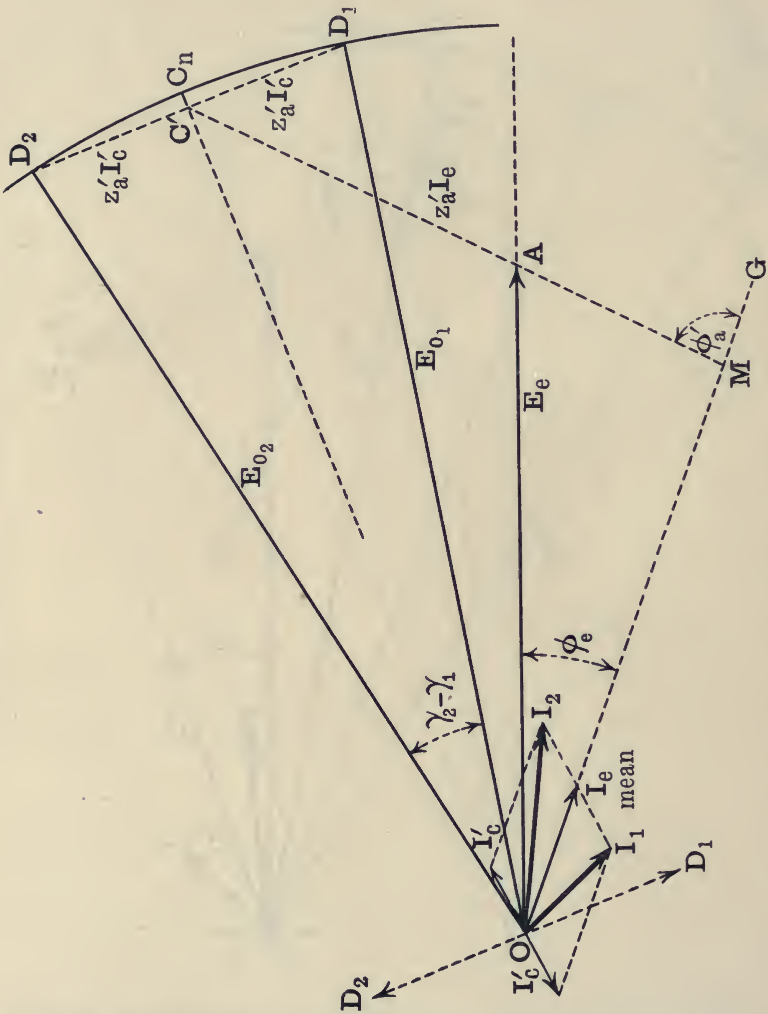


FIG. 556.—Preceding diagram in terms of I_c.

But even in the case of permanent equality of the loads, the present method affords convenient expressions for examining the case of two alternators with unequal steam supplies.

The vector sum of the two armature currents I_1 and I_2 is always equal to the total external current, *i.e.* to 2 I_e (mean), while half the vector difference between I_1 and I_2 is equal to the current I_c' which flows only through the armature. It is now seen that this latter component is nearly in phase with the E.M.F. of the leading machine, and nearly opposed to the E.M.F. of the lagging machine. It thus alters the output of each machine into accordance with its steam supply. When a real current is resolved into any two components the rate of development of electrical energy by it in relation to any E.M.F. is always given by the algebraic sum of the products when each component is separately projected on to the E.M.F. and the projections are multiplied by the E.M.F. Hence the power absorbed in each machine in the present case is equal to the sum of the products with the E.M.F. E_o of the projections upon it first of the mean external current I_e and then of the component I_c', *i.e.* in the machine of which the E.M.F. E_{o2} leads relatively to the mean vector OC

$$W_2 = mE_o\left[\text{mean } I_e \,.\, \cos\left(COG + \frac{\gamma_2-\gamma_1}{2}\right) + I_c' \,.\, \cos\left\{\frac{\gamma_2-\gamma_1}{2} - (90^\circ - \phi_a')\right\}\right]$$

$$= mE_o\left\{I_e \,.\, \cos\left(COG + \frac{\gamma_2-\gamma_1}{2}\right) + I_c' \,.\, \sin\left(\frac{\gamma_2-\gamma_1}{2} + \phi_a'\right)\right\}$$

and in the machine of which the E.M.F. E_{o1} lags relatively to OC

$$W_1 = mE_o\left\{\text{mean } I_e \,.\, \cos\left(COG - \frac{\gamma_2-\gamma_1}{2}\right) - I_c' \,.\, \cos\left(90^\circ - \phi_a' + \frac{\gamma_2-\gamma_1}{2}\right)\right\}$$

$$= mE_o\left\{I_e \,.\, \cos\left(COG - \frac{\gamma_2-\gamma_1}{2}\right) + I_c' \sin\left(\frac{\gamma_2-\gamma_1}{2} - \phi_a'\right)\right\}$$

Since $D_1C' = E_o \,.\, \sin\frac{\gamma_2-\gamma_1}{2}$, $I_c' = \frac{E_o}{z_a'} \,.\, \sin\frac{\gamma_2-\gamma_1}{2} = I_o \,.\, \sin\frac{\gamma_2-\gamma_1}{2}$, and the above relations can also be expressed as

$$W_2 = mE_o \,.\, I_e \,.\, \cos\left(COG + \frac{\gamma_2-\gamma_1}{2}\right) + \frac{mE_o^2}{2z_a'}\left\{\cos\phi_a' - \cos\left(\phi_a' + \frac{\gamma_2-\gamma_1}{2}\right)\right\}$$

$$W_1 = mE_o \,.\, I_e \,.\, \cos\left(COG - \frac{\gamma_2-\gamma_1}{2}\right) + \frac{mE_o^2}{2z_a'}\left\{\cos\phi_a' - \cos\left(\phi_a' - \frac{\gamma_2-\gamma_1}{2}\right)\right\}$$

The permanent difference between the two powers is thus

$$W_2 - W_1 = mE_o\left\{-2\, I_e \,.\, \sin COG \,.\, \sin\frac{\gamma_2-\gamma_1}{2} + \frac{E_o}{z_a'} \,.\, \sin\phi_a' \,.\, \sin(\gamma_2-\gamma_1)\right\} \quad . \quad . \quad . \quad (203)$$

Since $I_e = \dfrac{E_o \,.\, \cos\frac{\gamma_2-\gamma_1}{2}}{\text{joint impedance}}$ and $\sin COG = \dfrac{x_a' + 2x_e}{\text{joint impedance}}$, while $\sin\phi_a' = \dfrac{x_a'}{z_a'}$, we also have

$$W_2 - W_1 = mE_o^2\left\{\frac{x_a'}{(z_a')^2}\sin(\gamma_2-\gamma_1) - 2 \,.\, \sin\frac{\gamma_2-\gamma_1}{2} \,.\, \cos\frac{\gamma_2-\gamma_1}{2} \,.\, \frac{x_a' + 2\,x_e}{(r_a' + 2\,R_e)^2 + (x_a' + 2\,x_e)^2}\right\}$$

$$= mE_o^2\left\{\frac{x_a'}{(z_a')^2} - \frac{x_a' + 2\,x_e}{(r_a' + 2\,R_e)^2 + (x_a' + 2x_e)^2}\right\}\sin(\gamma_2-\gamma_1) \quad . \quad . \quad . \quad . \quad (204)$$

It is therefore for any given conditions of excitation and network equal to the sine of the angle of phase difference multiplied by a constant.

In Fig. 556, $\frac{\gamma_2-\gamma_1}{2} = 10{\cdot}8$, E_o for both machines $= 15{\cdot}85$, and the mean external current with the same data as in the preceding diagrams is

$$I_e' = \frac{15{\cdot}85 \cos 10{\cdot}8^\circ}{5{\cdot}68} = 2{\cdot}74 \,. \qquad I_o = \frac{15{\cdot}85}{2{\cdot}33} = 6{\cdot}8, \text{ and } I_c' = 6{\cdot}8 \sin 10{\cdot}8^\circ = 1{\cdot}275.$$

The two powers W_2 and W_1 are in the ratios 2·94 : 1·13, or by eq. (204)

$$W_2 - W_1 = mE_o^2\left\{\frac{2{\cdot}3178}{2{\cdot}33^2} - \frac{3{\cdot}774}{5{\cdot}68^2}\right\}\sin 21{\cdot}6^\circ$$

The maximum possible difference of steam supply, so long as both machines are to act as generators, is of course limited by such divergence of phase that the current of one machine is at right angles to its E.M.F., and hence is entirely neutral. On open circuit any divergence of phase means that the lagging machine is at least partially driven as a motor.

Eq. (203) has further a direct bearing upon the stability of running of alternators in parallel. For example, with two similar alternators it shows that if their phases diverge for any reason there is a direct electric effect assisting the action of the governors and opposing the change, the load being shifted from the lagging on to the leading machine and so tending to keep them in step. It also follows from eq. (203) or (204) that with given alternators the electric regulation is improved with larger values of E_o, *i.e.* with increased excitation.

A fairly long range of speed from no-load to full-load is desirable in the mechanical governor, so that any inequality of the loads on machines working in parallel may cause a definite change in each case and assist in bringing them into correct relation. The curves connecting load and speed, as well as those connecting load and excitation, should be similar in the various machines, and a very close mechanical regulation is not to be recommended, since with it a slight inaccuracy of speed means a large variation in the power delivered by the engine. With a number of alternators in parallel, the machine of which the prime mover has the most sensitive governor controls the speed of the rest, and unless they have similar characteristics for speed and load this may affect the division of the load in due proportion between them.

§ 16. **Sudden fluctuations of load.**—The free oscillations set up when through some momentary disturbance two alternators diverge from their true position for normal running may also be traced from eq. (203) as follows. Let their steam supplies be initially unequal, and correspondingly the phase difference of their E.M.F.'s E_{o_1} and E_{o_2} be $\gamma_2-\gamma_1$. Let this angle for some reason have increased to $\gamma_2-\gamma_1+\beta$; *i.e.* if two radii are marked on the two rotors when in normal work before disturbance, such that their mechanical distance apart is $\frac{\gamma_2-\gamma_1}{p}$, their altered distance after disturbance is $\frac{\gamma_2-\gamma_1+\beta}{p}$. With negligible damping the change of torque in each of the two machines separately is

$$\frac{\Delta W_2}{\omega_2}=M_2k_2^2 \cdot \frac{d^2\theta_2}{dt^2}, \text{ and } -\frac{\Delta W_1}{\omega_1}=M_1k_1^2 \cdot \frac{d^2\theta_1}{dt^2}$$

If θ_1 and θ_2 are the two mechanical displacements corresponding to the total electrical displacement β, *i.e.* to the amount of the abnormal difference of phase β between the two machines, β may be expressed in terms of θ_1 and θ_2 by multiplying each by its corresponding number of pairs of poles; thus $p_1\theta_1-p_2\theta_2=\beta$, the difference of electrical phase angle.

The changes of torque with unaltered mean speed and constant amount of energy in the system must have opposite signs, and ΔW_1 and ΔW_2 are here treated as not in themselves having any sign. Multiplying each expression by its respective value p_1 or p_2, we have

$$p_1M_1k_1^2\frac{d^2\theta_1}{dt^2}+\frac{p_1\Delta W_1}{\omega_1}=0$$

$$p_2M_2k_2^2\frac{d^2\theta_2}{dt^2}-\frac{p_2 \cdot \Delta W_2}{\omega_2}=0$$

Subtracting

$$\frac{d^2(p_1\theta_1-p_2\theta_2)}{dt^2}+\left(\frac{p_1 \cdot \Delta W_1}{\omega_1M_1k_1^2}+\frac{p_2 \cdot \Delta W_2}{\omega_2M_2k_2^2}\right)=0$$

Since $p_1\theta_1 - p_2\theta_2 = \beta$, and for small changes both ΔW_1 and ΔW_2 are proportional to β, this becomes

$$\frac{d^2\beta}{dt^2} + \left(\frac{p_1 \cdot \Delta W}{\omega_1 M_1 k_1^2 \beta} + \frac{p_2 \cdot \Delta W_2}{\omega_2 M_2 k_2^2 \beta}\right)\beta = 0$$

$$\frac{d^2\beta}{dt^2} + \mu\beta = 0$$

The expression in the bracket which has been symbolised by μ may also be expressed as $= \frac{\lambda_1}{M_1 k_1^2} + \frac{\lambda_2}{M_2 k_2^2}$, where λ_1 and λ_2 are the two values of the synchronising couple in each machine separately per unit of the total electrical divergence. The solution being

$$\beta = A \cdot \sin(\sqrt{\mu} \cdot t + \eta)$$

the motion is simple harmonic, and the periodic time $T_p = \frac{2\pi}{\sqrt{\mu}}$ where

$$\mu = \frac{\lambda_1}{M_1 k_1^2} + \frac{\lambda_2}{M_2 k_2^2} = \frac{1}{\beta}\left(\frac{p_1 \Delta W_1}{\omega_1 M_1 k_1^2} + \frac{p_2 \cdot \Delta W_2}{\omega_2 M_2 k_2^2}\right) \quad . \quad . \quad . \quad . \quad . \quad (205)$$

Now, from § 15, if the two alternators have similar electrical properties, and machine 2 leads while machine 1 lags, the change of power in the two machines separately is

$\Delta W_2 =$ (206)

$$mE_o \cdot I_e' \cdot \cos\left(COG + \frac{\gamma_2 - \gamma_1 + \beta}{2}\right) + \frac{mE_o^2}{z_a'} \cdot \sin\frac{\gamma_2 - \gamma_1 + \beta}{2} \cdot \sin\left(\frac{\gamma_2 - \gamma_1 + \beta}{2} + \phi_a'\right)$$

$$- mE_o \cdot I_e \cdot \cos\left(COG + \frac{\gamma_2 - \gamma_1}{2}\right) - \frac{mE_o^2}{z_a'} \cdot \sin\frac{\gamma_2 - \gamma_1}{2} \cdot \sin\left(\frac{\gamma_2 - \gamma_1}{2} + \phi_a'\right)$$

and

$\Delta W_1 =$ (207)

$$mE_o \cdot I_e \cdot \cos\left(COG - \frac{\gamma_2 - \gamma_1}{2}\right) + \frac{mE_o^2}{z_a'} \cdot \sin\frac{\gamma_2 - \gamma_1}{2} \cdot \sin\left(\frac{\gamma_2 - \gamma_1}{2} - \phi_a'\right)$$

$$- mE_o \cdot I_e' \cdot \cos\left(COG - \frac{\gamma_2 - \gamma_1 + \beta}{2}\right) - \frac{mE_o^2}{z_a'} \cdot \sin\frac{\gamma_2 - \gamma_1 + \beta}{2} \cdot \sin\left(\frac{\gamma_2 - \gamma_1 + \beta}{2} - \phi_a'\right)$$

Here I_e = the normal mean current, and I_e' = the new mean value which the external current takes. The change of power is not exactly alike in the two machines, being less in the leading than in the lagging machine.

If one alternator, although similar in electrical design, has a very large moment of inertia, so that, *e.g.*, $M_1 k_1^2$ renders the second term of eq. (205) negligible, the case is equivalent to a small alternator coupled to very large alternators having a constant vector of terminal P.D. The displacement is then entirely in one machine, and β becomes equal to θ_e of one machine. Eq. (205) would then reduce to the same as (199) for a single machine coupled to a large network.

If the two machines are identical, $\mu = \frac{p}{\omega \cdot \beta \cdot Mk^2}(\Delta W_1 + \Delta W_2)$, and from equations (206) and (207) if $\frac{\beta}{2}$ may be written for $\sin\frac{\beta}{2}$.

$$\Delta W_1 + \Delta W_2 = mE_o\left[\left\{\frac{E_o}{z_a'}\sin\phi_a' \cdot \cos(\gamma_2 - \gamma_1) - I_e' \cdot \sin COG \cdot \cos\frac{\gamma_2 - \gamma_1}{2}\right\}\beta \right.$$
$$\left. + 2\sin COG \cdot \sin\frac{\gamma_2 - \gamma_1}{2}\left(I_e - I_e' \cdot \cos\frac{\beta}{2}\right)\right].$$

For small oscillations the last term may be neglected, and

$$\Delta W_1 + \Delta W_2 = mE_o\left\{\frac{E_o}{z_a'} \cdot \sin\phi_a' \cdot \cos(\gamma_2 - \gamma_1) - I_e' \cdot \sin COG \cdot \cos\frac{\gamma_2 - \gamma_1}{2}\right\}\beta \quad . \quad . \quad . \quad (208)$$

while

$$T_p = 2\pi\sqrt{\frac{Mk^2}{\frac{p}{\omega} \cdot \frac{\Delta W_1 + \Delta W_2}{\beta}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (209)$$

The periodic time therefore alters with the difference in the phases of the E.M.F.'s of the two machines, being a minimum when $\gamma_2 - \gamma_1 = 0$; further, it depends upon the nature of the load as affecting the angle COG, but the influence of the second term in (208) is usually small as compared with the first.

§ 17. **Rhythmical fluctuation of speed.**—The free oscillations set up by fluctuation of the load or on switching machines into and out of parallel are not in themselves likely to prove dangerous or injurious, provided the initial disturbance does not itself throw the alternators out of step. If the regulation of the machines be good, so that the short-circuit current is large as compared with the full-load current, there may be large transfers of energy from one machine to another, but the amplitude of the free oscillations will usually become smaller and disappear.

There is, however, always the possibility that the period of the free oscillation may approach too closely to the period of the "forced" oscillation of speed due to the unequal turning moment of the steam- or gas-engine. In this case the rhythmic "hunting" or variation of speed which is now to be considered may increase to such an extent as to throw the machines out of step.

If two similar alternators equally excited and running at the same mean speed with equal steam supply vary rhythmically in driving torque and speed, the effect is at once seen from Fig. 556. Whatever the amount of the irregularity, it is always possible that the maximum velocity of one machine may coincide with the minimum velocity of the other machine, so that the displacement of each machine is a maximum but on opposite sides of the normal; unless, indeed, the moment of paralleling when the electrical phases were in synchronism happened or was arranged to coincide with synchronism of the mechanical phases of the prime movers. Synchronisation of the crank positions in the steam-engines, as by momentarily closing the circuit of a bell for a particular position, is only resorted to in exceptional cases of very slow-speed machines. Apart, then, from such devices, when one machine runs ahead of its mean position by the electrical angle $\frac{\beta}{2}$, the other machine may lag behind the mean position by the same angle, so that the total divergence of phase is β. Fig. 556, then, gives all the essential relations; it is only necessary to write β for $\gamma_2 - \gamma_1$. The line OC_n now bisects the angle β, and gives the normal E.M.F.; the normal current I_n is $\frac{A_nC_n}{z_a'}$, and the mean output is $mI_nE_o \cos COG$. When the phases diverge, the point C_n travels inwards towards the centre to the position C', so that the terminal E.M.F. is also slightly reduced. At the same time, the current component $\frac{D_1C'D_2}{2z_a'}$ arises, which is now synchronising and may be symbolised as I_s'. In combination with the mean external current I_e, it yields the total armature currents and their phases.

The mechanical oscillation due to want of complete uniformity in the driving torque of the prime movers may be described as follows :—When a periodic condition has been reached (cp. Fig. 557), the rotor passes its true normal position in one or other direction with either maximum or minimum angular velocity; assuming the damping to be negligible, the driving torque has then its mean normal value. Starting from this moment when the driving torque is normal and the angular velocity is a minimum, the rotor, say of machine 1, passes its normal position and falls back. The displacement backwards or lag of the rotor causes the rate at which electrical energy is developed by machine 1 to fall below the normal. At the same time, the driving torque from the steam engine is now above the average, so that both the actual excess of the steam torque above the average and also the reduction in the opposing electrical torque are available in combination to accelerate the rotor up to its maximum velocity when once again in its normal position, and with normal driving torque. There follows a displacement forwards or lead, and correspondingly the development of electrical energy by machine 1 is above the normal. At the same time, the driving torque is below the average, so that both the deficit in the work from the steam and also the increase of electrical work are made up by a gradual reduction of the kinetic energy stored in the rotor and fly-wheel of the leading machine. This goes on until the displacement forwards has been annulled, and the rotor of machine 1 has again its normal position with minimum angular velocity. In the rotor of machine 2 the same processes have been occurring, but in reverse order, so that there results a cyclical displacement of the relative phases with a total amplitude β.

Putting $\gamma_2 - \gamma_1 = 0$ in equations (206) and (207) since the normal difference of phase in the present case is zero, the changes of power are

$$\Delta W = \pm m E_o \left\{ I_e' \cos\left(\text{COG} \pm \frac{\beta}{2}\right) + I_s' \cdot \sin\left(\frac{\beta}{2} \pm \phi_a'\right) - I_n \cdot \cos \text{COG} \right\}$$

the upper + signs referring to the leading machine, and the lower − signs to the lagging machine. Or, in terms of the short-circuit current, since

$$I_s' = \frac{E_o}{z_a'} \cdot \sin\frac{\beta}{2} = I_o \cdot \sin\frac{\beta}{2},$$

$$\Delta W = \pm m E_o \left\{ I_o \cdot \sin\frac{\beta}{2} \cdot \sin\left(\frac{\beta}{2} \pm \phi_a'\right) \right.$$

$$\left. + I_e' \cdot \cos\left(\text{COG} \pm \frac{\beta}{2}\right) - I_n \cdot \cos \text{COG} \right\}$$

When the full expression for the change of load or the synchronising power in two similar machines of equal excitation, coupled in parallel and with equal steam supply, is examined in reference to its dependence upon the angle β, it will be found that in the case of the leading machine in the first term the slight drop in the proportionality to the angle in $I_o \sin \frac{\beta}{2}$ is up to a certain point largely corrected by the rise in $\sin\left(\frac{\beta}{2}+\phi_a'\right)$, so that the first term varies nearly directly as the angle $\frac{\beta}{2}$; on the other hand, the difference between the second and third terms which is negative increases faster than in proportion to the angle, so that on the whole the synchronising power increases less rapidly than the angle of phase divergence. In the lagging machine the first negative term is not nearly so proportional to the angle, since $\sin\left(\frac{\beta}{2}-\phi_a'\right)$ decreases instead of rising; but the positive difference between the second and third terms by no means increases as the angle, so that the net result is much more nearly proportional to the angle of divergence. Thus in both cases the synchronising power is rather proportional to the sine of the angle; but just as for small values of the angle the sine is proportional to the angle, so also for small values of $\frac{\beta}{2}$ the synchronising power is fairly proportional to $\frac{\beta}{2}$ or θ_e.

Assuming, therefore, proportionality to the angle, from either the last expressions for ΔW or from eq. (208), since $\gamma_2-\gamma_1=0$, we have

$$\Delta W_1+\Delta W_2=2W_s=mE_o\,\{I_o \,.\, \sin\phi_a'-I_e' \,.\, \sin COG\}\,\beta \quad . \quad . \quad . \quad (210)$$

From the value $\frac{2W_s}{\beta}$ or $\frac{W_s}{\theta_e}$, where W_s is the synchronising watts in either machine separately, the periodic time of a free oscillation is found, and also $\frac{T_s}{\theta_e}=T_{eu}$, of which more will be said later.

The mean E.M.F. OC′ is $E_o \,.\, \cos\frac{\beta}{2}$, and the mean external current I_e' corresponding to this reduced E.M.F. is $\frac{E_o \,.\, \cos\frac{\beta}{2}}{\text{joint impedance}}$; since $\sin COG=\frac{x_a'+2x_e}{\text{joint impedance}}$, eq. (210) becomes

$$\Delta W_1+\Delta W_2=2W_s=mE_o^2\left\{\frac{x_a'}{(z_a')^2}-\frac{\cos\frac{\beta}{2} \,.\, (x_a'+2x_e)}{(r_a'+2R_e)^2+(x_a'+2x_e)^2}\right\}\beta$$

For small oscillations which are alone considered, $\cos\frac{\beta}{2}$ nearly $=1$, and the expression then becomes exactly analogous to eq. (204) for the difference $W_2 - W_1$ between the powers of the two machines.

By comparison of eq. (210) with eq. (196) it is evident that the periodic time of a free oscillation with two identical alternators supplied with the same normal steam torque is less than for an alternator on no-load coupled to a network to which such large machines are coupled that the velocity of the terminal P.D. vector cannot vary, in which case we should have by eq. (196)

$$\frac{W_s}{\theta_e} = mE_o \, . \, I_o \sin \phi_a'$$

But when the two alternators are on open circuit or on light load, so that I_e' is negligible, the case is again identical with a single alternator coupled to a large network, so that in general we have approximately with sufficient accuracy

$$\frac{W_s}{\theta_e} = mE_o \, . \, I_o$$

§ 18. **The angular displacement due to inequality of driving torque in an isolated alternator.**—The complete process of determining the variation in speed and the angle of displacement in a single alternator working against a constant resisting torque as caused simply by the unequal driving torque of the steam engine will be as follows:—From the indicator diagrams,* combined with a consideration of the inertia of the reciprocating parts, is obtained a curve of the driving torque, and thence a curve of the instantaneous value of the angular acceleration or retardation in relation to time. Two successive integrations will then enable the velocity and the space traversed to be obtained; for the instantaneous value of the angular velocity is the integral of the acceleration in relation to time, and finally the time-integral of this last curve plotted above or below the mean velocity as a base line will give at any moment the angle of displacement (alternately lead and lag) which has been traversed by the alternator rotor on either side of the mean position corresponding to perfectly uniform rotation.†

* See especially Kruesi, "Speed Variations of Engines Direct-connected to Alternators," *Electr. World*, vol. xxxvii. p. 591 ff.; *Bull. Soc. Intern. des Electriciens*, vol. i. 2nd series (1901), pp. 536 and 555; P. O. Keilholtz, *Trans. Amer. Inst. Electr. Eng.*, 25th October 1901; and A. R. Horne, *Engineering*, vol. lxxxvii. p. 719.

† Cp. M. David, *Bull. Soc. Intern. des Electriciens* (last cited), p. 503, where it is shown that the actual displacement as measured experimentally is almost always larger than that obtained by the calculation described above, and the reason for this discrepancy is found in the mean speed not being strictly constant. For the methods of M. David and M. Cornu for measuring the degree of uniformity of speed or the angular variation in actual practice, see *Bull. Soc. Intern. des Electriciens* abstracted in *Electr. Eng.*, vol. xxviii. p. 728, and a paper by Dr. Rudolf Franke, translated in *Electr. Review*, vol. l. p. 897.

The process described above is laborious, and also demands exact knowledge of the working of the steam engine, so that an approximate simplification may meet all the needs of practice. It is evident that the variation of the speed and the angular displacement will depend upon the time during which the acceleration or retardation lasts, since their effects are cumulative; hence for a given permissible maximum angle of displacement the maximum permissible variation of speed may be higher in an engine with two cranks than in one with a single crank, since in the former case the excess speed only lasts half the time of the latter case. It will also depend upon the law which the fluctuations of the driving torque or of the acceleration follow. The curve of the driving torque or tangential effort relatively to time may be regarded as made up of a variable portion, alternately positive or accelerating and negative or retarding, superposed upon a constant torque as a base line. A simple assumption will then be that the variable part of the driving torque may be replaced by a sine curve, and so far as this is dependent on the successive steam impulses its periodic time in seconds will be

$$T_e = \frac{60}{N \times n_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (211)$$

where n_c is the number of steam impulses per revolution, and $N =$ revs. per min.

With a single-cylinder double-acting steam-engine there are two points of maximum speed and two minimum points in each revolution, so that the periodic time of the forced oscillation is $T_e = \frac{30}{N}$ (Fig. 558) in a 2-crank engine with cranks at 90°, $T_e = \frac{15}{N}$ and in a 3-crank engine with cranks at 120°, $T_e = \frac{10}{N}$.

The instantaneous value of the driving torque being $T_n + t_v$, where T_n is the mean value averaged over an entire cycle, the above simple assumption gives

$$t_v = T_o \sin\left(\frac{2\pi}{T_e} \times t\right) = T_o \sin q_1 t$$

where T_o is the maximum value of the variable t_v, and time within the cycle of duration $T_e = \frac{2\pi}{q_1}$ is reckoned from the moment when the torque has its normal value. The angular acceleration $\frac{d\omega}{dt} = \frac{t_v}{Mk^2}$ is also a sine curve in phase with t_v.

Similarly, the instantaneous angular velocity is then represented by the superposition of a variable sinusoidal portion upon a mean constant velocity Ω, or $\omega = \Omega + \omega_v$, and

$$\omega_v = -\omega_o \,.\, \cos q_1 t = \omega_o \,.\, \sin\left(q_1 t - \frac{\pi}{2}\right)$$

where the maximum value ω_o is $\frac{T_o}{Mk^2 . q_1}$. The variable part of the space traversed, or the angular displacement from the position for truly uniform motion, is then

$$\theta = -\theta_o . \sin q_1 t = \theta_o . \sin (q_1 t - \pi)$$

The maximum angular displacement, or the amplitude of the curve, is $\theta_o = \frac{T_o}{Mk^2 q_1^2}$, or since $\frac{1}{q_1} = \frac{T_e}{2\pi} = \frac{1}{\Omega . n_c}$, it may also be expressed as

$$\theta_o = \frac{T_o}{n_c^2 \Omega^2 M k^2}$$

Thus, while the curve of instantaneous angular velocity lags 90° behind t_v, the curve of instantaneous angular displacement lags 90° behind ω_v and 180° behind t_v. When the torque has its mean value, the angular velocity is at its minimum or maximum, and the angular displacement from the position of uniform rotation is zero. When the variable torque is positive or accelerating, and the angular velocity is rising, the angular displacement increases on the negative or lagging side and passes through its negative maximum when the torque is a positive maximum and the angular velocity is at its mean value. *Vice versa*, when the variable torque is negative or retarding, the angular displacement is positive or leads (Fig. 557).

$$\text{Since } \delta = \frac{\omega_{max.} - \omega_{min.}}{\Omega} = \frac{(\Omega + \omega_o) - (\Omega - \omega_o)}{\Omega} = \frac{2\omega_o}{\Omega}, \text{ we have } \omega_o = \frac{\delta\Omega}{2}$$

$$\text{Now, } \theta_o = \omega_o . \frac{T_e}{2\pi}; \text{ therefore } \theta_o = \frac{\delta\Omega}{2} . \frac{T_e}{2\pi}$$

But $\Omega . T_e$ = the angle turned through in the time of one cycle $= \frac{2\pi}{n_c}$

therefore $\theta_o = \frac{\delta}{2n_c}$ radians $= \frac{\delta}{2n_c} \times \frac{360°}{2\pi}$ mechanical degrees, or

$$\theta_o = \frac{\delta}{2n_c} \times \frac{360°}{2\pi} \times p \text{ electrical degrees} \quad . \quad . \quad . \quad . \quad . \quad (212)$$

In reality the curve of the variable portion of the driving torque, with the effects of inertia taken into account, being of an irregular shape in the actual steam engine, can only be strictly expressed by a Fourier series of sine terms; yet one of these terms may very greatly predominate over the others. In the single-crank engine this is the term having the periodic time

$$T_e = \frac{2\pi}{n_c . \Omega} = \frac{2\pi}{2\Omega}, \text{ so that } t_v = T_o . \sin 2\Omega t$$

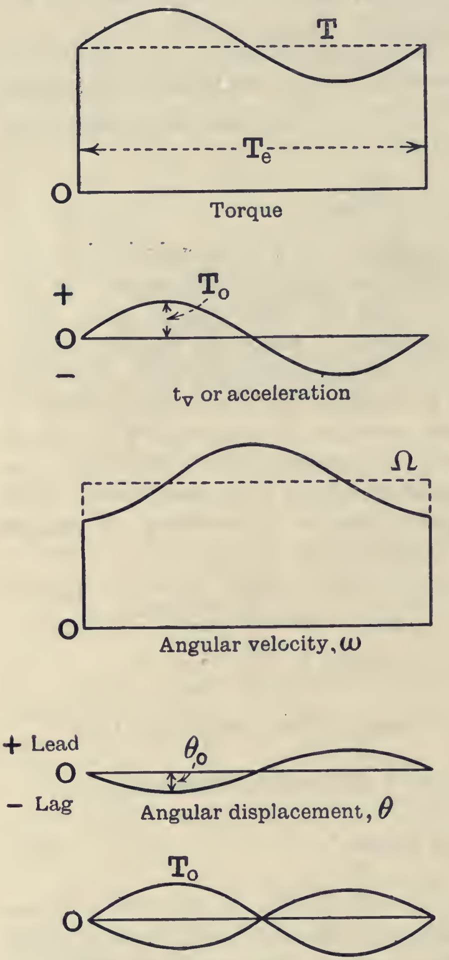
Analysis of actual results * from a number of single-crank engines

* Boucherot, *Bull. Soc. Intern. des Electriciens*, November 1901, vol. i. 2nd series, p. 537, and *Trans. Intern. Electr. Congress St. Louis*, 1904, vol. i. p. 695,

working against a constant resisting torque shows that except under extreme conditions of very light loads or exceptional inertia of the reciprocating parts, the displacement may safely be deduced from the coefficient of speed variation as by eq. (212) with $n_c = 2$. The remarkable result was also found that the same expression, namely,

$$\theta_o = \frac{\delta}{4} \times \frac{360^\circ}{2\pi} \text{ degrees,}$$

could at least, as an approximation, be extended to the case of engines with two or more cranks, regardless of the fact that n_c is then more than 2. Thus in conclusion it may be said that with some caution the hypothesis of a sinusoidal variable torque as replacing the actual varying torque may generally be adopted in practice.



Combination of variable driving torque and change of resisting torque.

FIG. 557.—Torque, acceleration, velocity, and displacement curves.

§ 19. **Calculation of necessary fly-wheel effect for given value of δ.**—The fly-wheel effect required to keep the variation of speed within any assigned limits is calculated as follows. Let a complete cycle of speed be performed in y revolutions; usually y is one revolution or some fraction of a revolution, but in the case, *e.g.*, of a single-cylinder gas engine, after one explosion two revolutions may be required to complete a cycle of operations before the second explosion occurs. The mean power is $\text{HP} \times \frac{33{,}000}{2240}$ ft.-tons per minute, and the

which are in the main followed throughout the subsequent sections. Cp. also Ceytre, *Rev. Electrique*, 1908, ix. p. 29.

mean energy developed in the y revolutions is $E = y \times \frac{33{,}000}{2240} \times \frac{HP}{N}$ ft.-tons. The crank-effort curve, in which the tangential force in tons on the driving point or crank-pin of the engine is plotted as ordinates with the linear feet moved through by the crank-pin as abscissæ, is determined from the combined indicator diagrams of the several cylinders, the effect of inertia in the moving parts being also taken into account (Fig. 558). A mean effort-line being drawn corresponding to the mean energy E, the largest of the areas of excess lying above or of deficiency lying below this line represents the *fluctuation of engine energy* ΔE.

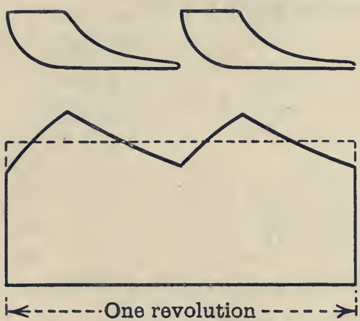


FIG. 558.—Single-cylinder steam engine.

If W be the weight of the fly-wheel, and V be the mean velocity in ft. per sec. of its rim at the radius of gyration, the energy stored in the fly-wheel at its maximum velocity $= \frac{1}{2} \cdot \frac{W}{g} V^2_{max.}$, and at its minimum velocity $= \frac{1}{2} \cdot \frac{W}{g} \cdot V^2_{min.}$, so that the energy received and stored by it as its speed rises from $V_{min.}$ to $V_{max.}$, or the *fluctuation of the fly-wheel energy*, is

$$\frac{1}{2} \frac{W}{g} \cdot (V^2_{max.} - V^2_{min.}) = \frac{1}{2} \cdot \frac{W}{g} (V_{max.} + V_{min.}) (V_{max.} - V_{min.})$$

For small changes of speed, $V = \frac{V_{max.} + V_{min.}}{2}$, so that the above fluctuation in the fly-wheel energy is $\frac{W}{g} \cdot V \cdot V\delta = \frac{W}{g} \cdot V^2\delta$

$$= \frac{W}{g} \cdot \frac{\pi^2 D^2 N^2}{60^2} \cdot \delta \text{ ft.-tons, if W is in tons} \quad . \quad . \quad . \quad . \quad (213)$$

Or in rotational measure, if $\frac{W}{g} \cdot r^2 =$ the moment of inertia,

$$\frac{1}{2} \cdot \frac{W}{g} \cdot r^2 (\omega^2_{max.} - \omega^2_{min.}) = \frac{1}{2} \cdot \frac{W}{g} \cdot r^2\omega^2\delta$$

This quantity must then be equal to ΔE, or the average excess of the tangential pressure in tons on the driving-point or crank-pin of the engine multiplied by that portion of the y revolutions during which it lasts, expressed in feet travelled by the crank-pin. Or if it be a negative area which is the largest, and the fly-wheel gives out energy as it falls in speed from $V_{max.}$ to $V_{min.}$, it is then the product of the deficit of the tangential pressure by the feet through which it lasts. The ratio of the fluctuation of energy ΔE to the mean energy E developed in the period of y revolutions (also expressed in ft.-tons) is the *coefficient of fluctuation of energy*, and forms a constant x for the engine in question, of which the value will depend largely upon the number of cylinders and their arrangement. Thus

$$\frac{\Delta E}{E} = x = \frac{\frac{W}{g} \cdot \frac{\pi^2 D^2 N^2 \delta}{60^2}}{y \times \frac{33{,}000 \text{ HP}}{2240 \text{ N}}}$$

whence

$$WD^2 = \frac{17 \cdot 4 xy \times HP \times 10^4}{\delta N^3} \qquad (214)$$

or combining together $17 \cdot 4xy$ into one constant c

$$WD^2 = \frac{c \times HP \times 10^4}{\delta N^3} \qquad (215)$$

where W is in tons and D is the mean diameter of the rim in ft.

The determination of ΔE or of c is by no means easily made, but certain figures taken from Herr O. Lasche's above-quoted paper * on the "Construction and Manufacture of Alternators" may be given to illustrate the subject. In particular cases the values of c were

for a single-cylinder steam-engine . . . . . . 3·65
for a compound engine with cranks at right angles . . 1·6
and for a triple-expansion engine with cranks at 120° . . 0·93

From eq. (215) such values must be taken for W and D as will give a convenient size of fly-wheel. Or, if the size of fly-wheel is given, the value † of δ is found.

§ 20. **Effect of the synchronising torque in increasing the displacement.**—In both of the preceding sections the reaction of the alternator upon the prime mover has in no way been taken into account. Since T_o is the maximum variable portion of the steam-driving torque when acting alone, and only the mechanical data of the steam engine and alternator have been under consideration, the displacement θ_o and the fluctuation of speed δ upon which it is dependent is simply that

* In footnote Chap. xxiv. § 5.

† See M. David, *Bull. Soc. Intern. des Electriciens*, vol. i. 2nd series (1901), pp. 504, 505, where an example is worked out.

occurring in a single steam-driven alternator feeding lamps alone which for any given mean load would yield a constant resisting torque.

But when the same engine and alternator are coupled to a network containing synchronous motors, or to other generators, the conditions are altered, and a further force arises owing to the resisting torque not being constant. It has already been shown (§ 17) that, as the armature E.M.F.'s of two similar machines equally excited diverge in phase from their normal position of coincidence, the electrical resisting torque of one rises and of the other falls, and the normal *minus* the actual resisting torque in either case gives the synchronising torque acting upon the machine in question. The change of the resisting torque is in phase with the displacement, so that, when a machine leads, the electrical resisting torque is above the mean, or the variable part of the resisting torque is positive, but when a machine lags, the electrical resisting torque is below the mean, or the variable part is negative. A change of torque, alternately positive and negative, may therefore be regarded as superposed upon the normal constant torque corresponding to coincidence of phase in the two machines, and since its alternations coincide in phase with the displacement, it lags 180° behind the variable part t_v of the driving torque (Fig. 557), provided there is no damping. The synchronising torque is the same as the change of resisting torque but with reversed sign, so that it then coincides exactly in phase with t_v. The sum of the two is therefore available for accelerating or retarding the rotor. Thus, although the synchronising torque is always acting to reduce any difference of phase between two alternators running in parallel, the work which it does in accelerating or retarding the rotating mass has the effect already pointed out in § 17 of actually increasing the angular displacement which would be due solely to the unequal turning moment of the steam engine. Take the moment when the crank-pin of a single-cylinder steam engine is passing the dead centre; the rotor is then in advance of the position corresponding to uniform speed owing to the previous accelerating action of the excess driving torque, hence the synchronising torque is holding back the rotor; at the same moment the driving torque is a minimum, or, in other words, the variable part of the driving torque is negative and also retarding the rotor. Or, when the driving torque is normal the speed is a maximum or a minimum, but there is no displacement and no synchronising torque to check it. Thus the synchronising torque acts in harmony with the variable part of the driving torque, and the amplitude of the oscillations is increased.*

* Boucherot, *Bull. de la Soc. Intern. des Electr.*, 2nd series, vol. i. pp. 529–553 (abstracted in *L'Éclairage Électrique*, vol. xxx. p. 287); Longwell, "Paralleling of Alternators," *Electr. World and Engineer*, vol. xxxix. p. 958, and *Str. Rly. Rev.* (1902), xii. p. 358: Oscanyan, *Electr. World and Engineer*, vol. xl. p. 416.

The first four diagrams of Fig. 557, which assume a constant resisting torque, must therefore be modified so as to show the increase of the displacement by the synchronising torque. As already mentioned, the curve of the variable driving torque can only be accurately represented in practice by a Fourier series of sine waves, although one of these may be predominant. But if we isolate any one of these components we can deduce the initial displacement due to it as arising from the steam engine alone, and also the initial synchronising torque which it calls into action; the combined action of the driving torque and of this initial synchronising torque will then give an increased acceleration, and larger variable velocity and displacement diagrams; these again will give a larger synchronising torque (although the increase except under special conditions will be less), so that the diagrams must be step by step rectified until all the conditions are satisfied. But since the component of the variable driving torque which is in question is sinusoidal, this gradual process of correction may be at once abbreviated by mathematical analysis. Let the variable component in question be expressed by $T_o \sin (q_1 t)$ or $T_o \sin (n_c . \Omega\, t)$, so that its periodic time is $T_e = \frac{2\pi}{q_1} = \frac{2\pi}{n_c . \Omega}$, the symbol n_c being now not simply the number of steam impulses per revolution, but being extended to cover the number of complete waves of the component in question per revolution.

Under the action of an external periodically varying torque, of which the instantaneous value is $T_o \sin q_1 t$, where $q_1 = \frac{2\pi}{T_e}$, the three terms on the left-hand side of equation (194) are no longer equal to zero, but to the instantaneous value of the varying torque, or

$$Mk^2 \frac{d^2\theta}{dt^2} + b . \frac{d\theta}{dt} + T_u . \theta = T_o . \sin q_1 t$$

The solution of the equation is

$$\theta = \frac{T_o}{\sqrt{b^2 q_1^2 + (T_u - Mk^2 . q_1^2)^2}} . \sin (q_1 t - \eta)$$

$$\text{where } \tan \eta = \frac{b q_1}{T_u - Mk^2 . q_1^2}$$

$$\text{and } \theta_{mf} = \frac{T_o}{\sqrt{b^2 q_1^2 + (T_u - Mk^2 q_1^2)^2}}$$

The full expression for θ must also contain terms which define the time in which any particular value of the displacement is reached, and theoretically the final value of the displacement is only reached after an infinite number of cycles, but except under special conditions the final value as given above is closely attained after a few oscillations.

In practical cases, bq_1 is small * as compared with T_u, but in the first place we will assume damping to be entirely absent, so that

$$\theta = \frac{T_o}{T_u - Mk^2 q_1^2} \cdot \sin q_1 t$$

The final maximum for the mechanical angular displacement is then

$$\theta_{mf} = \frac{T_o}{T_u - Mk^2 \cdot q_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (216)$$

At the same time, θ_o without the action of the synchronising torque would be $-\frac{T_o}{Mk^2 \cdot q_1^2}$, so that the ratio of the final to the initial displacement with negligible damping is

$$\sigma = \frac{\theta_{mf}}{\theta_o} = \frac{Mk^2 \cdot q_1^2}{Mk^2 \cdot q_1^2 - T_u} = \frac{1}{1 - \frac{T_u}{Mk^2 \cdot q_1^2}}$$

and it is evident that the increase is entirely dependent upon the ratio of T_u to $Mk^2 \cdot q_1^2$. Since $T_u = \frac{4\pi^2 Mk^2}{T_p^2}$ and $Mk^2 q_1^2 = \frac{4\pi^2 Mk^2}{T_e^2}$, we also have

$$\frac{T_u}{Mk^2 q_1^2} = \frac{T_e^2}{T_p^2}, \text{ and } \sigma = \frac{1}{1 - \left(\frac{T_e}{T_p}\right)^2}$$

Thus the increase in the final displacement is dependent upon the square of the ratio of the periodic times of the forced and natural oscillations.

This may also be expressed in other terms by substituting $p \cdot T_{eu}$ for T_u and $-\frac{T_o}{\theta_o}$ for $Mk^2 q_1^2$. The ratio then becomes $\frac{p \cdot T_{eu}}{-\frac{T_o}{\theta_o}} = \frac{-p \cdot T_{eu} \cdot \theta_o}{T_o}$.

For any angle of displacement θ, the synchronising torque $T_s = -T_u \cdot \theta = -p \cdot T_{eu} \cdot \theta$. Therefore $-p \cdot T_{eu} \cdot \theta_o$ is the initial value of the synchronising torque or T_{so} as called into action by the initial displacement, so that

$$\frac{T_u}{Mk^2 q_1^2} = \frac{T_e^2}{T_p^2} = \frac{T_{so}}{T_o}$$

The ratio of the initial synchronising torque from the initial θ_o due solely to a variation of the steam-driving torque of sinusoidal law, to the variable part of the driving torque, is therefore another expres-

* For values of the ratio $\frac{b}{T_u}$ have been given the following figures, namely: (1) with laminated pole-pieces, 0·003–0·007; (2) with solid pole-pieces, 0·01–0·025; (3) with Leblanc dampers, 0·01–0·2.—O. Weisshaar, *Elektrotech. u. Maschinenbau*, vol. xxvi. p. 555 ff.).

sion for the square of the ratio of the two periodic times, and if $Q=\frac{T_{so}}{T_o}=\left(\frac{T_e}{T_p}\right)^2$, upon the quotient $\frac{1}{1-Q}$ depends the final value of the displacement. If T_{so} is $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{3}$ or $\frac{1}{2}$ of T_o, θ_{mf} is $\frac{10}{9}$, $\frac{5}{4}$, $\frac{3}{2}$, or $\frac{2}{1}$ times θ_o, and the coefficient of speed variation is by the same amount larger than that which would be due simply to the steam engine alone. Using the subscript letters *f* and *o* to indicate final and initial values,

$$\sigma=\frac{1}{1-Q}=\frac{\theta_{mf}}{\theta_o}=\frac{\omega_{vf}}{\omega_{vo}}=\frac{T_{sf}}{T_{so}} \quad . \quad . \quad . \quad . \quad . \quad (217)$$

$$Q \text{ is also } =\frac{T_{sf}}{T_o+T_{sf}} \text{ and } \sigma=\frac{T_{mf}}{T_o}=\frac{T_o+T_{sf}}{T_o} \quad . \quad . \quad . \quad (217a)$$

The final synchronising T_{sf}, to which the fluctuation of output is proportional, being σT_{so}, is also $=\frac{Q}{1-Q}\,.\,T_o$ or $(\sigma-1)T_o$, so that if the ratio which T_o bears to the normal torque is known, the fluctuation of output is thence directly found. The action is clear if it is considered that to the initial value of the synchronising torque there must be added an amount QT_o as increasing the displacement; this again will cause an additional synchronising torque $Q(QT_o)=Q^2\,.\,T_o$, and so on indefinitely. The total variable torque, or the sum of the initial variable torque and the additional increments from the synchronising action, is therefore

$$T_{mf}=T_o+Q\,.\,T_o+Q^2\,.\,T_o+\quad . \quad . \quad . \quad =T_o\left(\frac{1-Q^n}{1-Q}\right)$$

If Q be less than unity, the series is convergent and approaches indefinitely close to the final value $T_{mf}=T_o\,.\,\frac{1}{1-Q}$, and $\theta_{mf}=\theta_o\frac{1}{1-Q}$

When $Q>1$, *i.e.* $T_e>T_p$, σ becomes negative, which means that the final displacement, the final velocity and the final synchronising torque are all in the *opposite* direction to the initial. T_{sf} is then negative or opposed to T_o, and so also is T_{mf} since T_{sf} is $>T_o$, *i.e.* at the moment of greatest lag, the engine torque is a minimum, and the forward synchronising torque more than counterbalances the deficit in the torque from the engine.

It will now be evident that when $Q=1$, *i.e.* when $T_{so}=T_o$, or $T_u=Mk^2\,.\,q_1^2$, the oscillations without damping grow indefinitely and the mechanical displacement becomes so great that the alternator cannot be run in parallel. This critical condition occurs when $T_e=T_p$, or the period of the forced oscillation due to the component of the variable driving torque which is in question exactly coincides with the natural period of free oscillation of the rotor of the alternator.

This may again be brought to mind if it be remembered that the synchronising torque, being assumed to be proportional to the electrical angle of displacement, $T_s=-T_{eu}\,.\,p\theta_o$ where T_{eu} is the synchronising torque per unit angle of electrical displacement, or the proportionality factor between any two corresponding values of synchronising torque and electrical displacement of phase; it has already been shown that, under the action of the variable steam torque alone, $\theta_o = -\frac{T_o}{Mk^2} \times \frac{T_e^2}{4\pi^2}$, so that the initial synchronising torque is $T_{so} = T_{eu} \cdot p \times \frac{T_o}{Mk^2} \times \frac{T_e^2}{4\pi^2}$.[1] If now $T_{so} = T_o$, we have the special condition

$$T_e = 2\pi\sqrt{\frac{Mk^2}{pT_{eu}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (218)$$

But it is not only when exact resonance occurs that danger is to be feared; if the critical condition is too nearly approached, the final displacement is greatly magnified by the imperfect resonance, and the oscillations once started by the small amount of speed variation which must persist even in an exceptionally good engine, will increase to such an extent from the pendulum action that the machine is eventually thrown out of step entirely. If $Mk^2q_1^2 > T_u$, but is too nearly equal to it, the remedy must be to increase the fly-wheel effect; but in the reverse case of $Mk^2q_1^2 < T_u$, a decrease in the fly-wheel will cause a greater divergence between the two quantities, and in certain exceptional cases in practice a reduction of the rotating masses has been found to be attended with improved results.

The extreme rapidity in the increase of the displacement near the point of resonance is shown by the following table of σ for various values of $\frac{T_e}{T_p}$.

$\frac{T_e}{T_p}$	σ	$\frac{T_e}{T_p}$	σ
0·0	1·00	0·9	5·26
0·2	1·04	1·0	∞
0·4	1·19	1·1	−4·75
0·6	1·56	1·2	−2·27
0·7	1·96	1·3	−1·45
0·8	2·78	2	−0·333
		∞	0

The conditions are, however, much modified when there is damping present. The damping torque is proportional to the vector of the varying speed, and the initial varying torque must contain a component to balance it. The result is that only the remaining component at right angles to that which balances the damping torque is effective in causing the initial displacement. In relation to this latter component the speed and displacement curves still continue to lag by 90° and 180° respectively as before, but the total initial variable torque, owing to the damping component now present in it, is no longer at right angles to the speed and exactly opposite in phase to the displacement. As Q is increased its vector draws nearer and nearer into phase with the varying speed, until, when $Q=1$, they coincide, and the whole of the initial variable torque now exactly balances the damping torque. The speed is not then infinite, but just rises to such a value that this equality holds. When $Q>1$, the vector of the initial variable torque lags behind the speed, and draws nearer in phase to the final displacement.*

If we still retain T_p to represent the natural periodic time without damping, then even when damping is added on, all the equations for Q, σ, and equations (216), (217), (217*a*) still hold good provided that throughout both in the original and final expressions without and with synchronising action we substitute for the full value of T_o only the component which is at right angles to the damping torque and is therefore unbalanced by it. The amplitude of this unbalanced component is

$$T_o \cos \eta = T_o . \frac{T_u - Mk^2 . q_1^2}{\sqrt{b^2 q_1^2 + (T_u - Mk^2 . q_1^2)^2}}$$

and when $Q=1$, this becomes infinitely small, so that although σ is infinite, the final values remain finite. The amplitude of the damping torque $T_d = b \times \omega_{vf}$ is

$$T_o \sin \eta = T_o . \frac{bq_1}{\sqrt{b^2 q_1^2 + (T_u - Mk^2 . q_1^2)^2}}$$

The true oscillation of output is then proportional to a fluctuating torque of which the amplitude is $\sqrt{T_d + T_{sf}^2}$.

§ 21. **Maximum permissible value of phase displacement.**—From the point of view simply of steadiness of the light, the coefficient of speed variation or δ in an engine driving an alternator might often be as high as $\frac{1}{100}$, but in practice smaller figures are in most cases obtained; the more usual figures which may be obtained without unduly heavy fly-wheels give limiting values of say $\frac{1}{180}$ or 0·55 of 1 per cent. for small machines, and $\frac{1}{250}$ for larger machines.

But the needs for parallel working are really the decisive considera-

* A convenient graphical method which also takes into account the effect of damping has been given by E. Rosenberg, *Journ. Inst. Electr. Eng.*, vol. xlii. p. 533, "The Parallel Operation of Alternators." This paper, which appeared after the above was written, brings before English readers the main points which Dr. Rosenberg has published in foreign periodicals together with additional matter.

For other articles dealing with the subject of the parallel working of alternators, the reader is referred to P. Boucherot, *La Lumière Électrique*, vol. xlv. pp. 201 and 260, and *Bull. Soc. Inter. Electr.*, 1904, iv. p. 495; A. Blondel, *Bull. Soc. Inter.*, 1893, p. 147; Kapp, *E.T.Z.*, 1899, vol. xx. p. 134; L. Wilson, *Journ. Inst. Electr. Eng.*, 1899, vol. xxviii. p. 395; G. Benischke, *E.T.Z.*, 1899, vol. xx. p. 870; H. Görges, *E.T.Z.*, 1900, vol. xxi. p. 188 and vol. xxiv. p. 561; *Proc. Amer. Inst. Electr. Eng.*, October 25, 1901, E. Rosenberg, *E.T.Z.*, 1902, vol. xxiii. p. 450; *Zeitschrift Vereines Deutsch. Ing.*, 1904, xlviii. p. 793, and I. Döry, *Elektrotechnik u. Maschinenbau*, vol. xxvii. p. 315.

tions, and it is therefore the permissible fluctuation of output which is paramount rather than the actual variation of speed. Owing to their perfectly uniform speed of rotation, the water-wheel and turbine are the best prime movers for alternators which are to be run in parallel. But even the steam turbine may in exceptional cases have forced oscillations set up if the steam is admitted in recurring puffs by the action of the governor. In the steam engine, after every precaution has been taken to balance the reciprocating parts and to distribute the load properly between the several cylinders and within each cylinder, the want of uniformity in the speed which still remains must be brought within certain practical limits by adding sufficient fly-wheel effect in the machine as a whole. The heavy revolving mass of the magnet system may require to be supplemented by a fly-wheel proper, but any weight of fly-wheel greater than is really called for by genuine electrical reasons is not only an unnecessary expense, but is prejudicial to the proper action of the engine governor. It therefore becomes of great importance to determine the maximum displacement of the electrical phase from the position for perfectly uniform speed which is compatible with good parallel working.

What the designer has to secure is, in the last resort, that the final displacement, being the vectorial sum of the displacements due to all of the component harmonics of the variable driving torque combined with the effect of the synchronising torque, *i.e.* θ_{ef}, should never exceed the maximum found permissible in practice. Obviously, with two similar alternators in parallel, if θ_{ef} ever exceeded 45°, the maximum theoretical synchronising power on our approximate assumptions would have been passed, and the alternators must fall out of step. But even with values much short of this, parallel working may be attended with serious trouble. It may again be here mentioned that the possibility of one machine's variation of speed coinciding with that of another, but in the reverse sense, must be foreseen, so that the total divergence of phase β may be twice the divergence θ_e of either machine from an assumed vector of fixed position. As a practical limit, perhaps a maximum electrical displacement θ_{ef} of 6° may be adopted,* although further evidence from theoretical analysis of actual cases (troublesome or otherwise) is much to be desired. The nature of the load requires too to be taken into account; rotary converters, for instance, form a most delicate load, and are peculiarly liable to rhythmic hunting or phase-swinging, which may upset the generators if their natural period approaches too closely to resonance with any speed-variation in the engines.

* This assumption follows the suggestion of A. S. Barnes, Jun. (*Trans. Amer. Inst. Elect. Eng.*, vol. xxi. p. 343 ff. Even figures as large as ± 22·5° have been regarded as permissible, cp. *Bull. Soc. Intern. des Electriciens*, vol. i. 2nd series, 1901, p. 557. M. Boucherot allows ± 10 as a limit.

Since $I_s' = \frac{E}{z_a} \sin\frac{\beta}{2}$, by the reasoning of § 17, the maximum synchronising current with a phase displacement of $\frac{\beta}{2} = \pm 6°$ is $0·1045\ I_o$, and if the short-circuit ratio $\frac{I_o}{I_n}$ have the very usual value of 2·5, the maximum synchronising current is 26 per cent. of the full-load current, an amount which need not be feared on the score of its effect on the efficiency.

In the next place, the permissible final displacement in mechanical units must vary inversely as the number of pairs of poles, or $\theta_{mf} = \sigma\theta_o = \frac{6°}{p}$. In other words, if a point on the revolving portion of the alternator or on the fly-wheel were marked, and its position relatively to that which it would have with perfectly uniform rotation could be followed by the eye, its maximum angular displacement must not exceed $\frac{6°}{p}$. With a standard speed for a large 1500 or 2000 kw. engine-driven alternator of about 75 revs. per min. 40 poles are required for 25 cycles, or $p = 20$, and θ_{mf} must then be $\leq$ 0·3 degree. If the same output is to be given at a frequency of 60, the number of poles must be increased to 96, and for the same want of uniformity of engine speed and ratio of T_{so} to T_o or of T_e to T_p, the phase displacement would now be $6° \times \frac{96}{40} = 14·4$ electrical degrees, which might render parallel working difficult. Hence in the latter case, for the same phase displacement to be again maintained, a much heavier fly-wheel must be employed. A low frequency is therefore advantageous, and for a given frequency the advantage of a high speed is evident. Thus, as has already been stated (chap. xxiv. § 13), the greater the number of poles the less must be θ_{mf}. But it does not necessarily follow that the initial mechanical displacement θ_o, as due solely to inequality of the driving torque, must be less with the greater number of poles, or the rotation due to the engine alone more uniform. The value of σ for any particular component of the variable driving torque of periodic time T_e being $\frac{1}{1 - \left(\frac{T_e}{T_p}\right)^2}$, the most important factor is the ratio of the periodic times of the forced and free oscillations; for if the conditions of resonance are approached, the increase of the displacement is so great that the value of the initial displacement θ_o becomes of but minor importance.

If the various harmonics of the variable torque and their percentages of the normal full-load torque are obtainable, the full process should be to deduce the displacement and oscillating output due to each, with allowance for damping, and adding them vectorially to limit the total fluctuating output, say to 25 per cent. of the full load. It will be found that σ decreases very rapidly for the terms of the series containing 3Ω, 4Ω, etc., *i.e.* when in $T_e = \frac{2\pi}{n_c . \Omega}$, n_c is 3, 4, or more. Thus even with multi-crank engines it is chiefly the components in Ω and 2Ω which require attention, and if these give satisfactory results the higher harmonics may be safely neglected. With single-crank engines the component with $n_c = 2$ must be carefully examined. But above all in every case the first term with $n_c = 1$, which is chiefly due to the obliquity of the connecting rod to the piston rod, to unbalanced weights, and to inequality between the impulses on the two sides of a double-acting piston, is the most important; in the first place, it is most likely to be resonant with the natural period, and also it may in multi-crank engines cause an initial displacement four times that due to the term in 2Ω.

It is best not to work * with a negative value of σ for the term in Ω, *i.e.* with $T_p < T_e$, since there is then a danger of resonance with the term in 2Ω when T_p approaches $\frac{T_e}{2}$, the more so because T_p is itself not a constant, but varies somewhat with the load and voltage. As a general rule, the natural periodic time of the alternator is in practice made greater than even the longest periodic time of any forced oscillation from inequality of the driving torque.

The exact curve of the driving torque is seldom available for resolution of its variable portion into the various components by the usual methods of analysis. We then have to fall back upon examination of the periods for Ω and 2Ω, and in each case deduce the initial displacement from extreme values of T_o, such as 0·16 and 0·9 of the normal torque in single-crank engines, or 0·1 and 0·5 of the normal torque in multi-crank engines; or even in both cases to deduce it from the coefficient of speed variation, δ, by the empirical expression, namely, $\theta_o = \frac{\delta}{4} \times \frac{360°}{2\pi}$ mechanical degrees. The error is then at least on the safe side.

As a practical guide, without allowance for damping, it may perhaps be laid down that it is unsafe for any T_e to be greater than 0·7 T_p; or conversely, the frequencies of the oscillations being the reciprocals of their periodic times, the natural frequency should not be more than 70 per cent. of any forced frequency. The final displacement is then practically double the initial displacement, for $\sigma = \frac{1}{1-(0{\cdot}7)^2} = 1{\cdot}96$. Lastly, with this condition must be combined some further assumption as to the amount of the maximum permissible initial displacement, and it follows from our first rule that this must be given as $p\theta_o = \pm 3$

* Unless for cogent reasons; cp. E. Rosenberg, *Journ. Inst. Electr. Eng.*, vol. xlii. p. 543.

electrical degrees.* The total displacement when the alternator is working in parallel is then $\frac{6°}{p}$ on either side of the normal.

If η = the ratio of the terminal voltage at full-load to that on open circuit, the normal watts of output per phase $= \eta E_o . I_n . \cos$ COG, and if the angle COG be identified with ϕ_e, by which it is mainly determined,

$$E_o = \frac{\text{normal watts per phase}}{I_n . \cos\phi_e \times \eta}$$

$$\frac{W_s}{\theta_e} = mE_o . I_o . \sin\phi_a' \text{ is thus} = \frac{\text{KW} \times 1000 \times \sin\phi_a'}{\cos\phi_e \times \eta} \times \frac{I_o}{I_n}$$

and since $\sin\phi_a'$ and η are not far different, $= \text{KVA} \times 1000 \times \frac{I_o}{I_n}$. The periodic time of a free oscillation may then be expressed in terms of apparent output, frequency, moment of inertia, and ratio of short-circuit to full-load current as

$$T_p = 1{\cdot}27\ N \sqrt{\frac{WD^2}{f \times \text{KVA} \times 1000 \times \frac{I_o}{I_n}}} \quad . \quad . \quad . \quad . \quad (219)$$

In this expression it will be seen that the only electrical quantity that the designer can modify if the moment of inertia is fixed, is the ratio $\frac{I_o}{I_n}$. This ratio is dependent upon the excitation, and the greater that it is, the shorter the natural period, which is to the disadvantage of parallel working. An excitation lower than is usually employed lessens the ratio and increases the natural period, so that an alternator may perhaps be safely run in parallel at a pressure lower than the normal for which it is designed. But any such decrease in $\frac{I_o}{I_n}$ is in direct conflict with the requirements of inherent regulation in the alternator; in fact, the natural period is proportional to the square root of the inductance, and the higher this is the better from the points of view of safety during short-circuit and of parallel running, but the worse as regards regulation. A compromise must therefore be struck, and in practice this is set at $\frac{I_o}{I_n}$ = from $2\frac{1}{4}$ to $3\frac{1}{2}$, or on an average $= 2\frac{1}{2}$. There is thus but little scope for modification of the short-circuit ratio, and since the forced period is not to be more than 70 per cent. of the natural,

$$\frac{60}{N . n_c} = 0{\cdot}7 \times 1{\cdot}27\ N \sqrt{\frac{WD^2}{f \times \text{KVA} \times 1000 \times \frac{I_o}{I_n}}}$$

$$\frac{4550}{N^4 . n_c^2} = \frac{WD^2}{f \times \text{KVA} \times 1000 \times \frac{I_o}{I_n}}$$

* Since the above was written the report of the Engineering Standards Committee on Reciprocating Steam-engines (1909) confirms the same recommendation.

in which the only quantity that permits of considerable modification is the moment of inertia of the whole of the rotating parts, since this can be affected by alteration of the fly-wheel effect. The fly-wheel must therefore be designed so that, in conjunction with the other rotating parts, in British units of tons and feet

$$\mathrm{WD}^2 = \frac{f \times \mathrm{KVA} \times \dfrac{\mathrm{I}_o}{\mathrm{I}_n}}{\mathrm{N}^4 \times n_c^2} \times 4.55 \times 10^6 \quad . \quad . \quad . \quad (220)$$

from which the great advantage of a quick-revolution engine in reducing the necessary size of fly-wheel for a given horse-power and frequency is obvious. With the fly-wheel and rotor thus dimensioned the initial displacement must again be checked, to see that its value does not exceed the assumed permissible maximum of $\pm 3°$ of electrical phase.

§ 22. **Influence of the governor.**—In addition to the rhythmical variation in the driving torque of steam engines, there may also be forced oscillations set up by the governors under sudden variations of load. Since the period of such governor oscillations is fairly long, there is the more likelihood of resonance with the natural period of the alternator, and the only remedy is then to damp the action of the governor by means of a dash-pot. An unduly heavy fly-wheel with long natural period is especially to be avoided in this connection, as well as from the fact that it prolongs the time required for synchronising when switching into parallel.* The action of the mechanical governor introduces so many complexities, since there is a certain lag between the change of steam pressure or steam supply and the change of speed to which it is due, that space prevents a further consideration of the subject.

§ 23. **Influence of the shape of the E.M.F. curve.**—So far the E.M.F. waves of the alternators have been assumed to have the same sinusoidal shape, but when, as is often the case, they are very unlike in form, the resulting cross E.M.F. acting round the circuit of the armatures may cause such a powerful cross current and such violent oscillations as to render parallel working impossible. It is evident that a slight divergence of phase between two very peaked waves of E.M.F. will produce a much larger resultant cross E.M.F. than the same divergence between smooth sine waves. Further, the oscillograph shows that on light loads there may in such cases of different wave-

* For further discussion on the influence of governor and fly-wheel, see Van der Stegen, *Soc. Belge Electr. Bull.*, 1902, xix., p. 14; C. F. Guilbert, *Bull. Soc. Intern. Elect.*, 1902, ii. p. 281; P. Boucherot, *Trans. Intern. Electr. Congress St. Louis*, 1904, vol. i. pp. 697–705, and *Bull. Soc. Intern. Electr.*, 1905, v. p. 509; H. H. Barnes, *Trans. Amer. I.E.E.*, vol. xxi. p. 343, "Notes on Flywheels"; de Marchena *Bull. Soc. Intern. Electr.*, 1902, ii. p. 754; C. E. Lucke, *Trans. Amer. I.E.E.*, vol. xxvi. part i. p. 1.

forms be alternations of the current more rapid than the true frequency, and consequently rapid changes from generator to motor action. If the frequency of the reversals of power is such that they are reinforced by any approach to resonance with the natural period, hunting may result, even though there is no periodic fluctuation of driving torque from the prime mover.*

§ 24. **Damper winding.**—For the purpose of rendering polyphase machines more stable when run in parallel, and more independent of any fluctuation of the prime movers' speed, M. Leblanc introduced the device of an additional *damper* or "*amortisseur*" winding. This consists in principle of a number of copper bars embedded in holes close to the faces of the pole-pieces parallel to the armature wires or slots, and riveted at their projecting ends to two solid rings of copper or bronze.† The damper system is thus closely analogous to the squirrel-cage armature of an asynchronous induction motor and its action is to be explained on precisely the same principles.‡ The eddy-currents which would flow in a solid pole-piece are canalised or confined to paths at right angles to the flux-direction, and are rendered fully effective.

As explained in Chapter XXIII. § 9, the progressive rise and fall of the current in the armature windings of a polyphase generator sets up a fundamental wave of M.M.F. which may be taken into account by regarding it as causing a rotary field similar to that due to the primary coils of an induction motor. When only one generator is at work this field rotates in exact synchronism with the field-magnet, so that the bars of the damper winding are in no way cut by it, and the same is true if several machines run in parallel with perfectly uniform velocity.

But if a free oscillation is set up in any machine, the rotating field-magnet regarded as a secondary lags or leads at regular intervals, and so has a certain slip relatively to the armature regarded as a primary. The secondary currents set up in the bars of the damper winding then have the effect described in § 10 as due to currents crossing the pole-faces, so that their function is to make b positive and damp out the oscillation. Since the amount of damping is proportional to the amplitude of the changes of induction and to their phase difference with the E.M.F. causing them, and the amplitude is diminished approximately in proportion to the conductivity of the damper winding, Professor B. Hopkinson § has pointed out that it is possible to increase the conductivity too far, when the diminished amplitude outbalances

* *See* especially A. F. T. Atchison, *Journ. Inst. Electr. Eng.*, vol. xxxiii. p. 1119.

† Cp. *Electr. Eng.*, 20th November 1896, vol. xviii. p. 580; *Electrician*, 28th September 1900, vol. xlv. p. 844.

‡ Cp. Fischer-Hinnen, *Electr. World and Eng.*, vol. xxxviii. p. 1058.

§ Professor B. Hopkinson, *Electr. Eng.*, vol. xxxii. p. 471.

the increased phase difference; in other words, it is possible to put too much copper into the damper, and the extent to which it is embedded in the iron admits of some variation in the design so as to secure the right relation between its inductance and its resistance to exert maximum effect for the frequency of the oscillations which it is especially to damp out. The forced oscillations due to the hunting of the steam-engine governor may likewise be reduced by damper winding, but if of short period, as due to irregularity of speed, the usefulness of damper winding is more open to question, except when resonance is closely approached and the damping torque is nearly equal to the initial variable torque.*

It is not necessary that the rings short-circuiting the damper bars should be continuous, provided that there is no interference in the paths round which the secondary currents flow, and this will be ensured if there are only small gaps occurring midway between the poles. Further, the bars of copper may in the interpolar gaps be replaced by

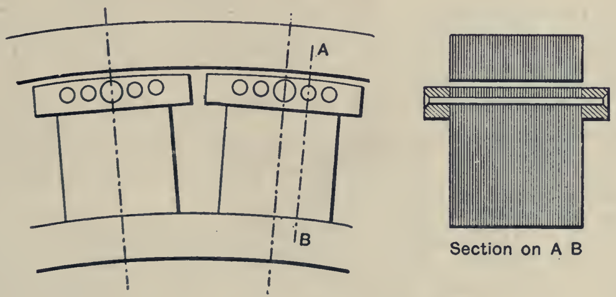


FIG. 559.—Damper winding.

solid brass pieces. Fig. 559 shows a form of damper winding employed by the Allgemeine Elektricitäts Gesellschaft, in which each pole is self-contained; the pole-faces are encircled by brass castings, into which a few round copper bars passing through the laminations of the pole are riveted. The casting at the same time serves to hold in place the winding of the field-coils.

In the single-phase alternator the action of the damper winding is somewhat different. The secondary currents set up in it have the effect of partially neutralising the armature ampere-turns, and so of reducing the armature reaction or of increasing the output for the same drop of volts. The self-induced field from the armature ampere-turns as acting on the main magnetic circuit does not rotate, yet although stationary it pulsates in strength; it may therefore, as already explained in Chap. xxiii. § 12, be replaced in imagination by two rotary fields, each of constant strength equal to half the maximum of the varying field. The

* E. Rosenberg, *Journ. Inst. Electr. Eng.*, vol. xlii. pp. 535 and 547.

one component field when the speed is perfectly steady rotates synchronously with the magnet or the damper, and has therefore no effect on it, but when free oscillations are set up the currents set up in the damper by this component have a damping effect and render the running stable. The other component rotates at the same speed of synchronism, but in the opposite direction, so that its velocity relatively to the damper is $2\omega_1$, and currents are set up in the damper of twice the periodicity of the machine. These currents are nearly opposed to the ampere-turns causing the field to which they are themselves due, so that in the ideal case the inductance of the armature should be halved. In practice so large a reduction is not fully realised owing to the leakage of lines in the relatively large air-gap between the armature coils and the damper, but there is a marked reduction of the value of λ as compared with the same machine without the damper winding. Since the damper bars demand a considerable amount of copper (in many cases more than half of that of the exciting coils) and add to the expense of manufacture, it is an open question whether the same results in the single-phase case could not be more cheaply obtained by increasing the field strength and by adding part only of the extra weight of copper to the magnet-coils.*

If it is not required to reduce the virtual inductance of the armature, the damper winding may be embedded more deeply in the pole-faces, so that only its function of damping oscillations of long period for which it is practically non-inductive remains.

Both in polyphase and single-phase machines, reducing the resistance of the exciting circuit by adding more weight of copper to it would in some cases have the same effect of damping out free oscillations and rendering them stable as are produced by an actual damper winding.

* Cp. Fischer Hinnen, *Elec. World and Eng.*, vol. xxxviii. p. 1058; and *Elec. Eng.*, vol. xx. pp. 597 and 654.

CHAPTER XXVII

DESCRIPTIONS OF TYPICAL DYNAMOS

So varied are the dynamos now manufactured, that no attempt can here be made to describe all of even the leading types. It is only proposed in the present chapter to describe a few typical continuous and alternating-current dynamos in order to familiarise the student with the way in which the principles that have been discussed in preceding chapters are carried out in practice, and these examples are chosen not specially on account of their electrical design and dimensions, but rather as illustrating the mechanical construction and general appearance of complete machines.

§ 1. **Multipolar traction generator of English Electric Manufacturing Company.**—The generator of the English Electric Manufacturing Company Limited, of Preston, is an example of a multipolar continuous-current dynamo with slotted armature for direct coupling to the crank-shaft of the driving engine. Two such machines, each of 1100-kilowatts output, are shown in Fig. 560, as coupled together for a Hopkinson test under full-load on the testing bed. For the purpose of carrying out the test the armature hubs are mounted temporarily on a common shaft by means of wedges driven into large keyways cut in the shaft.

The general description of the machine is as follows:—The rotating armature is carried on a massive cast-iron hub, bored out so that it may be forced directly on to the engine shaft, where it is further secured by two keys. In order to avoid shrinkage strains when cooling, the several arms of the hub, each carrying a segmental portion of the rim, are when cast unconnected except through the nave of the hub (Fig. 561). In the forked end of each arm a pair of holes is cast, so arranged as to drive air through radial ducts in the core when the armature is rotated. The armature core is formed of thin annealed sheet-steel laminations, of segmental shape, each thoroughly varnished on both sides, and carrying on the inner edge at least two wedge-shaped keys fitting into dovetailed grooves on the rim of the hub. During the process of assembling the laminations there are introduced at intervals of about three or four inches special spacing discs which have projecting bosses punched on them, and in which the teeth are twisted

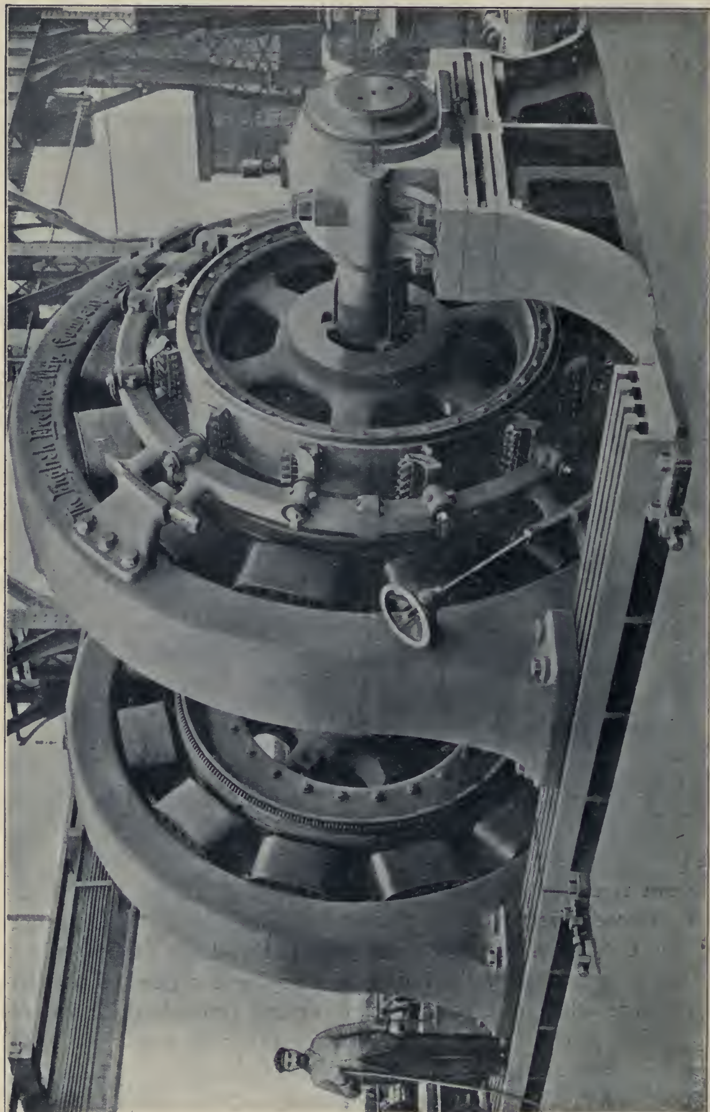


FIG. 560.—Traction generators of English Electric Manufacturing Company Limited, arranged for Hopkinson efficiency test.

at right angles to the flat surface of the plate. By this means the plates are kept about $\frac{1}{2}''$ apart, and an air-duct is formed in the core. The complete core is clamped together between two end-rings by bolts passing through the interior below the discs. A flange on the end-plate at the engine side, perforated to give good ventilation, serves to support the ends of the barrel-wound armature coils, while the end-plate next to the commutator has a number of clamps fastened to facings on its outer surface by which equalising rings are held securely in place. These rings, seen in Fig. 562, are electrically connected at intervals with the commutator sectors, and equalise the voltage and currents of the several parallel paths of the winding (Chap. XI. § 20).

FIG. 561.—Armature hub.

The hub of the commutator is of cast iron, and is keyed to an extension of the armature hub, so that no movement can take place between the two. The hard-drawn copper sectors are insulated from one another by plates of soft amber mica $\frac{1}{32}''$ thick, and are clamped firmly between steel end-rings, from which they are insulated by moulded rings of mica, $\frac{1}{8}''$ thick and projecting beyond the edge of the copper an inch or more. The numerous bolts which pass from end to end through the commutator under the sectors are threaded through mica insulating tubes. The wearing depth of the copper is not less than one inch.

When the commutator is in place on the hub the armature core is ready to receive the coils. These are of copper bent on edge at the centre so as to form two layers, and shaped on a former into a hexagon, so that there are no joints except at the end where they are soldered into the commutator lugs. The latter are of laminated copper riveted and soldered into slots sawn in the commutator sectors. The armature coils are covered with mica, red

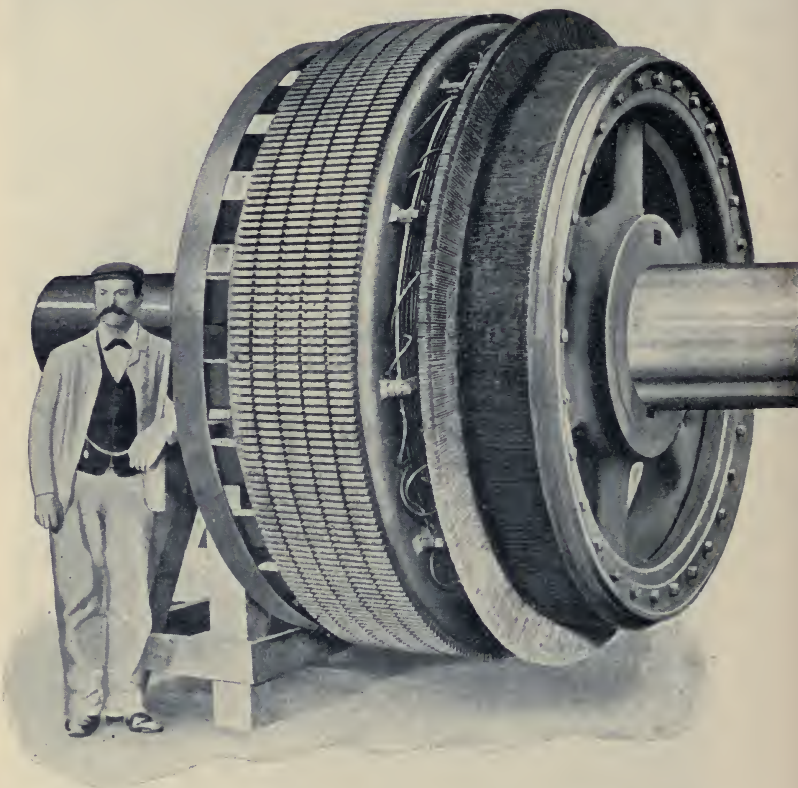


FIG. 562.—Armature core and commutator. (English Electric Manuiacturing Company Limited.)

rope paper, and Japanese paper, and painted with varnish; two, and sometimes three, according to the nature of the design, are taped over with oiled linen, dipped and baked to form a composite coil, which is then placed in the slots on the surface of the core. Bands of binding wire at intervals along the length of the armature complete the winding.

The magnet frame (Fig. 563) consists of a circular yoke-ring of cast iron, divided on a horizontal plane, and bolted together by four bolts let into pockets on the inside of the circle. The radial poles project inwards, and are composed of rectangular sheet-steel laminations, about $\frac{1}{4}''$ thick, cast into the yoke-ring. A number of the laminæ, punched to the correct shape, are assembled and clamped between two thicker steel plates to which they are riveted. The required number of poles thus formed are placed in position in the mould prepared for the casting, and have their outer ends coated with a flux to assist the union of the molten metal and the steel;

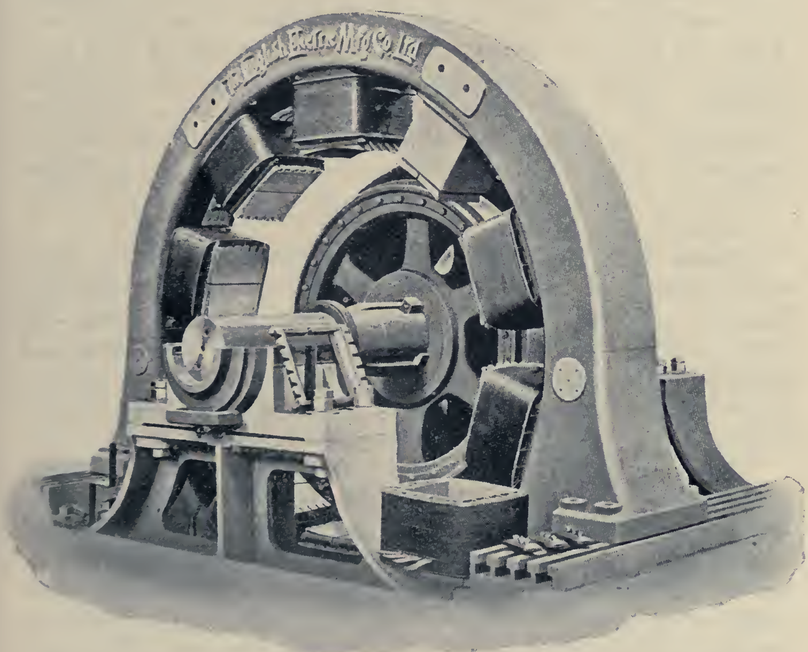


FIG. 563.—Ten-pole magnet of traction generator. (English Electric Manufacturing Company Limited.)

the outer ends are left rough, and are further provided with a longitudinal wedge-shaped groove on either side, to ensure their being held firm in the casting. After the poles have been heated up to prevent chilling of the metal at the first contact, the cast-iron is run into the mould, and the whole is left to cool untouched in the sand, in order to thoroughly anneal it and secure a soft casting easy to machine and of high permeability. When the two half-rings have been bolted together, the pole-faces are turned to the correct bore, and at the same time there is turned in four cast-iron brackets bolted to the face of the yoke, the groove which is subsequently to receive the rocker ring carrying the brushes. The inner end of each pole has a dovetailed groove on either side parallel to the axis of the armature, and on this is clamped a detachable cast-iron frame * in two halves, which forms a polar extension and at the same time holds the magnet coil in place. The magnet coils are wound on a rectangular sheet-steel shell, just large enough to slip easily over the pole, light open-work brass flanges pierced with holes being riveted to each end of the shell. The spool is insulated with mica and paper, and the shunt coil, which occupies about two-thirds of the winding space, is kept apart from the series coil which occupies the remaining third by $\frac{1}{4}''$ of insulation. Double-cotton-covered wire is used for the shunt, while the series coil is composed of a few turns of wide copper ribbon wound on the flat. As a further mechanical protection the centre is covered with braided rope, filled with waterproofing, and thoroughly dried by baking; it is then tested for insulation by an alternating difference of potential of 2500 volts. When placed on the poles the several sets of shunt and series coils are respectively connected in series.

On the lower half of the magnet frame are cast two feet, which when planed rest on a pair of horizontal foundation girders and carry the whole magnet. Adjustment of the clearance between the poles and the armature is obtained horizontally by screws, and vertically by thin packing-pieces inserted under the feet. In order to expose the armature or to withdraw a magnet coil, the whole magnet frame can be moved lengthwise along the girders.

The brush rocker is a cast-iron ring in halves bolted together, of channel section, with projections fitting into the groove on the four brackets on the magnet frame, and provided with a screw and hand wheel for moving it concentrically with the commutator to the best position for sparkless commutation of the current. Concealed in the inside of the channel are two cross-connecting rings, tapped at intervals with leads to the brushes of the same sign. The frame carrying each set of brush-holders terminates in a spindle passing through an insulated hole in the rocker ring, so that the frame can be turned round through 180° to obtain the right set of the brushes for either direction of rotation. The carbon brushes are in holders attached to laminated copper strips, and as many sets of brushes are employed as there are poles, the armature winding being simplex lap.

The output of the machines shown in Fig. 560 is 2000 amperes, 550 volts at 100 revs. per min., the field being over-compounded so that the voltage rises from 500 volts at no-load to 550 at full-load. After a prolonged run at full-load the highest rise of temperature of the armature was 47° F., and of the field was 58° F. The total weight

* One such is seen in Fig. 563 resting against the plummer-block.

of the machine giving 11,000 watts per rev. per min. was 56 tons, divided into 21 tons for the armature and 35 tons for the magnet. The efficiency was 95 per cent., and by calculation from the tests the following approximate division of the losses in watts was made :—

	¼-Load.	½-Load.	¾-Load.	Full-Load.
Hysteresis and eddy-currents	15,000	16,680	18,370	20,000
Armature resistance and brushes	1,310	5,250	11,800	21,000
Shunt coils	8,000	8,820	9,530	10,000
Series coils	310	1,250	2,800	5,000
Total losses apart from friction	24,600	32,000	42,500	56,000
Output	257,000	526,000	800,000	1,100,000
Input	281,600	558,000	842,500	1,156,000
Efficiency percentage	87·5	94·5	95	95

§ 2. **2700-KW. generator of General Electric Company of New York.**—One of the largest continuous-current generators for traction work is a 2700-kilowatt machine built by the General Electric Company of New York, and installed at Lincoln Wharf, Boston Elevated Railway. For the details of this fine machine, and for the accompanying photographs which illustrate it when completed and at various stages in the process of erection, we are indebted to the courtesy of the British Thomson-Houston Company.

The machine is a 36-pole generator with slotted armature, and has an output at 75 revs. per min. of 575 volts 4700 amperes, or 36,000 watts per rev. per min. The diameter of the armature is 21′ 8″, and of the outside of the magnet frame is 31′ 0″. The length of the armature core is 1′ 4″. The weight of the armature complete is 71½ tons, or nearly half the total weight of the entire machine, which is 138 tons. Fig. 564, from a photograph taken while the generator is running, shows the completed machine mounted on the crank-shaft between the standards of its vertical engine.

The magnet frame (Fig. 565) is of cast iron of deep section with lightening holes, and is divided horizontally so that the upper half can be lifted for inspection or repair to either armature or magnet coils. On account of the large size of the frame, the lower and upper halves were each subdivided into three sections in order to facilitate transportation, and the junctions of the sections are registered by feathers sunk into the adjacent surfaces of contact. Adjusting screws are provided for the purpose of centering the field with the armature. The magnet-cores, 36 in number, are of cast steel with laminated pole-pieces securely fastened to them by screws; they are bolted to the yoke in such wise that any pole-piece with its winding can be readily withdrawn after removing the fastening bolts. The laminations of the pole-faces are of two lengths, arranged alternately, so as to produce the graduated field effect given by a chamfered pole-edge. The field coils are wound on spools, having a double sheet-iron body and malleable iron flanges; the

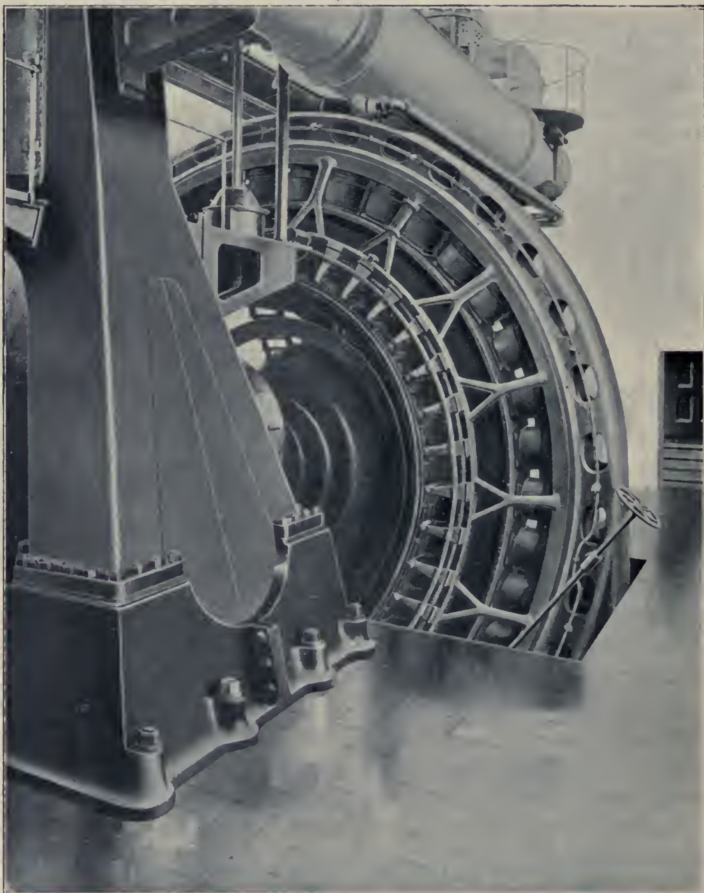


FIG. 564.—2700-KW. traction generator at Lincoln Wharf, Boston Elevated Railway. (General Electric Company of New York.)

shunt and series windings are kept separate from one another in each bobbin.

The armature spider (Fig. 566) is of cast iron, with arms divided at the periphery so as to avoid shrinkage strains; its two halves are bolted together and the whole keyed to the engine crank-shaft. The iron core is built up of laminations japanned prior to being assembled; these are secured to the spider by dovetail projections, and are held in place by end-flanges which also serve to support the end-windings (Figs. 567 and 568). Spacing bolts are inserted at equal intervals to provide five ventilating ducts between the laminations. The armature spider is so

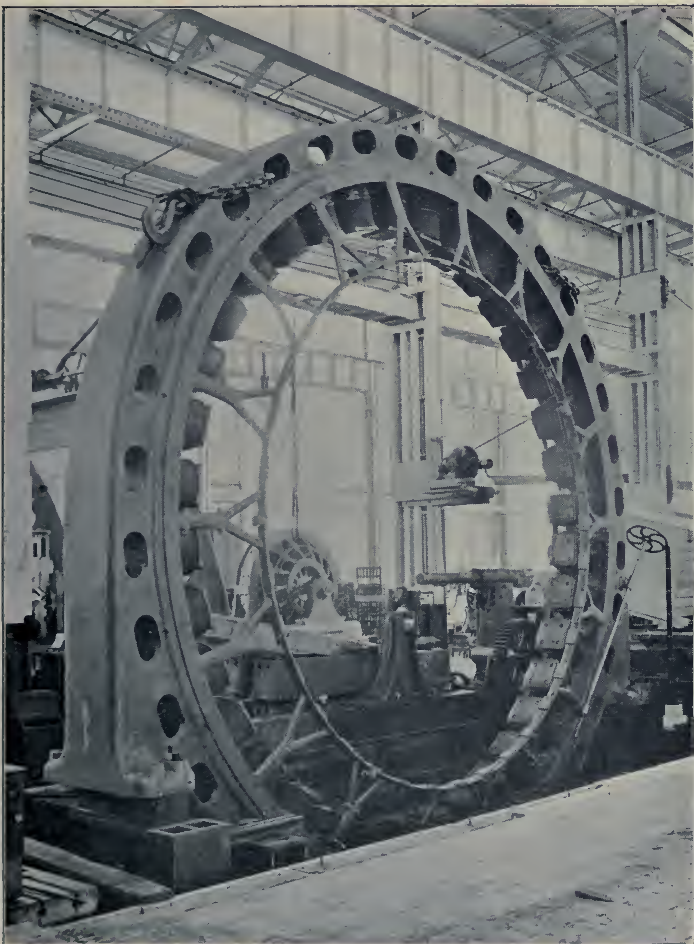


FIG. 565.—Field frame of 2700-KW. generator. (General Electric Company of New York.)

arranged that it produces a fan effect, forcing air through the ventilating spaces. The barrel-winding is that of a simplex lap-wound drum, and has equaliser rings connected to points of equal potential. These rings are secured by insulating pieces attached to the arms of the spider at the back of the armature similarly to the armature shown in

Fig. 229. The armature coils are made in halves, the back connection being made by soldering two ends together, and metal bands bind the conductors firmly to the end-flanges.

The commutator is 15′ 7″ in diameter, with sectors of hard-drawn copper made with a dovetail on the lower edge to hold them in place.

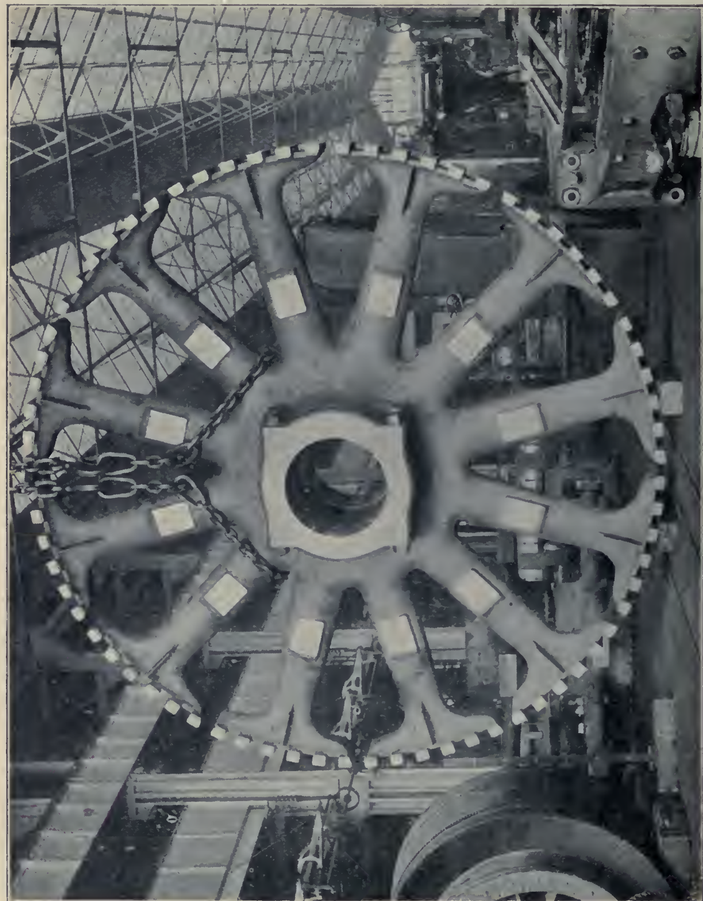


FIG. 566.—Armature spider casting for 2700-KW. generator. (General Electric Company of New York.)

The mica between the sectors is of a soft quality, and as thin as the requirements for insulation will permit, so as to ensure even wear of mica and copper. The whole is carried on a cylinder with internal arms bolted to the spokes of the armature hub (Fig. 569).

The large circular frame carrying the 36 sets of brushes is moved in a groove on the magnet frame by means of a hand-wheel at one side of the yoke-ring (Figs. 565 and 570). The carbon brushes are inclined slightly forward of a radial position, and are each furnished with a flexible copper conductor connecting it to the brush-holder.

The insulation between the field coils and magnet frame, and also between armature winding and armature core, is tested with an alternating difference of potential of 4000 volts for 10 seconds, or with

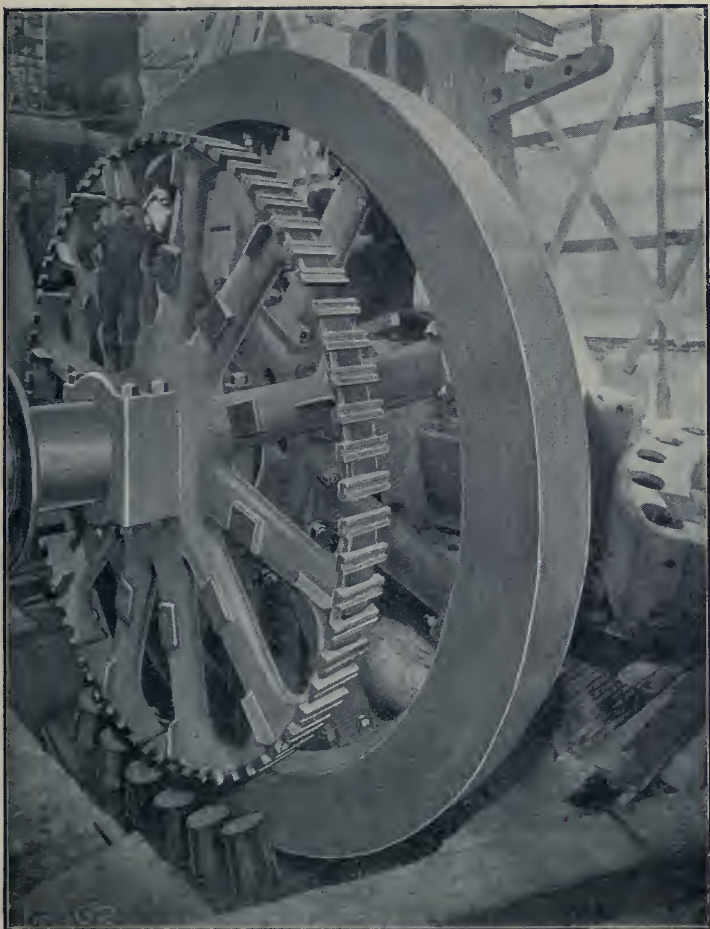


FIG. 567.—Armature spider of 2700-KW. generator during erection. (General Electric Company of New York.)

2000 volts for 60 seconds. The rise of temperature of the generator is guaranteed not to be more than 63° F. after a run of 24 hours at full-load, and an overload of 50 per cent. for two hours following a run of 24 hours at full-load will not cause the rise to exceed 99° F. The generator is guaranteed to carry an overload of 50 per cent. at the rated voltage for two hours, and of 100 per cent. momentarily without movement of the brushes and without injurious sparking. The compounding is arranged to give 525 volts at no-load and 575 volts at full-load. The efficiency rises from 94 per cent. at half-load to as much as $95\frac{1}{2}$ per cent. at full-load.

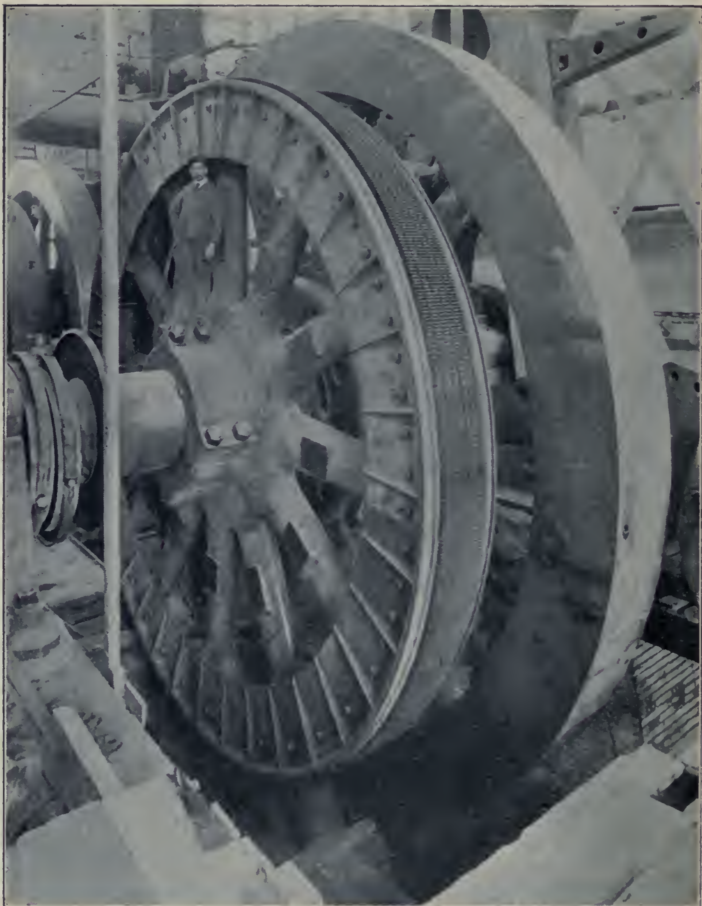


FIG. 568.—Armature core of 2700-KW. generator during erection. (General Electric Company of New York.)

Later machines by the same makers for the same output and overloads, installed in the Charlestown and Harvard stations of the Boston Elevated Railway Co., are furnished with commutating poles, and although giving the overloads above-mentioned with a fixed brush position, are nearly $4\frac{1}{2}$ ft. smaller in diameter.

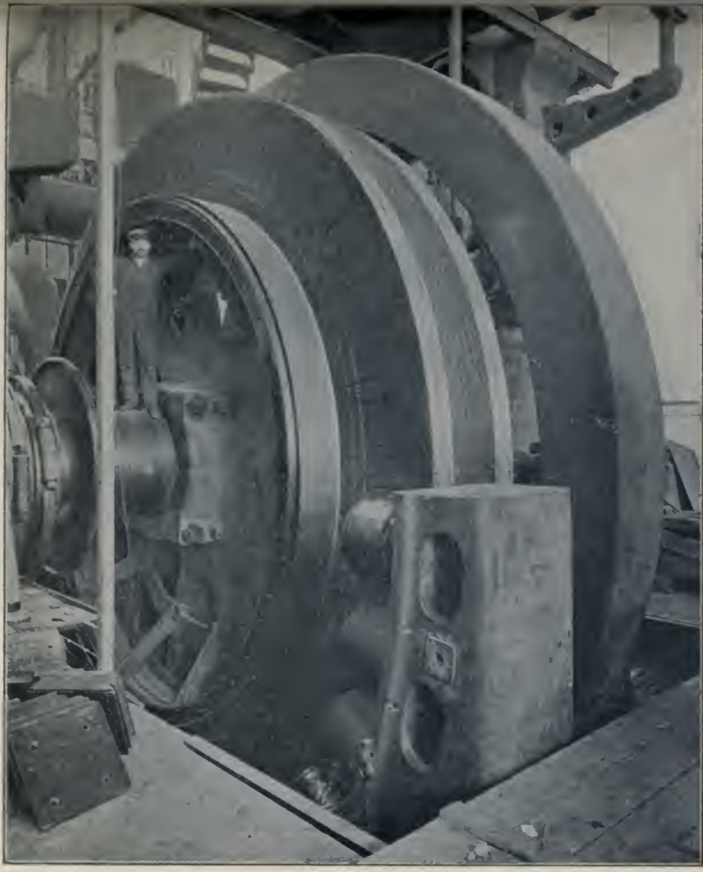


FIG. 569.—Complete armature of 2700-KW. generator during erection. (General Electric Company of New York.)

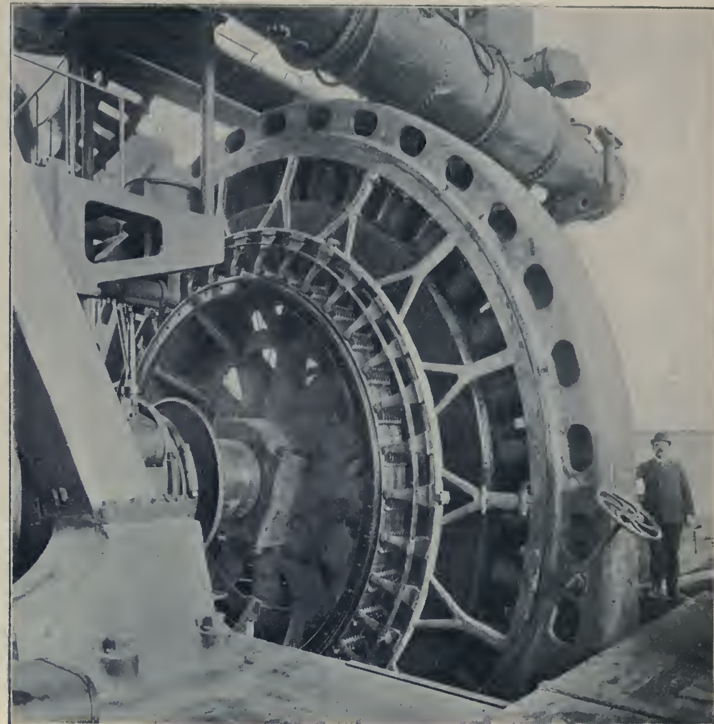


FIG. 570.—2700-KW. generator erected. (General Electric Company of New York.)

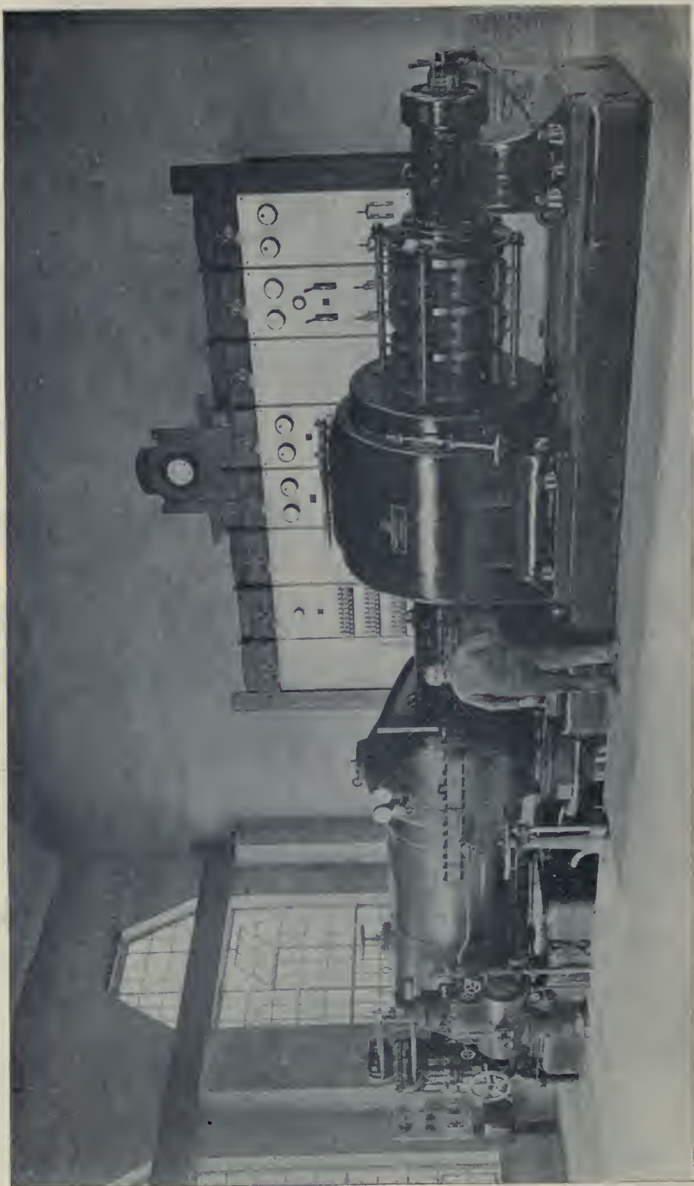


FIG. 571.—1100-KW. turbo-dynamo of Brown Boveri & Co.

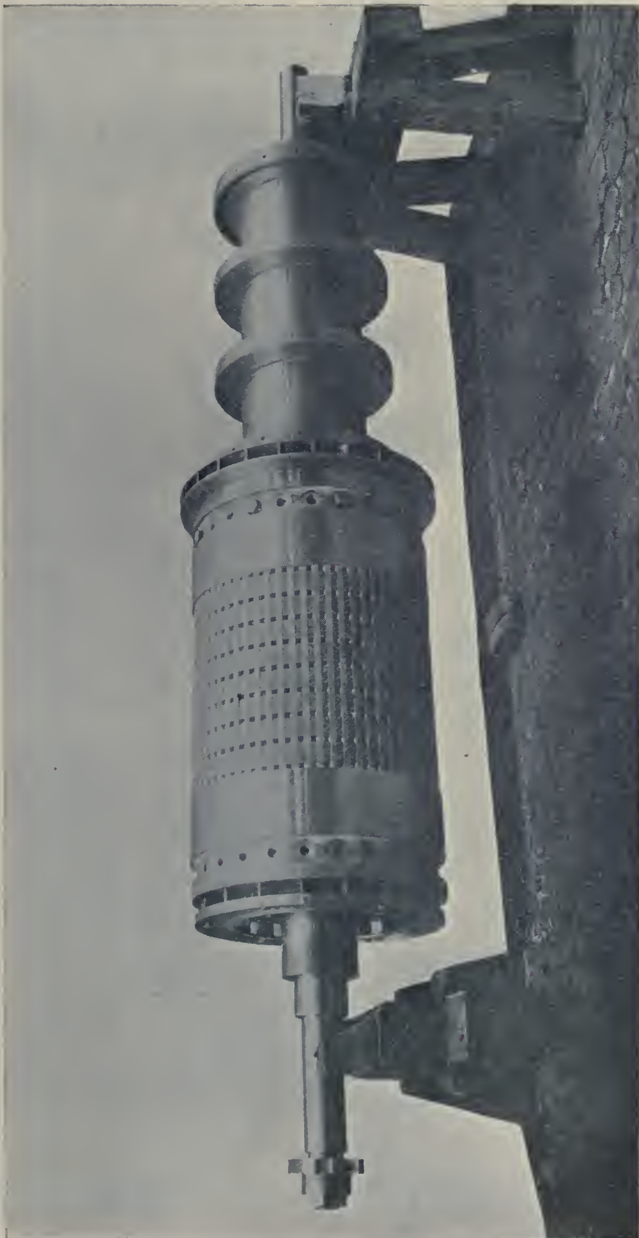


Fig. 572.—Armature of turbo-dynamo. (Brown Boveri & Co.)

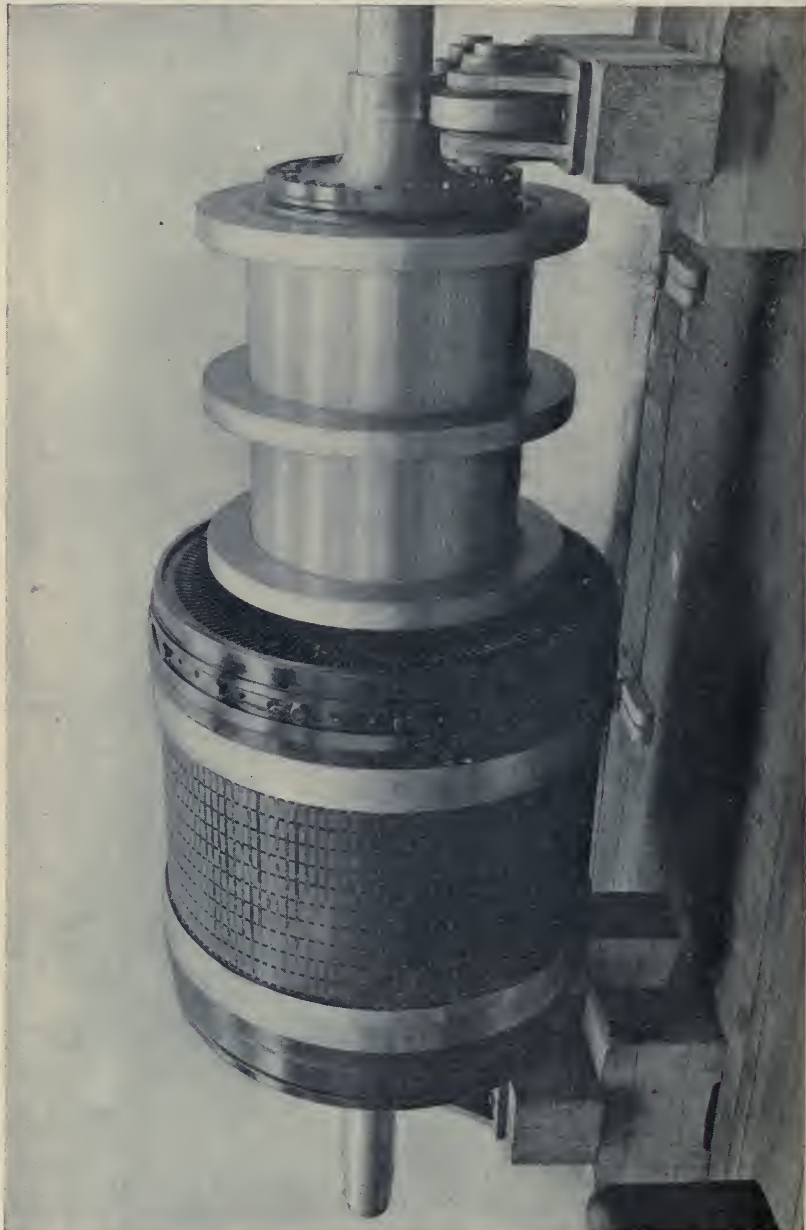


FIG. 573.—Armature of 1500-KW. 1000 revs. per min., 600-volt turbo-dynamo. (Brown Boveri & Co.)

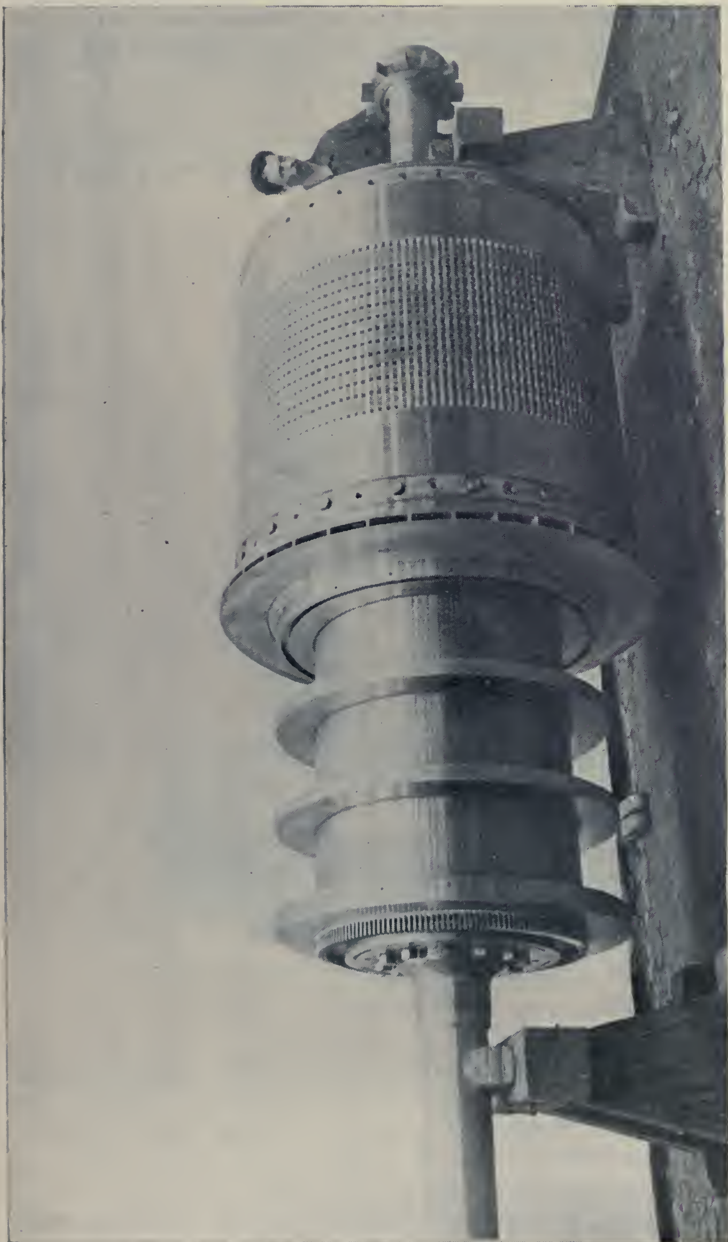


FIG. 574.—Armature of 1500-KW. turbo-dynamo. (Brown Boveri & Co.)

§ 3. **Turbo-dynamos of Brown Boveri & Co.**—In the construction of continuous-current dynamos for direct coupling to steam turbines, Messrs. Brown Boveri & Co. have adopted many of the same features as in their turbo-alternators. The external casing which is now the field-magnet is entirely enclosed, and a similar system of assisted ventilation is employed, by which the air is drawn in at the bottom at the end remote from the commutator and discharged at the top through one or two chimney openings. The field-magnet core is built up of

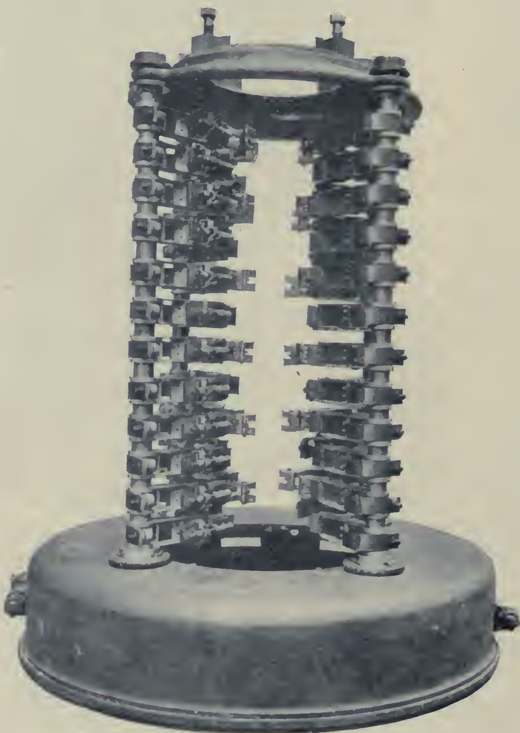


FIG. 575.—Brush-gear of Brown Boveri & Co.'s turbo-dynamo.

discs slotted on the inner periphery to receive the exciting coils and also a compensating winding on the Deri principle, so that when finished it has a nearly smooth interior and is not unlike the stator of an induction motor. Further, a direct-coupled exciter is employed outside the commutator bearing, as seen in Fig. 571, which shows a machine of 1100 kw. 550 volts, running at 1250 revs. per min. The use of separate excitation is to ensure the correct direction of the actual field when working in parallel with other machines, especially in the case when the compensating turns are numerous and are arranged to produce an over-compounding effect; under such circumstances there is a possibility of the main field becoming reversed and of the machine attaining a dangerously high speed, or of a periodic variation in the power being set up,—tendencies which are held in check by the separate exciter.

The general construction of the revolving armature is seen in Figs. 572–574; the latter is the armature of a 1500-kw. 1000 revs. per min.

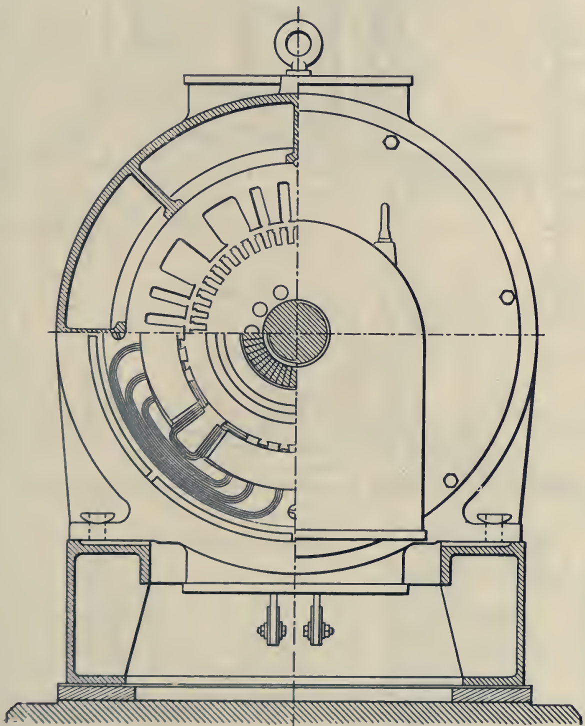


FIG. 576A.—250-KW. turbo-dynamo of Brown, Boveri & Co. End elevation.

550–650 volts dynamo at the Rotterdam Electricity Works, and shows well the ventilating fan at the commutator end. Screws are inserted in the end-shields of the winding to balance the armature, and a groove is cut in the massive shrink rings of the commutator, into which small plates are screwed for the purpose of separately balancing this portion of the rotor, as is better seen in the smaller armature of Fig. 572. The brushes are carried on rods between an end-plate which abuts against the field frame and a ring resting on the outer plummer block, and are

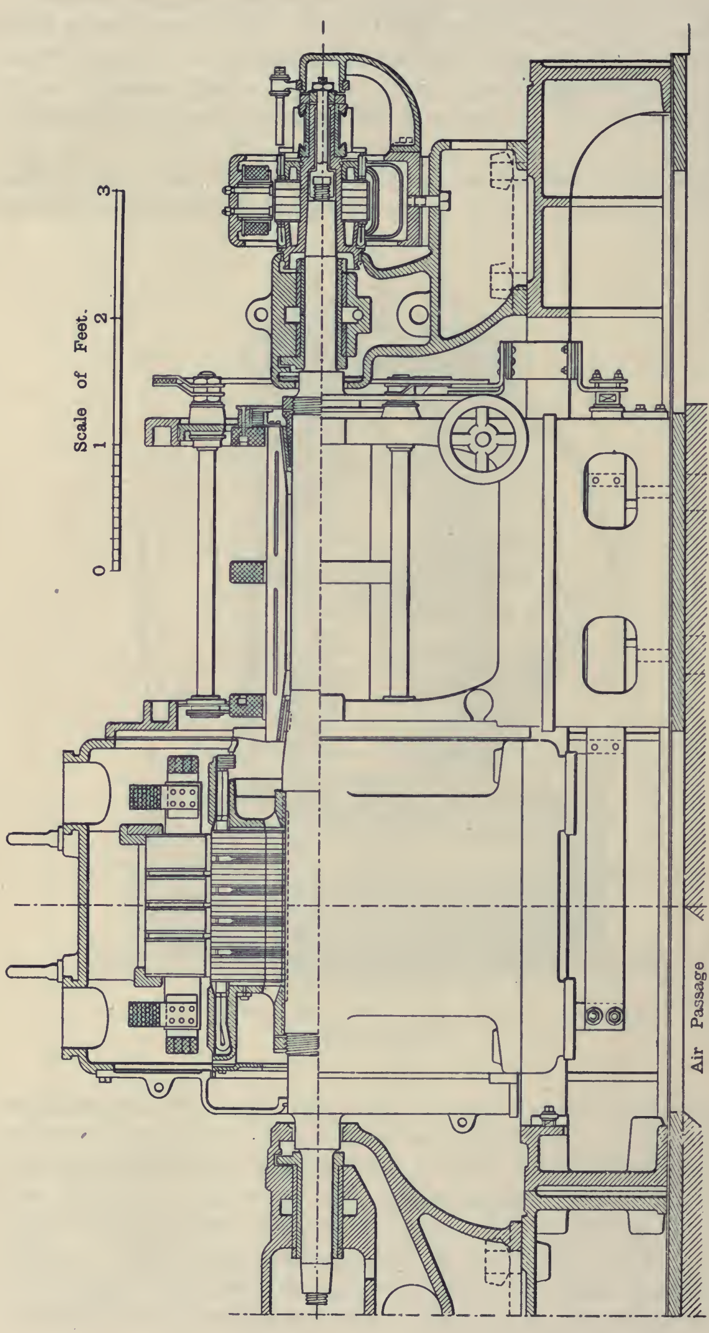


FIG. 576B.—250-KW. turbo-dynamo of Brown, Boveri & Co. Longitudinal section.

of a special type with copper and carbon tips alternating along the length of the commutator (Fig. 575).

Fig. 576 shows in section a turbo-dynamo built by Messrs. Brown Boveri & Co., to give 250 kw. 150 volts 1660 amperes at 2700 revs. per min., or 92·5 watts per rev. per min. The dimensions of the armature core are 20·43″ diameter × 14·55″ length, so that the peripheral speed is 14,500 ft. per min. The specific dimensional output is 0·0152 watts per rev. per min. for each cubic inch of D^2L,—a low figure such as is usually found in the case of turbo-dynamos with Deri compensating field-winding wherein a small value is employed for B_g. The commutator is $10\frac{1}{4}$″ diameter, giving a peripheral speed of 7250 ft. per min., and has an effective length of 21″. The bearings are 3·15″ in diameter, with a peripheral speed of 2220 ft. per min. The single air-gap is 1 cm. = 0·394″. The gross length of the magnet core is $13\frac{3}{4}$″; the discs of which it is composed have a radial depth of 5·7″, and are housed within a cast-iron frame having a large annular space round them, with two chimney openings at the top, one at

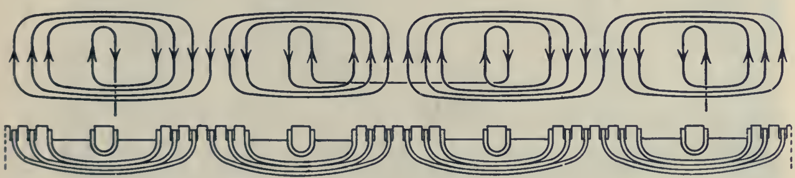


FIG. 577.—Diagram of Brown-Boveri Deri compensating field-winding for turbo-dynamos.

each end. There are four poles at opposite ends of a vertical and horizontal diameter, embraced by the four main exciting coils, between which in each interpolar gap lies a broad commutating tooth encircled by series ampere-turns. Each pole-face has four half-closed slots to receive the Deri compensating turns of copper strip placed edgewise in the slots and locked therein by wedges. The principle of the winding is shown diagrammatically in Fig. 577. The main exciting coils are secured on either side of the commutating tooth by metal wedges.

The large 10,000 h.p. 1000 revs. per min. steam turbine, built by Messrs. Brown Boveri & Co., for Elberfield, is shown in Fig. 578, which illustrates the outward similarity of the alternating and continuous-current generator designs. Next to the turbine is seen the 3-phase 6250 KVA 5000 volt alternator, and on the extreme right the field frame of the 1500-KW 600-volt continuous-current machine with the same arrangement of commutating teeth and slots for compensating winding as described above.

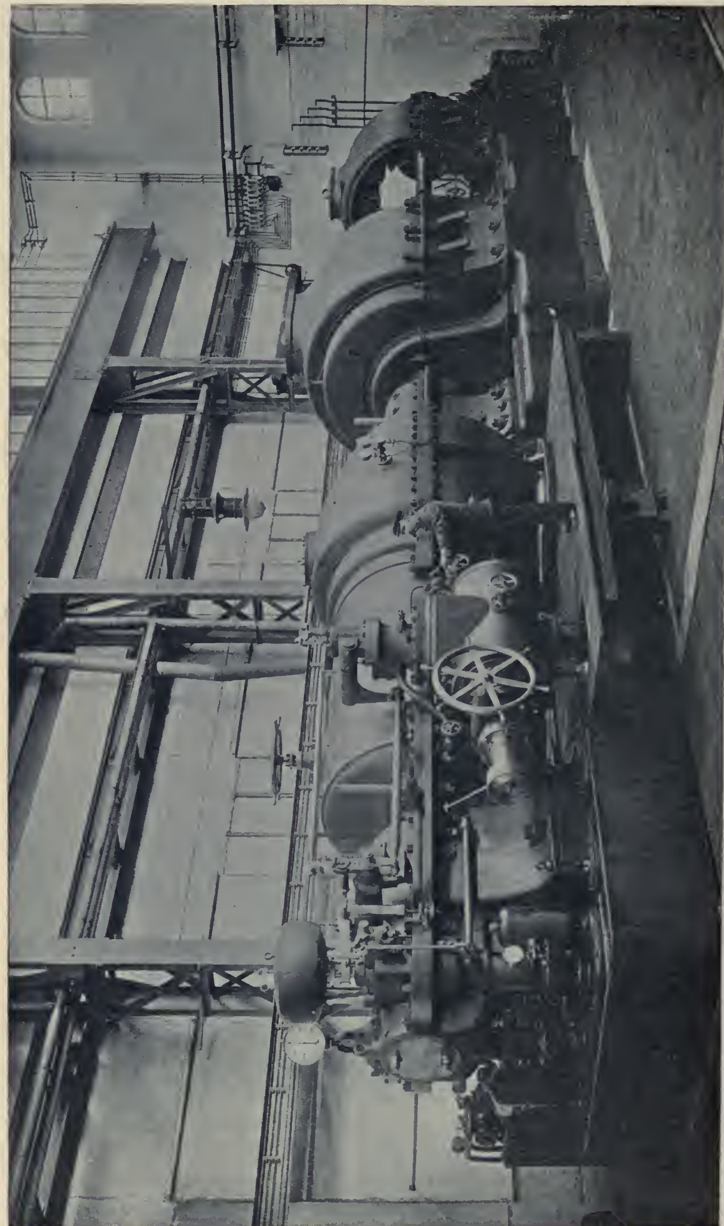


FIG. 578.—10,000-h.p. steam-turbine, with 6250-KVA alternator and 1500-KW. generator.

§ 4. **7500-KW. 3-phase alternator of the General Electric Company of Schenectady, N.Y.**—An example of an alternator with vertical shaft for direct coupling to a water-turbine is found in the large 3-phase 7500-KW. machines constructed by the General Electric

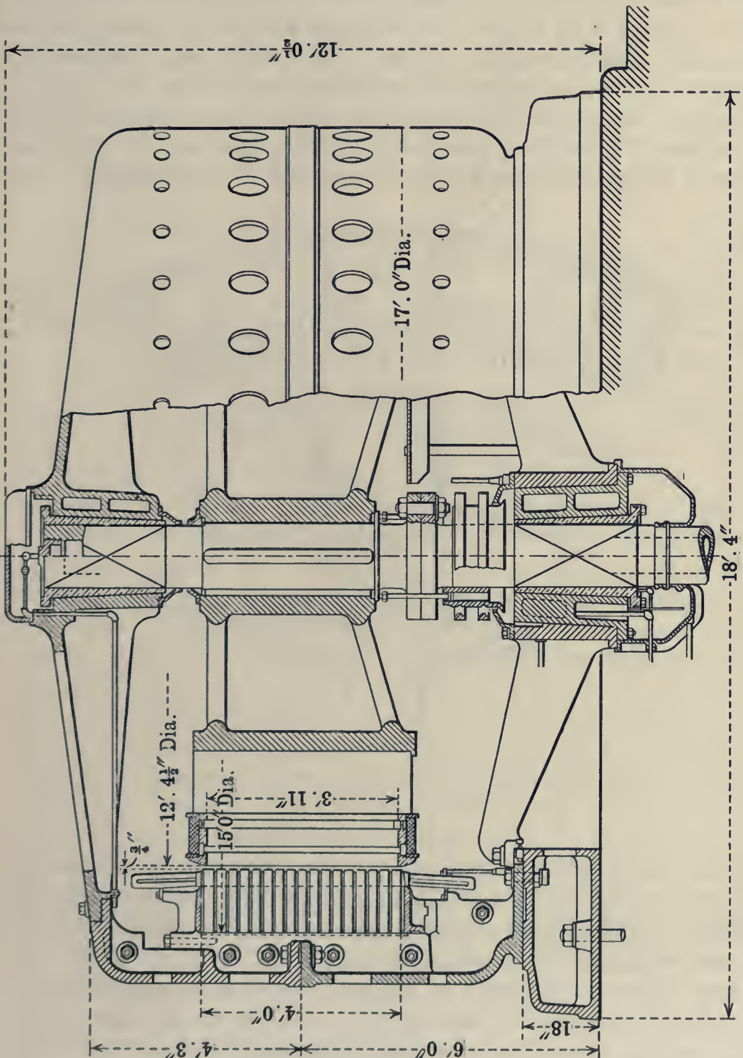


Fig. 579.—General Electric Company's 7500-KW. alternator. Sdie section.

Company of New York for the Canadian Niagara Power Company's power-house in Queen Victoria Park, about half a mile above the Horseshoe Falls on the Canadian side. The output of the machine (Figs. 579 and 580) is 11,000 to 12,000 volts interlinked pressure and 360 amperes per phase; with its 10,000 horse-power turbine it constitutes one of the largest single units which have yet been constructed. Its speed is 250 revs. per min., which with 12 poles gives a periodicity of 25 cycles. The diameter of the pole-faces is 12 ft. $4\frac{1}{2}$ in., and the peripheral speed of the rotor is thus 9700 ft. per min. The gross length of the armature core is as much as 4 ft., and the watts per rev. per min. being 30,000, its specific output per cubic inch of D^2L is 0·0284; it is therefore very compact, and the outside diameter of its casing, being 17′ 1″, is but little larger than that (14 ft. 10 in.) of the earlier 2-phase machines of half the output, or 3750 KW., installed on the American side in the power-house of the parent company, the Niagara Falls Power Company. The

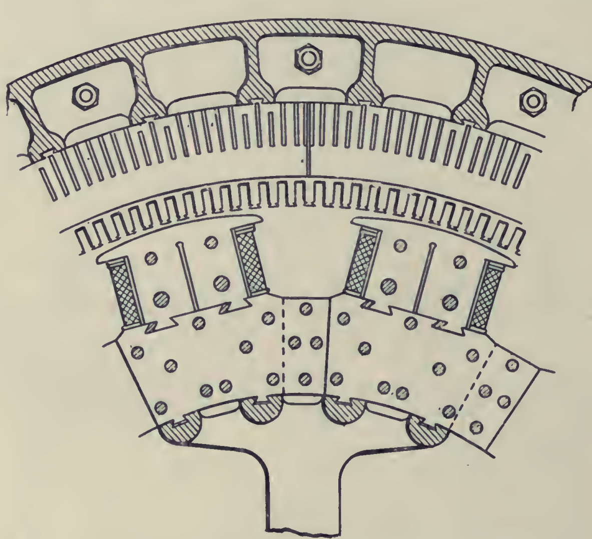


Fig. 580.—General Electric Co.'s 7500-KW. alternator. Section viewed from above.

adoption of the larger unit of 10,000 as against 5000 horse-power has therefore resulted in a considerable reduction in the length of wheel-pit and power-house for a given horse-power, as well as in the first cost of the generators and turbines.

The stationary armature core is built up of sheet-steel segments, 0·014″ thick, dovetailed to the cast-iron casing, which is made in two halves, an upper and a lower portion, pierced with holes and with ample ventilation space at the back of the discs. The external and internal diameters of the discs being 15 ft. and 12 ft. 6 in. respectively, their radial depth is 15 in. or $11\frac{3}{8}''$ at the back of the slots, which are $3\frac{5}{8}''$ deep. Sixteen air-ducts, $\frac{1}{2}''$ wide, together with interspersed laminae of paper, reduce the effective width of iron core to about 36 in. The end-plates which serve to compress the core are separate castings. There are 5 slots per pole and per phase, or 180 in all, each $1\frac{3}{8}''$ wide, with parallel sides and closed at the top by wedges engaging in notches. The tooth tapers nearly from $1\frac{3}{8}''$ to $1\frac{1}{4}''$. There are 4 conductors per slot, or 240 per phase, each conductor measuring bare $1'' \times 0{\cdot}3''$, connected as a wave winding in two layers. While the lower conductors are solid, the upper layer is of stranded cable compressed to the same rectangular dimensions, in order to reduce the loss by eddy-currents due to the alternating field which crosses the top of the slot. The pole-pitch is 39 in., and, owing to the wide span of the end-connections of the coils, they are anchored to rings on the end-plates.

The shaft diameter at the centre of the rotor is 16 in., and upon it is pressed a cast-iron spider with hub $27''$ in diameter and six arms; an oil-way down the inner surface of the hub permits of the drainage of waste oil from the upper bearing through a pipe to the lower bearing, the hub itself abutting upon a lead joint $\frac{1}{16}$ in. thick. Upon the arms of the hub are dovetailed steel laminations $\frac{1}{8}$ in. thick and 14 in. radial depth, with a number of rivets passing through the 53 in. width of the plates to hold them together; a yoke-ring is thus formed of high mechanical strength and magnetic permeability. Upon the ring are dovetailed the poles of similar laminations, with overhanging pole-tips to retain the exciting coils, the length of the pole-cores parallel to the shaft being 47 in., their width 16 in., and their length in a radial direction where encircled by the bobbin 10 in. Stout end-plates with bolts running between them compress each pole-core, the outer pair of bolts being insulated from the core. The air-gap varies from $\frac{3}{4}$ in. at the centre of the pole to nearly double this amount at the edges, the pole-face being shaped to a radius so as to give as nearly as possible a sine wave of E.M.F. The ratio of polar arc to pole-pitch is approximately 0·666. The weights of the poles and yoke are about 18 and 31 tons respectively, and the entire weight of the rotor is 63 tons. The armature and frame weighs about 80 tons.

The ampere-wires per inch of circumference of the stator are 550, and an approximate calculation gives $Z_a = 53 \times 10^6$ C.G.S. lines, average $B_g = 6130$, $B_a = 9800$, and B_t from 15,200 to 16,700. With allowance for leakage, the densities in pole-core and yoke are approximately $B_m = 12{,}600$ and $B_y = 6500$. The voltage of the exciters, which are also turbine-driven, is 125 volts.

When the finished winding of one phase is followed out, a number of tours of the armature is made, until half the bars of the phase have been traversed, and these fall in twelve groups, alternately in the upper and lower layers; the winding then turns back upon itself, and the same number of tours of the armature is made in the opposite direction, filling up the slots, with groups of conductors alternately in the lower and upper layers. The beginnings of the phases and their leads to the common junction are roughly at 120° apart.*

§ 5. **Three-phase generators of Allgemeine Elektricitäts Gesellschaft.**—One of the largest alternators exhibited at Paris in 1900 was the 3000 kilovolt-ampere 3-phase machine of the Allgemeine Elektricitäts Gesellschaft of Berlin. A large number of these

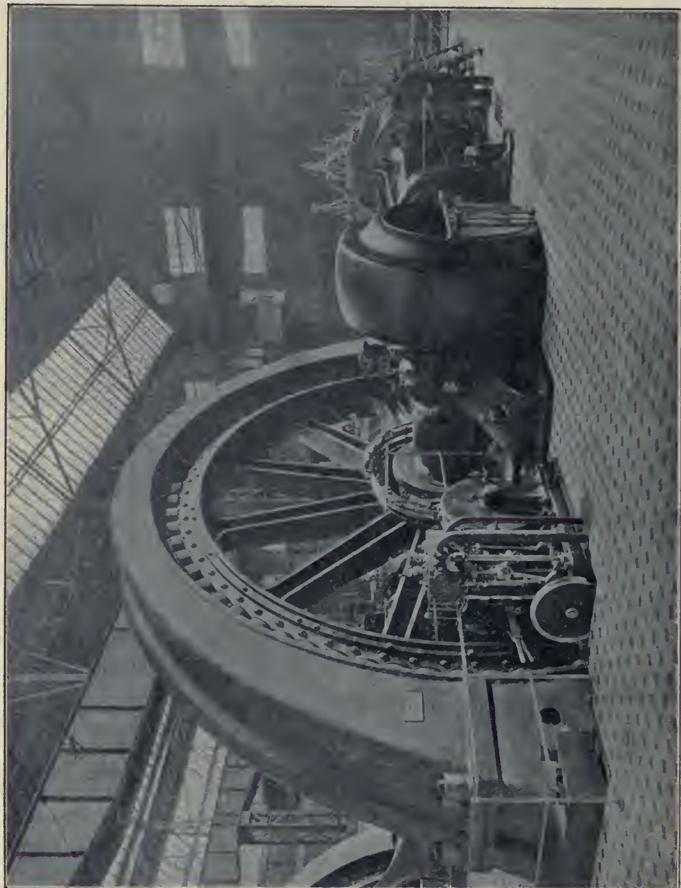


FIG. 581.—3000-KVA. 3-phase generator of Allgemeine Elektricitäts Gesellschaft.

alternators are now at work in the Oberspree and Moabit stations of the Berlin Electricity Works, one such being shown in Fig. 586. The

* An interesting comparison may be made with the 6500/7300-kw. alternator 3-phase 25-cycle 300 revs. per min., coupled to a horizontal water-turbine of 11,000 h.p. at the Niagara Falls Hydraulic Power and Manufacturing Company's Works, of which a full description has been given by Mr. B. A. Behrend, *Trans. Amer. I.E.E.*, July 1908.

machine illustrated is directly coupled to a 4000-B.H.P. horizontal steam engine, running at 83·3 revs. per min., and with its 72 poles gives a frequency of 50. The winding of the stationary armature is star-connected, each phase giving 3460 volts, so that the interlinked pressure between any pair of terminals is $3460 \times \sqrt{3} = 6000$ volts, while the current per phase is 290 amperes. It is designed for a power-factor $\cos \phi_e = 0·9$, so that its normal output is 2700 kilowatts, or 32,500 watts per rev. per min., while in addition the overload capacity is considerable. Although the pressure is high, yet the machine is of so large a size that massive copper bars are employed for the winding; it is further noteworthy for its large number of slots per pole, namely, 15, or 5 per pole and per phase, and also from the employment of the Hutin and Leblanc damper winding on the poles. The external diameter of the outer armature casing is 28 ft., and its over-all width nearly 4 ft. The complete lower half has already been shown in Fig. 496. On the inside of the outer casting is cast a ring which forms the foundation on which the segmental laminations are built up by being threaded over a number of bolts. In the centre of the casing is cast a second ring or rib, and when half the axial width of the armature core has been built up the laminations are tightly compressed by means of steel segments screwed to the central internal rib. When the second half of the core has been threaded over the bolts, the whole is again tightly compressed by means of segmental end-plates screwed to the inside of the casing. The central segments are grooved to allow of air being forced through behind the discs, and each half of the core is divided into four packets by metal distance-pieces placed round the circle at frequent intervals, to ensure good ventilation, while the outer surface of the casing is pierced with numerous holes to allow of the air escaping. The packets are arranged so as to break joint, and when the two halves of the armature are finally combined, cross one another and are locked together. The whole rests on three pairs of feet, one pair a little below the horizontal diameter, the second and most important about half-way round the lower quadrants, the two being further reinforced by a pair of smaller adjustable feet at the bottom. Two tension rods, one on either side of the machine, screwed into the upper feet, can be tightened up so as to preserve the roundness of the frame on the horizontal diameter. The gross width of the armature core is 22·8″, and the diameter of the bore is 24 ft. 4 in. nearly.

The slots in the armature, 1080 in all, are open, of rectangular shape, $1\frac{1}{2}$ in. deep $\times \frac{7}{16}$ in. wide, and semicircular at the bottom (Fig. 582).* The total radial depth of the armature laminations, including the slots, is 10 in. In each slot is a micanite tube 0·138 in. thick, with a rectangular copper bar threaded through it; the dimensions of the bar are 4×25 mm.^{2}, with rounded corners, giving an area of 0·144 sq. in., the

* Reproduced by permission from *Engineering*, 12th October 1900.

whole being held in place by a wooden wedge dovetailed into grooves at the sides of the slot. The bars are united by forked or V-shaped end-connectors held well apart by the bars and nearly equal to them in sectional area. Each coil per pole consists of two lap-wound turns and one bar or half-turn, by which the passage is made to the opposite side of the armature; the second coil then takes up the winding, and after describing two complete turns brings the winding back to the same side as the starting-point by means of the single bar or half-turn, so that there are ten active bars per pair of poles. Since the E.M.F. induced

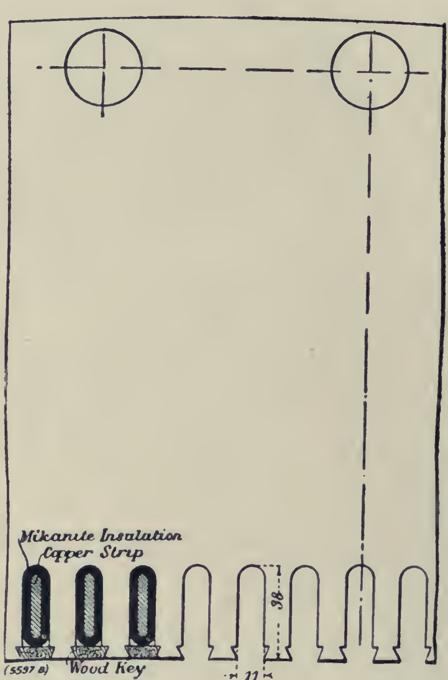


FIG. 582.—Armature lamination of 3000-KVA. generator.

per bar is about ten volts, the difference between adjacent end-connectors is only some twenty volts, so that they are only lightly insulated with varnish. The resistance of each phase when hot is 0·098 ohm, whence the loss of E.M.F. is $28·5 \times 1·73 = 49·5$ volts of interlinked pressure, and the copper loss on full-load is about 25 kilowatts or 0·93 per cent. of the net output. The total weight of the armature is about 90 tons.

The flywheel is cast in four quarters, and united by bolts and by two rings shrunk on the nave which is keyed to the shaft by two keys at right angles (cp. Fig. 583).* Each quadrant is carried by three arms, and the junction of two quadrants at the rim is effected by two bolts, and in addition by the introduction of a circular core held by cotters at each end. The yoke of the magnetic circuit is formed of segmental laminations, each covering the arc corresponding to three poles, traversed by bolts and compressed between a ring cast in one

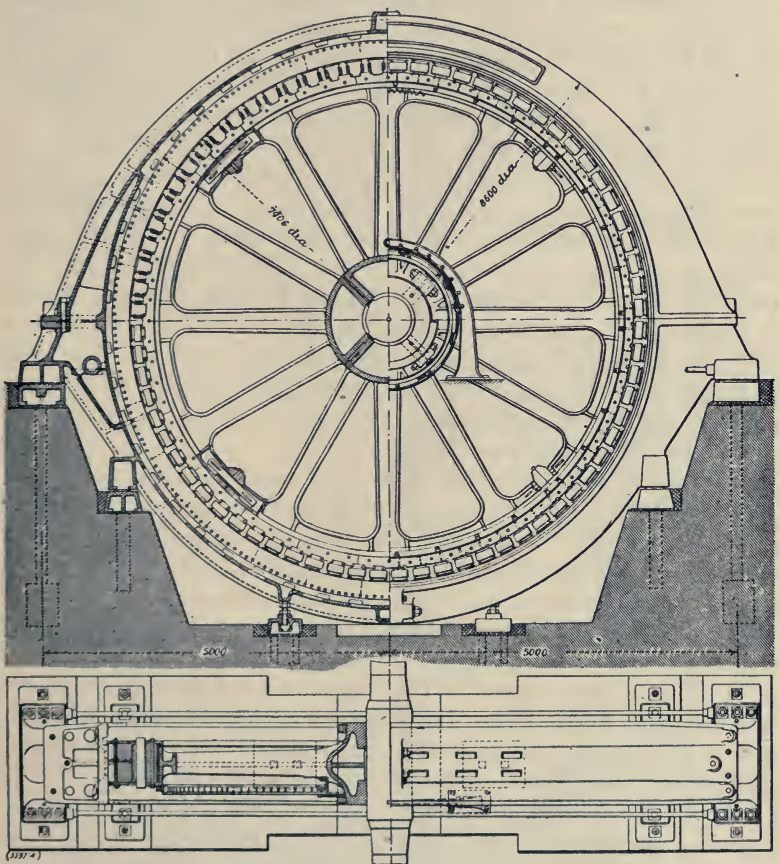


FIG. 583.—3000-KVA. 3-phase generator of Allgemeine Elektricitäts Gesellschaft.

with the fly-wheel rim and segments engaging in a turned recess on the other side of the rim. The magnet cores are also laminated, and are dovetailed into the yoke-ring; each core is fastened by two wedges, which are driven in from opposite ends and themselves locked by a rod passing through a half-round hole in the adjacent sides of the pair. The diameter over the pole-faces is nearly 291″ (the single air-gap being

* Reproduced by permission from *Engineering*, 12th October 1900.

only 1 cm. = 0·394″), giving a peripheral speed of 6340 ft. per min. The axial length of the pole-faces is 22″ and their width 9·45″, or nearly 75 per cent. of the pole-pitch. Each pole 22″ × 5·3″ in section is wound with a rectangular cable coiled on the flat in 10 layers of 9 turns each, and the poles are grouped in two parallels. The excitation current is 130 amperes at 220 volts, and the loss at full-load amounts to about 29,000 watts, or 1·07 per cent. of the net output. The damper winding at the tips of the poles is formed of six rectangular bars and two round bars at the corners of each pole, all of them riveted to two segments of copper one on either side of each pole. The weight of the complete fly-wheel magnet is 70 tons, making a total weight for the machine with foundation plates of 160 tons.

The flux per pole is about 8,800,000 lines, the density averaged over the area of the air-gap opposite each pole being approximately $B_g = 6250$. The minimum area of the teeth being 610 sq. cm., the induction in them of $B_t = 14,500$ is low. If the leakage between the pole-tips is reckoned as 855,000 lines, and between the magnet-cores as 2,030,000, so that the total Z_m at the root of the poles = 11,700,000 = $1·33 Z_g$, the density in the magnet cores of area 717 sq. cm. rises from a minimum of $B_m = 13,350$ to a maximum $B_m = 16,300$, so that the magnets are but little saturated. The constant of size = 59·5, and the specific dimensional output = 0·0168.

Two later machines which have since been built for the Berlin Electricity Works, also with 72 poles for 50 cycles at 83·3 revs. per min., illustrate the advance in economy of design, and especially the saving of weight effected by the tie-rod construction described in Chapter XXIV. § 5. The bore of the stator is 24 ft. 5 in., or nearly the same as in the 3000-KVA. alternator above described, and its greatest width over the outer edges of its feet 32 ft. 10 in.; yet its apparent output is 4700-KVA., or, 6000 volts 450 amperes, and its true output at cos $\phi_e = 0·9$ is 4230 KW. with an input of 6000 H.P. Its total weight with bed-plate is 180 tons; that of the rotating magnet is 118 tons, while the stator frame weighs only 49 tons as against 80 tons in the older 3000-KVA. design.

The frame is divided into four parts, and has a double system of tie-rods arranged on the back of the armature discs which are exposed to the air, with numerous air canals interspersed. The star-winding is embedded in micanite tubes, with wood supports for the end-connections against the sides of the frame.

The magnet system is also in four sections, with a laminated chain of discs compressed between side-cheeks, and the whole secured by 24 keys to the rim of a cast-iron wheel (cp. Figs. 514, 515). The pole-cores are laminated, and dovetailed into the laminated ring with double wedges. The pole-shoes are solid, and fastened by screws to the pole-cores. The exciting coils are of flat copper strip 1·77″ wide × 0·11″ thick, the excitation required for a non-inductive load being 42 KW., or

1 per cent. of the output. The $WD^2 = 3{,}800{,}000$ kilogramme-(metres)^2, whereby $\delta = \frac{1}{320}$ is obtained.

§ 6. **5000 KW. Westinghouse alternator.**—The 5000-KW. 3-phase generator constructed by the Westinghouse Company for

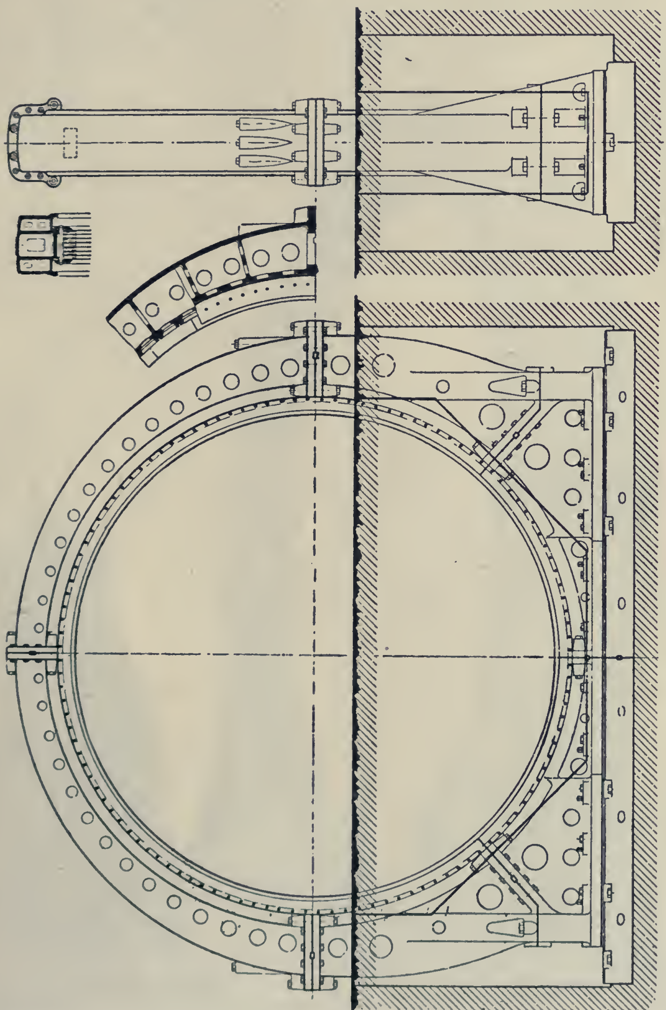


FIG. 584.—Westinghouse 5000-KW. alternator.

the Manhattan Railway Company (74th Street Power Station) is one of the largest alternators that have yet been built, and is illustrated by Figs. 584–586. The height of the great frame of the stationary armature is 42 ft., while the diameter of the rotating field is 32 ft., and its weight 166 tons out of a total for the machine of nearly 400 tons. The speed being 75 revs. per min., the peripheral speed is as much as 7500 ft. per min., and in order to reduce the length between the bearings to a minimum, the weight and design of the rotating part is such as to render any auxiliary fly-wheel unnecessary. The fly-wheel effect from

FIG. 585.—Armature frame of Westinghouse 5000-KW. alternator.

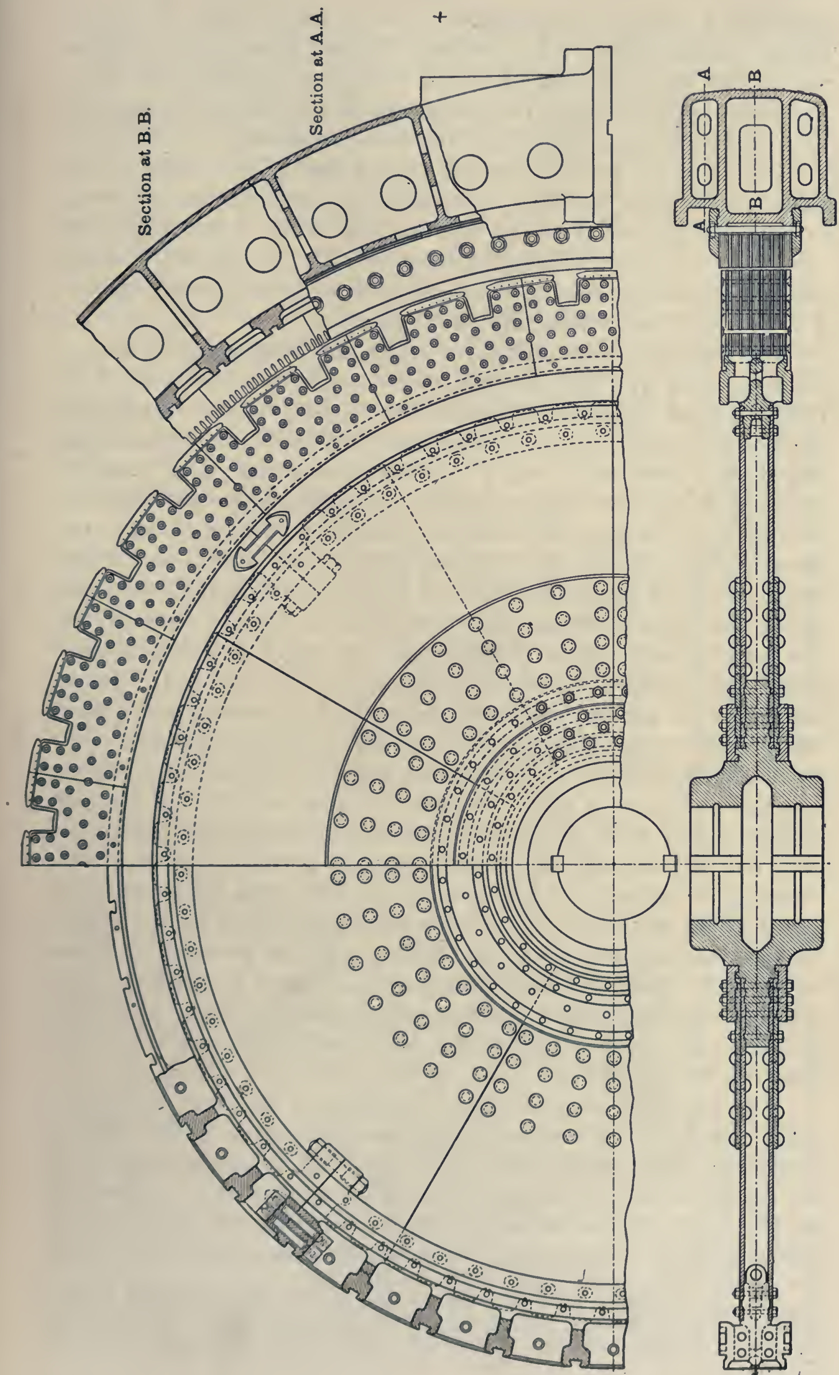


FIG. 586.—Magnet-wheel construction of 5000-KW. Westinghouse alternator.

the magnet system is calculated to be equal to its weight of 166 tons acting at a radius of 11·7 ft., or WD^2 in tons and ft.^{2} = 91,000. Further, the driving power is supplied by two compound engines, each with a horizontal high-pressure and a vertical low-pressure cylinder working on to a single overhung crankpin. The two cranks, one on each side of the alternator, are set at an angle of 135° to each other. Thus the four cylinders give eight impulses to the shaft at equal intervals of time in each revolution, and from the resulting uniformity of the crank effort combined with the fly-wheel effect from the rotor, no difficulty is anticipated in the driving even of rotary converters of 1500 kilowatts output.* The number of poles being 40, the frequency is 25.

The shaft is a hollow forging of fluid-compressed steel, the external diameter being 37″, and the internal 16″. The cast steel hub of the revolving field is keyed to this shaft, and in place of arms two annular webs of rolled steel plates are bolted and recessed into its central flange, and at their outer edge carry a cast-iron driving rim. The poles and yoke $23\frac{1}{2}$ in. wide are composed of thin sheet-steel laminations dovetailed into the cast-iron rim, and farther traversed by a number of bolts compressing them between two steel end-plates. The laminations overlap in neighbouring layers, and the butt joint of two adjacent laminations in the same layer is arranged to occur down the centre of a pole, so that the flux does not cross the dividing lines, the length of each lamination being equal to the width of two poles (cp. Figs. 512 and 513). In the laminated rim are six ventilating ducts about 3 inches apart, passing inwards from the periphery to large openings in the cast-iron rim.

The field coils are of copper strip, wound on edge, with insulating material cemented in place between the turns, the edges of the strip being exposed freely to the air. No supporting case is used for the coil, but the thoroughness of the insulation provided by the internal covering is ensured by testing each coil with 2500 volts alternating potential for one minute. The coils are held in place by cast copper bridges which are slid under the bevelled edges of the poles, as shown in Fig. 518; these bridges also serve the purpose to a certain extent of a damper winding, their overhanging lips retarding in virtue of induced eddy-currents any tendency of the field flux to shift across the pole-edge (cp. Chap. XXVI. § 25).

The stationary armature frame, weighing in all some 234 tons, is supported by an exceptionally stiff cast-iron casing in six segments (Figs. 584, 585); the section of the casing and armature core with its six ventilating ducts opposite to those in the magnet wheel is illustrated in Fig. 491. The width of the core is $23\frac{1}{2}''$ with $8\frac{1}{4}''$ depth of iron below the slots. The winding is composed of bars, three per slot, with

* L. B. Stillwell, "The Electric Power Plant of the Manhattan Railway Company," *Street Railway Journal*, January 1901, from which many of the following particulars are derived.

separate end-connectors. As the pressure of 11,000 volts is directly generated, the insulation of the armature bars is very high, and was tested with a puncture test of 25,000 volts alternating potential for 30 minutes at 25 cycles. The pole-tips are bevelled at the edges to procure such a distribution of the field as will give a sinusoidal shape to the curve of E.M.F. at no-load, and especial care has been taken to secure the same shape under full-load. There are four slots per pole and per phase, so as to eliminate as far as possible any harmonics of higher frequency. The armature slots are partially closed.

The full-load output is 5000 kilowatts at 75 revs. per min., or 66,600 watts per rev. per min., and is made up of 263 amperes per phase at 11,000 volts between the terminals, the winding being star-connected, and each phase giving 6350 volts. The calculated exciting current was 225 amperes at 200 volts when the machine is giving its full rated current at 11,000 volts with cos $\phi_e = 1$ on a non-inductive load, with an increase of 15 per cent. for the full output at the normal voltage if the power-factor is 0·9. When the full-load at 11,000 volts and with unity power-factor is thrown off, the potential was guaranteed not to rise more than 6 per cent. with constant speed and field excitation, and from the data of the characteristic curves after test it was calculated that the actual regulation did not exceed $4\frac{1}{2}$ per cent. The overload capacity may be taken up as high as 7500 kilowatts, or 10,000 e.h.p., for a short period, and the efficiency without allowance for mechanical friction was guaranteed to be 90 per cent. at one-quarter load, 94·5 at half-load, 95·5 at three-quarters, 96·5 at full, and 97 per cent. at 25 per cent. over-load. The armature and field C^2R losses at full-load were respectively 24·16 and 35·54 kilowatts.

On a non-inductive load the alternator was guaranteed to give its full rated load with a rise of temperature in no part of the machine exceeding 63° F., or a 25 per cent. overload with a rise not exceeding 81° F., while with a current 50 per cent. greater than the normal, and with unity power-factor, the rise after two hours was not to exceed 99° F., all of the temperatures being as measured by thermometer. After a test run of 17 hours with an average load of 5000-kw., the increases of temperature above the surrounding air were actually for the field 40°·5 F., for the armature winding 40°·6 F., and for the armature laminations 46° F.

The machines for the same output more recently built by the Westinghouse Company for the Interborough Rapid Transit Company of New York City, and installed in the large power-house on the North River (58th Street) are similar in design, but have the external armature frame divided into seven segments. The upper segment forms as it were a small keystone, by the removal of which access can be obtained to any field coil for removal or replacement. The total weight of the magnet is given as 150 tons. The armature conductors are in this case formed of U-shaped coils which are slipped through the slots from both sides of the armature, their ends being then bent and soldered together.

An interesting comparison is afforded by the 5000-KW. alternators built by the General Electric Company of New York for the Philadelphia Electric Company; these also run at 75 revs. per min., but are 2-phase, 6000 volts, and 60 cycles with 96 poles and a diameter of 33 ft.

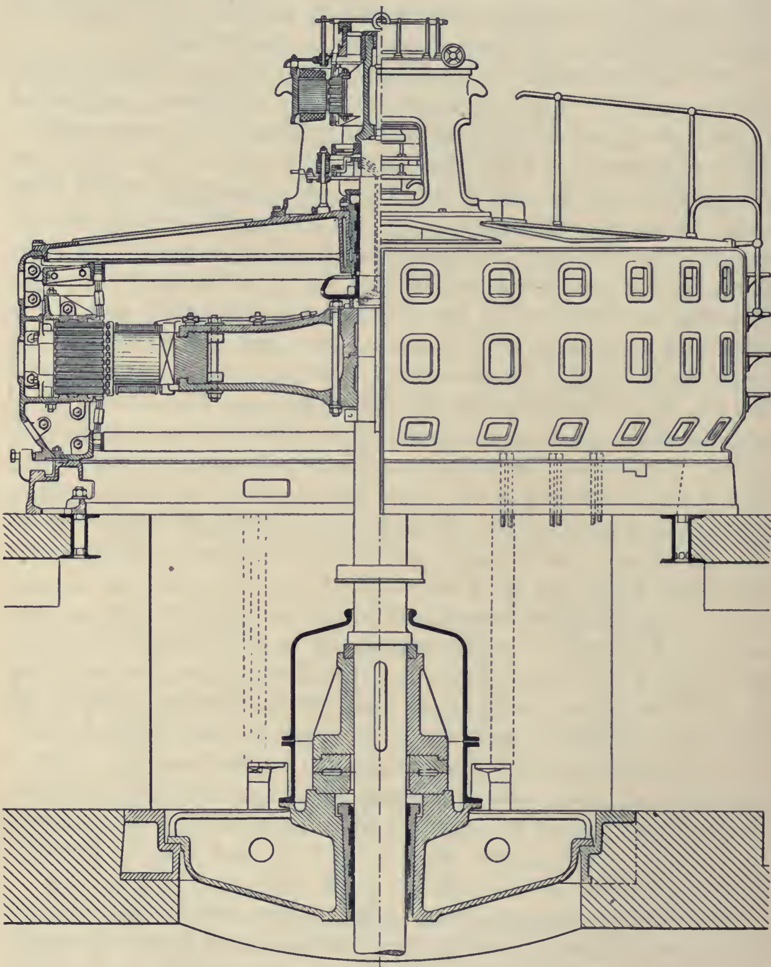


FIG. 587A.—Siemens-Schuckert 5000-KW 3-phase alternator. Elevation.

§ 7. **5000-KW Siemens-Schuckert 3-phase alternator.**—The description of a 5000 KW. 3-phase Siemens-Schuckert alternator, which has been given by Messrs. H. M. Hobart and F.

Punga,* deserves especial study from its fulness and from the interesting experiments which accompanied many of the tests. Six such machines driven by Pelton water-wheels were supplied by the Siemens-Schuckert Co.† for the Nexaca Falls station of the Mexican Light and Power Co.

At a speed of 300 revs. per min. and 50 cycles per sec., the machine was required to give as its full-load 5000 KW., or 4000 volts

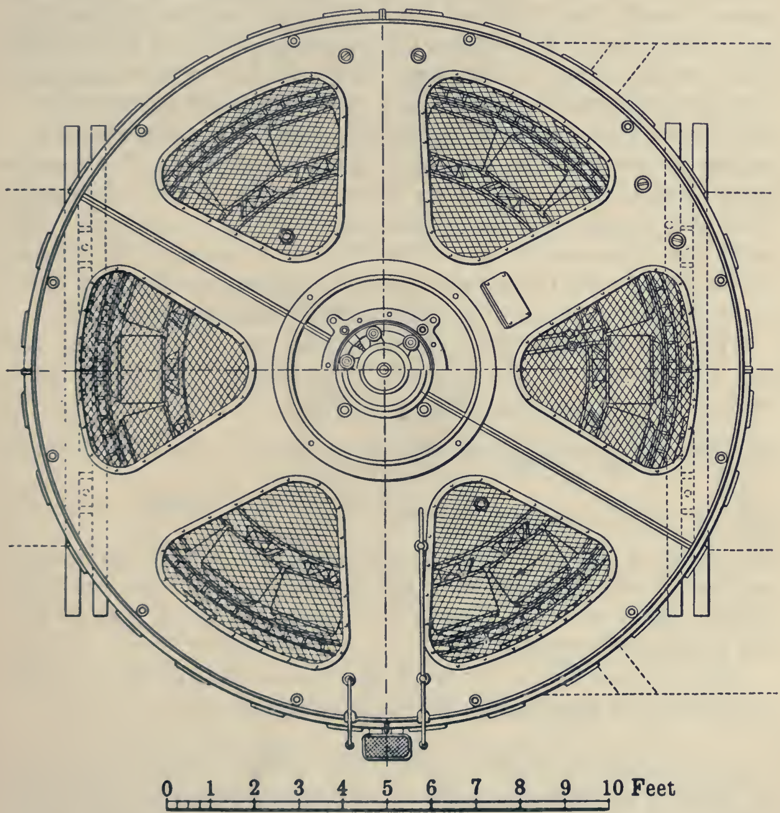


FIG. 587B.—Siemens-Schuckert 5000-KW 3-phase alternator. Plan.

interlinked pressure at its terminals and 730 amperes with cos $\phi_e = 0{\cdot}99$, with a temperature rise not exceeding 35° C. after 24 hours; at the

* *Elektrische Kraftbetriebe u. Bahnen*, vol. v. p. 541 ff., 4th October 1907. While, through the courtesy of the authors, availing himself of the dimensions and details there given, the present writer is alone responsible for the calculations which are based thereon.

† The writer has also to thank the Siemens-Schuckertwerke for the drawings which they kindly supplied to him.

conclusion of this test, it was for two hours to give an over-load of 25 per cent. or 6250 KW. also at 4000 volts and $\cos \phi_e = 0{\cdot}99$, without exceeding a temperature rise of 55° C. The rise of volts on switching off a non-inductive load of 5000 KW. at 4000 volts was not to exceed 8 per cent., and on switching off an inductive load of 4000 KW., or 4000 volts 730 amperes with $\cos \phi_e = 0{\cdot}8$, not to exceed 20 per cent., the excitation in each of the two cases remaining constant. The efficiency at full-load and at 25 per cent. over-load was not to be less than 97·3 per cent., and at half-load not less than 96 per cent., all friction and windage losses being excluded. The shape of the E.M.F. curve when plotted by polar co-ordinates was not to show a divergence of more than ± 3 per cent. from the radius of the circle, which would correspond to a pure sine wave. The insulation of the armature was to stand the application of 8000 alternating volts, and that of the field-winding 2000 volts between iron and copper, for one minute in both cases. The mechanical construction of the machine was to be such that it would be unharmed when run at twice its normal speed, and also such that it could be short-circuited without any damage or shifting of its parts.

The internal diameter of the stator was $147\frac{5}{8}''$ nearly, and the gross length of the armature core between end-flanges 20·1″, whence $D^2_{''}L_{''} = 440{,}000$. The volt-amperes per rev. per min. at full-load, or $\frac{VA}{N} = \frac{5{,}050{,}000}{300} = 16{,}800$, so that the dimensional torque $\frac{VA}{D_{''}^2 L_{''} N} = 0{\cdot}0382$, and its reciprocal or the size constant $= 26{\cdot}2$. On the interior of the stator were 210 open slots, each containing two active conductors one above the other, making $\tau = 420$, or 140 bars per phase. The frequency being 50, there were 20 poles, giving $3\frac{1}{2}$ slots per pole per phase.

Since the machine is to give a sine wave of E.M.F., $K = 1{\cdot}11$, and the equation of the armature E.M.F. per phase is

$$E_a = 1{\cdot}11 \times 2Z_a \times 50 \times 140 \times 10^{-8} = 155{\cdot}5\ Z_a \times 10^{-6}$$

With an interlinked pressure of 4000 volts on open circuit, or 2310 volts per phase, $Z_a = 14{\cdot}85 \times 10^6$, and the density in the air-gap is then $B_g = \frac{14{\cdot}85 \times 10^6}{1800} = 8250$. The ampere-wires per inch length of circumference are $a_w = \frac{420 \times 730}{463} = 662$.

The slots are 3·5 cm. = 0·92 in. wide and 5·6 cm. = 2·2 in. deep. The armature bars are 1·8 cm. square = 3·24 sq. cm., the lower layer at the bottom of the slots being solid, and the upper layer divided into 9 laminations each 0·2 cm. thick, in order to lessen the eddy-current loss (Fig. 588); they are locked in the slots by hard-wood keys, boiled in oil, 0·6 cm. = 0·236 in. thick.

The arrangement of $3\frac{1}{2}$ slots per pole per phase is obtained by a wave-winding with a uniform pitch of 21 at both ends, or reckoned in slots of 11 at the back end and of 10 at the front end, except at certain special places. Starting from a bar connected to the common junction of the star-winding, after passing through 19 end-connections and 20 bars, *i.e.* after completing a tour of the armature, the winding would close on itself, but this is avoided by shortening the last front pitch to 19 instead of 21 (cp. Chapter XXII. § 3 and Fig. 424). Similar second and third tours are then made, the pitch being shortened to 19 at the close of each tour. A fourth tour follows, at the close of which the pitch at the front end in the case of two phases is lengthened to 22; since this is an even number, it joins together two lower bars in the same layer, and so has the effect of reversing the direction of passing

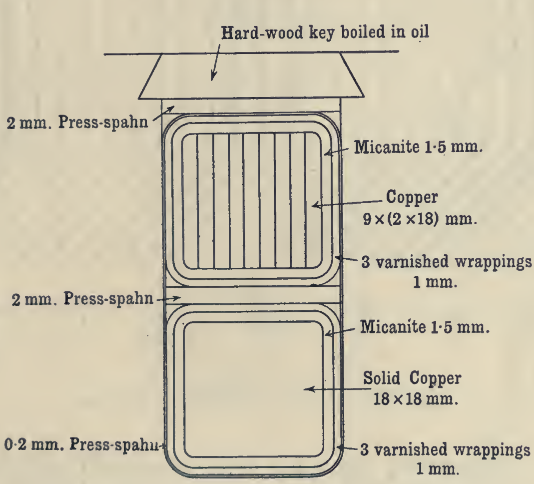


FIG. 588.—Armature slot and bars (full size).

round the armature. Three more tours complete the phase, the pitch being again twice shortened to 19. In the case of the third phase, after the fourth tour the winding is taken backwards through a pitch of 20, which again joins two bars in the same lower layer, and then makes three more tours in the reverse direction to the free terminal, the pitch being 19 at the completion of the 5th and 6th tours. The point of irregularity of the pitch is shown in Fig. 589.

The length of a bar within the core being 20·1″, and of one end-connection 35·8″, the length of a half-loop is 55·9″, and the resistance of one phase measured 0·011 ohm when cold, or say 0·0123 when hot. The weight of the armature copper is 374 lbs., of the armature iron core with teeth 12 tons.

The minimum air-gap is 8 mm. = 0·315″, but by rounding off each pole-shoe face to a radius of $49\frac{5}{8}''$ it is made to open out at each end to a maximum of 14 mm. (Fig. 590). The pole-shoe is divided into ten

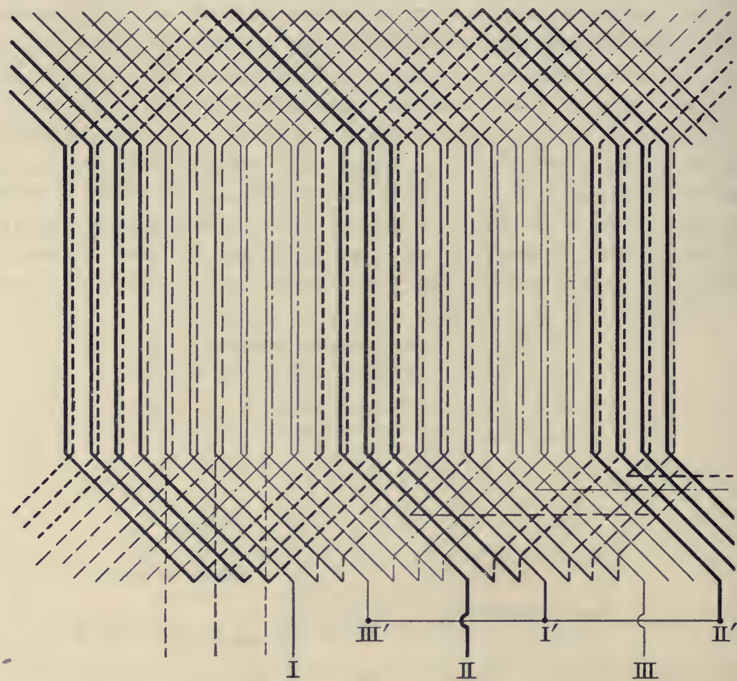


FIG. 589.—Portion of armature winding.

packets of laminations, eight 4·3 cm. wide axially and the two end-packets 3·25 cm. wide, with grooves 0·9 cm. wide between the packets

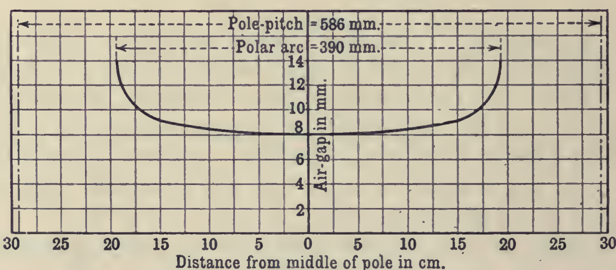


FIG. 590.—Air-gap in relation to polar arc.

exactly corresponding to 9 air-ducts through the stator core, so as to improve the ventilation.

The maximum diameter of the magnet-wheel with poles complete is 147″, and of the foundation rim is 117″, the radial depth of the poles below the pole-shoe being 12·6″. Each of the 20 poles is dovetailed into the rim with an angle of 65°, and locked by two steel keys with opposite tapers. On each pole there are 63 turns of flat copper

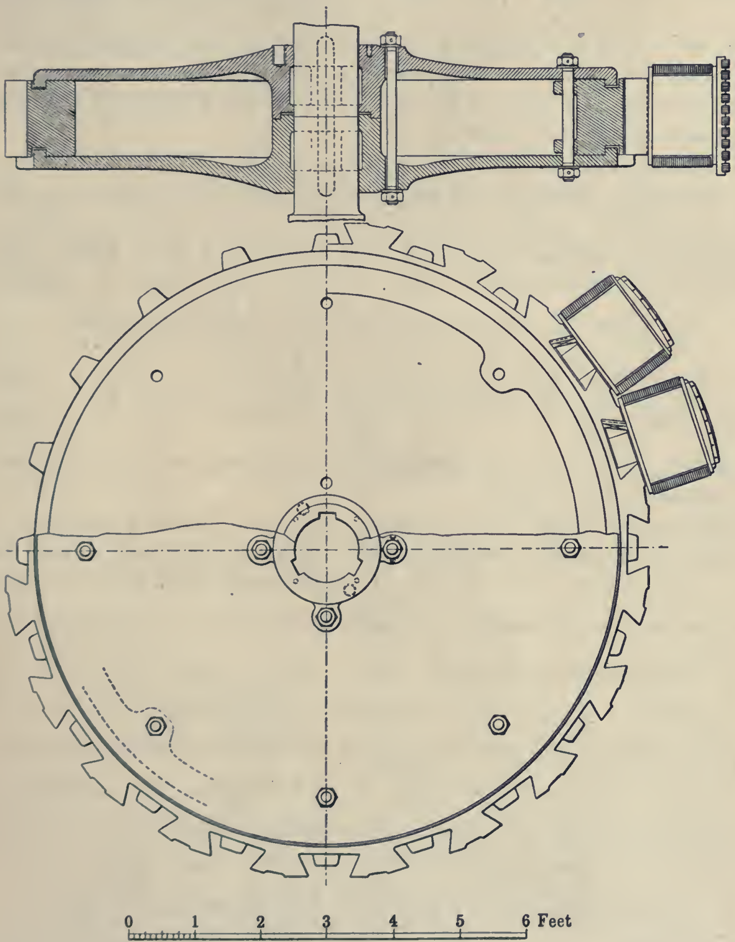


Fig. 591.—Magnet wheel construction.

3·5 cm. wide × 0·4 cm. thick, with intervening insulation 0·03 cm. thick. The coil 10⅝″ deep is secured in place by a wrought-iron and a silicium bronze washer at the inner and outer ends respectively, with press-spahn insulation round the pole and at the ends. The average length of a turn being 52·5″, the resistance of the 20 field-coils when warm is 0·24 ohm; the total number of turns 1260, and the weight of field copper 945 lbs.

The calculation of the air-gap reluctance is made as follows. From the axial length of the pole-shoe, which is the same as that of the armature core, must be deducted the equivalent width of the 9 air-ducts, each 0·9 cm. wide. The ratio $\frac{w_d}{l_g}$ being $\frac{0·9}{0·8}=1.125$, $K_3=0·16$, and $K_3 . nw_d=0·16 \times 9 \times 0·9=1·3$ cm. Owing to the shallow grooves dividing the packets of the pole-shoe laminations, some further deduction must be made. The axial length of the air-gap will therefore be taken as $51 - 2·3=48·7$ cm.

The polar arc is 39 cm., the pole-pitch being 58·5 cm. with a ratio of 0·666, but, owing to the rapid opening out of the air-gap towards each pole-tip, a graphic calculation gives for the effective polar arc reduced to a uniform air-gap of 0·8 cm. a breadth of about 37 cm., even with the fringe included. The equivalent polar area is therefore $48·7 \times 37=$ say 1800 sq. cm. with ratios $\frac{w_s}{l_g}=\frac{2·34}{0·8}=2·92$, and $\frac{w_{t1}}{w_s}=\frac{3·26}{2·34}=1·39$, $m=1·18$, and the equivalent air-gap is $ml_g=1·18 \times 0·8=0·945$ cm. The reluctance is thus $\frac{0·945}{1800}=0·000525$, and $\frac{X_g}{2} = 0·8Z_a \times 0·000525=0·00042\ Z_a$, which agrees very closely with the "air-line" of the no-load flux curve.

The radial depth of the armature core (not including the slots) is 28·4 cm., and the iron length $(51 - 9 \times 0·9) \times 0·9=38·6$ cm., so that $2ab = 2 \times 28·4 \times 38·6=2190$ sq. cm. The half-length of path may be taken as $\frac{l_a}{2}=32$ cm. The width of a tooth at the top of a slot being 3·26 cm., the iron area of the teeth under a pole is $7 \times 3·26 \times 38·6=880$ sq. cm., which is increased in the proportion $\frac{3·34}{3·26}$ to 900 sq. cm. at the bottom, while at the narrowest part of the notch for the wood key it is reduced in the proportion $\frac{2·62}{3·26}$ to 706 sq. cm. The uncorrected density at full-load is therefore about 17,000, rising to 21,000 at one spot, and the length of path through one tooth is $l_t=5·6$ cm.

The permeance between the tips of the pole-shoes is $\frac{4·75 \times 51}{17·5} \times 2 = 27·7$ and between their flanks $2·3 \times \frac{4·95}{\pi} \times \log \frac{\pi \times 19·5+17·5}{17·5} \times 4=9·45$. The poles being reckoned as equivalent to a square section of $33·6 \times 33·6$ sq. cm. at a mean distance apart of 19 cm., the permeance between the sides of the poles is $\frac{32 \times 33·6}{19·5} \times 2=110$; and between the flanks $2·3 \times \frac{32}{\pi} \times \log \frac{\pi \times 16·8+19·5}{19·5} \times 4=53·5$. Each of the two latter must be divided by 2, since they are only acted upon by a mean M.M.F. from $\frac{X_P}{2}$. The leakage permeance is therefore

$$\mathcal{P}_l = 27{\cdot}7 + 9{\cdot}45 + 55 + 26{\cdot}75 = 118{\cdot}9, \text{ say } 125$$

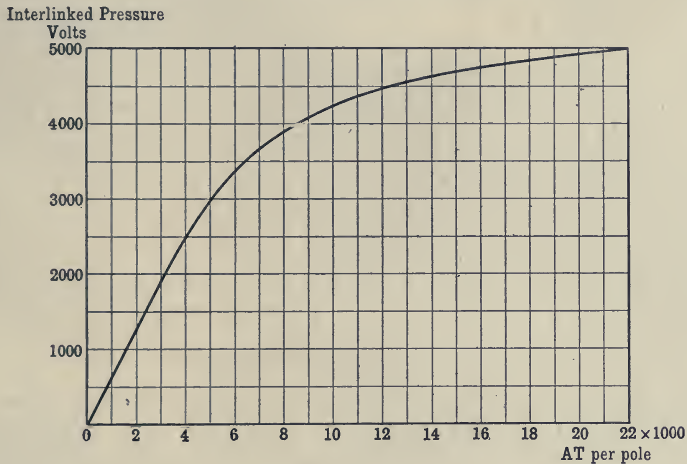


FIG. 592.—No-load characteristic.

Round each ingot-iron magnet core of 37 cm. diameter is a wrought-iron sheath 0·5 cm. thick, so that if the latter be reckoned as of only half-value, the magnetic cross-section is $37{\cdot}5^2 . \frac{\pi}{4} = 1100$ sq. cm. The length of path through one core and pole-shoe s $\frac{l_m}{2} = 37$ cm. The double section of the yoke is about 2000 sq. cm., and the half-length of path through it $\frac{l_y}{2} = 20$ cm.

An approximate division of the full-line open-circuit magnetisation curve (Fig. 592) into the two portions corresponding to the air-gap and armature core, and to the pole and yoke, is given in Fig. 593.

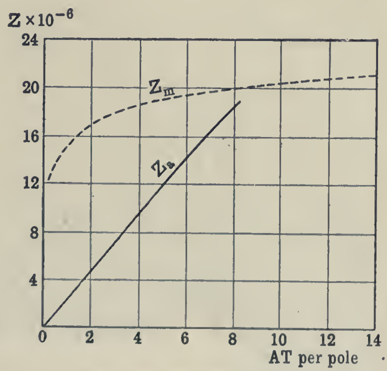


FIG. 593.—Partial magnetic characteristics.

To calculate the inductance of a phase, it is best to consider each large undivided coil of seven turns, and to take each side and end separately as they differ in the position or number of the bars. The influence of the self-induced flux through the solid bar shows itself in unequal current distribution and may be neglected; the self-inductance due to flux up to the level of the top of the laminated bar is per cm. length of core $12{\cdot}57\ \frac{2{\cdot}5}{2{\cdot}34} = 13{\cdot}4$, and the mutual inductance from the laminated bar is $6{\cdot}28\ \frac{1{\cdot}8}{2{\cdot}34} = 4{\cdot}8$. In the upper bar, even though laminated, it will be found that unequal current distribution causes considerable loss, and must therefore be appreciable, so that it is doubtful how far the effective self and mutual inductance is annulled; the mutual inductance from the lower bar $= 4{\cdot}8$ will therefore alone be taken. Above the laminated bar up to the slot opening both bars act equally, and their self and mutual inductance is $12{\cdot}57\ \frac{1{\cdot}1}{2{\cdot}7} \times 2^2 = 20{\cdot}5$. The inductance per cm. length of core for three completely filled slots is thus $3\ (13{\cdot}4 + 4{\cdot}8 + 4{\cdot}8 + 20{\cdot}5) = 130$; for the single solid bar it is $12{\cdot}57\ \frac{3{\cdot}6}{2{\cdot}34} = 19{\cdot}3$, and for the single laminated bar $12{\cdot}57\ \frac{1{\cdot}1}{2{\cdot}7} = 5{\cdot}1$. Hence the slot inductance of both sides of a complete coil is $(130 \times 2 + 19{\cdot}3 + 5{\cdot}1) \times 42{\cdot}9 = 12{,}200$.

For the surface-of-the-core inductance, owing to the coil-side being widely distributed in four slots, it is best to consider the opening of the slots as spanned by local flux in semicircles, and a joint flux in quadrants and straight lines extending beyond up to the limit of half the pole-pitch $= 29{\cdot}35$ cm. (cp. Chap. XVIII. § 24). The inductance from the local flux above each of the completely filled slots per cm. length is $2^2 \times 9{\cdot}2 \log. \frac{\pi w_t/2 + w_s}{w_s} = 4 \times 5{\cdot}72$ and above the single bar is $5{\cdot}72$; that from the joint flux is $7^2 \times 9{\cdot}2 \log. \frac{\pi(29{\cdot}35 - 11{\cdot}2) + 22{\cdot}4}{\pi \times 1{\cdot}63 + 22{\cdot}4} = 206$. Their total for both sides is

$$\{206 + 5{\cdot}72 + 3(5{\cdot}72 \times 4)\} \times 42{\cdot}9 \times 2 = 24{,}100$$

The V-shaped end-connections at either end are similarly best considered separately, since at one end there are four and at the other end three running side by side. The calculation is then to be made exactly as for the V-shaped end-connections of a continuous-current machine by formula (120). The length at one end, including the air-ducts of the armature core, is $l' = 91 + 8{\cdot}1 = 99{\cdot}1$; the equivalent diameter d_s of each packet is for 4 bars $\frac{2(18{\cdot}85 + 4{\cdot}25)}{\pi} = 14{\cdot}7$, and for 3 bars

$$\frac{2(17{\cdot}42 + 4{\cdot}25)}{\pi} = 11{\cdot}1$$

whence $(4{\cdot}6 \log. \frac{l'}{d_s} - 0{\cdot}9)$ in the two cases is $2{\cdot}9$ and $3{\cdot}47$ respectively The inductance is therefore $(2{\cdot}9 \times 16 + 3{\cdot}47 \times 9) \times 99{\cdot}1 = 7700$.

The sum of the three portions is $(12{,}200 + 24{,}100 + 7700) \times 10^{-9} = 0{\cdot}000044$ henrys, which, with an increase of 15 per cent. to allow for the partial mutual inductance of the other phases and for ten complete

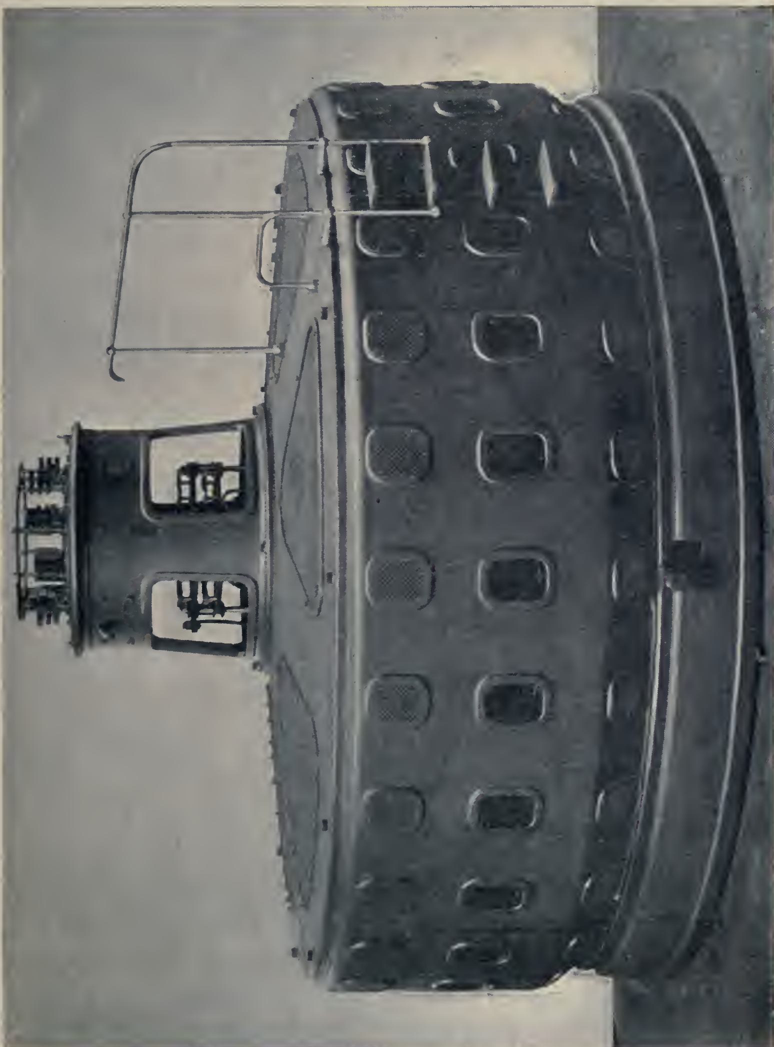


FIG. 594.—Siemens-Schuckert 5000-KW. alternator.

coils, gives as the inductance of one phase $L_a = 0{\cdot}000505$. The reactance $2\pi f L_a = 0{\cdot}159$, say $0{\cdot}16$ ohm.

A group of seven conductors, corresponding to one pole and in one phase, consists of three slots each filled with two bars and one single bar in either the upper or lower layer of a fourth slot. Taking the inner pair of slots with the smallest width of eight slot-pitches, we have in eq. (165)

$$2 \text{ bars} \times \sin \frac{\pi}{2} \cdot \frac{b'}{B} = \frac{2}{7} t \sin 90^\circ \times \frac{8}{10\cdot5} = 0\cdot266t$$

$$\text{for the next pair of slots} \ldots \frac{2}{7} t \sin 90^\circ \times \frac{10}{10\cdot5} = 0\cdot285t$$

$$\text{for the next pair of slots} \ldots \frac{2}{7} t \sin 90^\circ \times \frac{12}{10\cdot5} = 0\cdot273t$$

$$\text{and for the outer single bars } \frac{1}{7} t \sin 90^\circ \times \frac{14}{10\cdot5} = 0\cdot124t$$

$$\overline{\quad 0\cdot948t \quad}$$

$$\text{Therefore } k_d = m \cdot \frac{4}{\pi^2} \cdot \frac{B}{b} \cdot \sin \frac{\pi}{2} \cdot \frac{b}{B} \times 0\cdot948$$

$$= 3 \cdot \frac{4}{\pi^2} \cdot \frac{1}{0\cdot65} \sin (90^\circ \times 0\cdot65) \times 0\cdot948$$

$$= 1\cdot51$$

$$\text{and } X_{DM} = k_d \sqrt{2} t I \sin \phi_o = 1\cdot51 \times 1\cdot41 \times 7 \, I \sin \phi_o$$

$$= 14\cdot95 \, I \sin \phi_o$$

Similarly for the cross ampere-turns

$$k_c = m \cdot \frac{4}{\pi^2} \cdot \frac{B}{b} \cdot \left(1 - \cos \frac{\pi}{2} \cdot \frac{b}{B}\right) \times 0\cdot948$$

$$= 1\cdot51 \times \frac{1 - 0\cdot5225}{0\cdot8526} = 0\cdot845$$

and

$$X_{CM} = 14\cdot95 \times \frac{0\cdot845}{1\cdot51} I \cos \phi_o = 8\cdot36 \, I \cos \phi_o$$

Identifying the cross reluctance $\mathfrak{R}_c$ directly with that of the double air-gap which has been already calculated,

$$Z_c = \frac{1\cdot257 X_{CM}}{2ml_g} \times bL = \frac{1\cdot257 \times 8\cdot36 \, I \cos \phi_o}{0\cdot00105} = 10{,}000 \, I \cos \phi_o$$

$$= 0\cdot01 \, I \cos \phi_o \times 10^{-6}$$

$$E_{CM} = 155\cdot5 \, Z_c \times 10^{-6} = 1\cdot555 \, I \cos \phi_c$$

In order to allow for eddy-currents, the resistance of the armature per phase will be increased to $0\cdot0123 \times 1\cdot7 = 0\cdot021$ ohm.

The calculation of the necessary excitations may then be made as follows. With 4000 volts or E_e per phase $= 2310$, and with $\cos \phi_e = 1$ or $I = 722$ amperes, by equation (177)

$$\tan \phi_o = \frac{x_a I + \dfrac{E_{CM}}{\cos \phi_o}}{E_e + I r_a'} = \frac{115\cdot5 + 1120}{2310 + 15\cdot1} = 0\cdot53$$

whence $\phi_o = 28^\circ$. $OD = \sqrt{1235^2 + 2325^2} = 2630$, and E_a per phase $=$

$\text{OD} - \sin\phi_o \dfrac{E_{CM}}{\cos\phi_o} = 2630 - 525 = 2105$, whence $Z_a = 13{\cdot}55 \times 10^6$, $B_a = 6190$ and $B_t = 15100$

$$\begin{aligned}
\frac{X_g}{2} & \quad . \quad . \quad . \quad . \quad = 5700 \\
\frac{X_a}{2} &= 3 \times 32 \quad . \quad . \quad = 94 \\
\frac{X_t}{2} &= 20 \times 5{\cdot}6 \quad . \quad . \quad = 112 \\
\frac{X_{DM}}{2} &= 7{\cdot}475 \times 722 \times \sin 28^\circ = 2535 \\
& \qquad \frac{X_P}{2} = 8440
\end{aligned}$$

$\zeta = 1{\cdot}257 \times 8440 \times 2 \times 125 = 2{\cdot}65 \times 10^6$, and $Z_m = 16{\cdot}2 \times 10^6$, whence $B_m = 14{,}700$ and $B_y = 8100$

$\dfrac{X_m}{2} = 10 \times 37 = 370$, and $\dfrac{X_y}{2} = 56 \times 20 = 1120$, whence $X =$ say, 10,000.

For the same excitation the open-circuit voltage is 4260, so that the rise on switching off a non-inductive load is $6\frac{1}{2}$ per cent.

With $\cos\phi_e = 0{\cdot}8$, $\tan\phi_o = \dfrac{E_e \sin\phi_e + x_a I + \dfrac{E_{CM}}{\cos\phi_o}}{E_e \cos\phi_e + I r_a'}$

$$= \frac{1385 + 115{\cdot}5 + 1120}{1848 + 15} = 1{\cdot}4$$

whence $\phi_o = 54{\cdot}5^\circ$

$\text{OD} = \sqrt{2620^2 + 1863^2} = 3210$, and E_a per phase $= 3210 - 910 = 2300$. $Z_a = 14{\cdot}8 \times 10^6$, and the densities $B_a = 6750$, $B_t = 16{,}450$.

$$\begin{aligned}
\frac{X_g}{2} & \quad . \quad . \quad . \quad . \quad = 6210 \\
\frac{X_a}{2} &= 3{\cdot}5 \times 32 \quad . \quad . \quad = 112 \\
\frac{X_t}{2} &= 40 \times 5{\cdot}6 \quad . \quad . \quad = 224 \\
\frac{X_{DM}}{2} &= 7{\cdot}475 \times 722 \times \sin 54^\circ{\cdot}5 = 4400 \\
& \qquad \frac{X_P}{2} = 10{,}946
\end{aligned}$$

$\zeta = 1{\cdot}257 \times 10{,}946 \times 2 \times 125 = 3{\cdot}45 \times 10^6$. $Z_m = 18{\cdot}25 \times 10^6$ $B_m = 16{,}600$ and $B_y = 9120$

$$\begin{aligned}
\frac{X_m}{2} &= 40 \times 37 \quad . \quad . \quad = 1480 \\
\frac{X_y}{2} &= 100 \times 20 \quad . \quad . \quad = 2000 \\
& \qquad \frac{X}{2} = 14{,}426
\end{aligned}$$

For the same excitation the open-circuit voltage was 4700, or a rise on switching off full-load of $17\frac{1}{2}$ per cent.

On short-circuit $\tan\phi_o = \dfrac{0{\cdot}16 + 1{\cdot}555}{0{\cdot}021} = 81{\cdot}6$, and ϕ_o is $89^\circ{\cdot}3$, so that the impedance z' is practically the same as the numerator $= 1{\cdot}715$. Thence $E_a = \text{OD} - \sin\phi_o \cdot \dfrac{E_{\text{CM}}}{\cos\phi_o} = I_o(1{\cdot}715 - 1{\cdot}555) = 0{\cdot}16\ I_o$,

and $Z_a = \dfrac{E_a}{155{\cdot}5} \times 10^6 = 1030\ I_o$.

$$\text{For low saturations } \frac{X_g + X_a + X_t}{2} = 2{\cdot}83\ E_a = 440\ Z_a \times 10^{-6} \ldots = 0{\cdot}454\ I_o$$

$$\frac{X_{\text{DM}}}{2} \ldots\ldots = 7{\cdot}475\ I_o$$

$$\frac{X_{\text{P}}}{2} = 7{\cdot}929\ I_o$$

$\zeta = 1{\cdot}257\ \dfrac{X_{\text{P}}}{2} \times 2 \times 125 = 2490\ I_o$, and $Z_m = 3520\ I_o$. Thence

$B_m = \dfrac{3520\ I_o}{1100} = 3{\cdot}2\ I_o$, and $B_y = \dfrac{3520}{2000}\ I_o = 1{\cdot}76\ I_o$. If $f'(B_m) = \dfrac{B_m}{1200}$ and $f'(B_y) = \dfrac{B_y}{500}$,

$$\frac{X_m + X_y}{2} = \left(\frac{3{\cdot}2}{1200} \times 37 + \frac{1{\cdot}76}{500} \times 20\right)I_o = (0{\cdot}1 + 0{\cdot}07)I_o = 0{\cdot}17\ I_o,$$

$$\text{and } \frac{X}{2} = 8{\cdot}1\ I_o$$

Therefore, with an excitation of 10,000 AT per pole, $I_o = 1235$ amperes, or $1{\cdot}7$ times the normal current, which agrees closely with the experimentally ascertained value. The value of $\lambda = k \cdot k_d \cdot \sqrt{2} = \dfrac{X}{tI_o} = \dfrac{10{,}000 \times 2}{7 \times 1235} = 2{\cdot}32$, and $k = \dfrac{2{\cdot}32}{1{\cdot}51 \times \sqrt{2}} = 1{\cdot}09$.

Owing to the reluctance of the magnet becoming greatly increased between full-load and open-circuit conditions for the same excitation, the synchronous impedance rises, and the method based on it fails in the present case to give even approximately correct results for calculation of the regulation.

When suddenly short-circuited under full-load excitation, the armature current rose momentarily to a value about $2\frac{1}{2}$ times its steady short-circuit value, while at the same time the exciting current momentarily increased about $3\frac{1}{2}$ times (cp. Chap. XXIV. § 2).

Experiments were made to measure as closely as possible the separate losses from which the following conclusions are drawn.

In addition to the usual core-losses from hysteresis and losses, the conductors, even when laminated, are the seat of eddy-currents due to lines curving inwards into the slots through their open entrances. By an approximate calculation of the strength of this slanting field, Messrs. Hobart and Punga estimated that the eddy loss in the layer of solid and laminated conductors might probably amount to 8 and 3 KW. respectively. The total no-load loss in the armature of 93 KW. was experimentally measured.

The total armature loss when the machine was short-circuited and run with full armature current (730 amp.) showed, in addition to the ohmic loss over the armature resistance ($0{\cdot}0123 \times 730^2 \times 3 = 19{\cdot}6$ KW.), a further loss of 23·9 KW. The greater part of this is to be attributed to unequal current-distribution over the area of the laminated conductors near the openings of the slots, and in a lesser degree over the area of the solid conductors at the bottom of the slots; the relative amounts of these increases are given in Chap. XXV. § 9. The end-connections when carrying the full-current will show an increase from the same cause, and also from eddy-currents which their secondary leakage will set up in neighbouring iron masses.

The armature direct reaction varies about $\pm$ 8 per cent. with a sextuple frequency of 300 periods (cp. Chap. XXV. § 9). By analogy from experiments, a loss of about 400 watts at full-load with $\cos \phi_e = 1$ was indicated as probable from eddies set up in the magnet-cores by this pulsation in the effect of the armature back ampere-turns.

The excitation loss with $\cos \phi_e = 1$ is $(170 \text{ amp.})^2 \times 0{\cdot}24$ ohm $= 7$ KW., and with $\cos \phi_e = 0{\cdot}8$, $228^2 \times 0{\cdot}24 = 12{\cdot}5$ KW.

The measured loss of 52 KW. by friction and windage may be divided on the assumption that the former varied as the 1·5th, and the latter as the 3rd power of the revs. per min.

The efficiency is therefore to be calculated approximately as below :—

Hysteresis and eddies in armature core	82 KW.
Eddy-currents in solid conductors, approximately .	8 ,,
,, in laminated conductors, approximately	3 ,,

	$\cos \phi_e = 1$	$\cos \phi_e = 0{\cdot}8$
Measured no-load core loss	93·0 KW.	93·0 KW.
Ohmic loss over resistance of armature . . .	19·5 ,,	19·5 ,,
Additional loss in solid conductors within slots .	1·68 ,,	1·68 ,,
,, ,, in laminated conductors within slots	14·0 ,,	14·0 ,,
,, ,, from end-connections	7·52 ,,	7·52 ,,
Eddy-currents in magnet-cores from pulsation of field	0·4 ,,	0·4 ,,
Excitation	7·0 ,,	12·5 ,,
Friction of bearings	10·6 ,,	10·6 ,,
Windage	41·4 ,,	41·4 ,,
Total losses	195·1 KW.	200·6 KW.
Output	5000 ,,	4000 ,,
Input	5195·1 ,,	4200·6 ,,
Efficiency	96·2 per cent.	95.2 per cent.

Several tests were taken with different shapes and numbers of wing-shaped blades, which were attached to the rim and poles of the magnet-wheel for the purpose of causing an artificial draught of air through the machine.

The wave form was tested by oscillograph, and found to be very closely a pure sine curve.

LIST OF CHIEF SYMBOLS EMPLOYED

A . . . = amperes.
= radial depth of armature core including teeth.
= length of arc of pole-face.
AT . . . = ampere-turns.
AT_a . . . = total effective ampere-turns on alternator armature from all phases (Chap. xxiii. § 22).
AT_c . . . = ampere-turns in a coil.
AT_f . . . = total ampere-turns on all poles of field-magnet of hetero-polar alternator.
A_w . . . = ampere-wires on armature.
a_w . . . = ampere-wires per unit length of armature circumference (Chap. xviii. § 34).
a . . . . = number of pairs of armature paths $= \frac{q}{2}$ (Chap. xi. § 22).
= radial depth of armature core below slots.
a'' . . . = projection of straight armature bar from slot at one end (eq. 52).
α . . . . = temperature coefficient for increase of resistance by heating, per degree rise (Chap. xix. § 2).

B . . . . = magnetic induction or flux-density in C.G.S. lines per sq. cm.
= pole-pitch in equations for direct and cross magnetising ampere-turns of alternator armature (Chap. xxiii. § 8).
B_a . . . = flux-density in armature core below teeth.
B_g . . . = flux-density in air-gap.
B_g' . . . = flux-density in air-gap at leading edge of pole.
B_g'' . . . = flux-density in air-gap at trailing edge of pole.
B_s . . . = flux-density in slot.
B_t . . . = flux-density in teeth.
B_{t1} . . . = flux-density in teeth at top.
B_{t2} . . . = flux-density in teeth at bottom.
B_y . . . = flux-density in yoke.
B_m . . . = flux-density in field magnet.
= bending moment.
B_e . . . = equivalent bending moment.

B_o . . . = flux-density in interpolar gap from initial symmetrical field (Chap. xviii. § 29).
B_q . . . = flux-density in interpolar gap from cross-field of armature ampere-turns (Chap. xviii. § 29).
B_k . . . = flux-density in interpolar gap to give straight-line commutation (Chap. xviii. § 29).
B_c . . . = flux-density of commutating field at leading pole-tip in smooth-core armature (Chap. xviii. § 32).
B_r . . . = flux-density from commutating pole (eq. 132).
b . . . . = net axial length of iron in armature core.
= virtual width of field of one pole along pole-pitch (Chap. xxiii. § 8).
= width of set of brushes along axis of commutator.
b_1 . . . = width of brush in direction of rotation.
b' . . . = width between centres of coil-sides of alternator armature coil (Chap. xxiii. § 8).
β . . . = width of sector-pitch.
= ratio of pole-arc to pole-pitch.
= total electrical angle of displacement between two alternators in parallel (Chap. xxvi. § 17).

C . . . = continuous current in amperes.
C_a . . . = total current through armature.
C_e . . . = current in external circuit.
C_s . . . = shunt current.
c . . . . = half interpolar gap measured on the armature circumference.
= ratio of excitation of entire magnet circuit to excitation over air-gaps $= \frac{X}{X_g}$.

D . . . = diameter of armature core.
D_k . . . = diameter of commutator.
d_s . . . = diameter of circle having the same perimeter as a group of end-connectors simultaneously short-circuited (Chap. xviii. § 23).
d . . . . = distance between adjacent end-connectors in barrel winding.
d' . . . = diameter of journal in inches.
d_1 . . . = diameter of insulated wire.
δ . . . . = diameter of binding wire.
= ratio of greatest variation in speed to mean speed (Chap. xxvi. § 18).
= deflection of shaft or displacement of armature core from centre.
= total thickness of insulation.
$\triangle P$ . . . = loss of volts over brushes of one polarity.
$\triangle E$ . . . = E.M.F. set up between extreme edges of brush (Chap. xviii. § 30).
= fluctuation of energy from driving torque in steam-engine (Chap. xxvi. § 19).
$\triangle t$ . . . = difference of temperature.

E . . . = modulus of elasticity.
E . . . = E.M.F. in volts (R.M.S. value if alternating).
$\mathbf{E}$ . . . = maximum instantaneous value of alternating E.M.F. in volts.
E_a . . . = voltage generated in armature (R.M.S. value per phase in alternator armature).
E_b . . . = voltage at brushes of dynamo.
E_e . . . = voltage at terminals of external circuit (R.M.S. value if alternating).
E_x . . . = voltage of excitation.
e_i, E_i, $\mathbf{E}_i$. = instantaneous, effective, and maximum value of impressed alternating E.M.F.
e_r, E_r, $\mathbf{E}_r$. = instantaneous, effective, and maximum value of resultant or active alternating E.M.F.
e_s, E_s, $\mathbf{E}_s$. = instantaneous, effective, and maximum value of E.M.F. of self-induction.
e_s', E_s', $\mathbf{E}_s'$ = instantaneous, effective, and maximum value of E.M.F. consumed by self-induction.
ϵ . . . . = base of natural logarithms.
e_{sa} . . . = E.M.F. consumed by inductance from secondary leakage in local paths (Chap. xxiii. § 3).
e_{sb} . . . = direct magnetising effect of armature ampere-turns reduced to equivalent E.M.F.
η . . . . = hysteretic coefficient (Chap. xii. § 8).
= ratio of full-load to open-circuit volts in alternators (Chap. xxv. § 1).
= efficiency.
= coefficient of linear expansion by heat.
E_{CM} . . . = effective value of E.M.F. due to cross flux per phase of alternator (eq. 176).
E_t . . . = effective value of real induced E.M.F. per phase, being vector sum of E_a and E_{CM} (Chap. xxiii. § 11).
E_o . . . = effective value of E.M.F. on open circuit per phase of alternator.

F . . . . = mechanical force.
= eddy-current coefficient (Chap. xix. § 17).
F_u . . . = contact-surface of one row of brushes.
F_u' . . . = contact-surface of brush with leading sector.
F_u'' . . . = contact-surface of brush with trailing sector.
F_c . . . = total centrifugal force summed up round periphery of cylinder.
F_{sc} . . . = centrifugal force of whole commutator.
f . . . . = frequency in complete periods per second (Chap. viii. § 6, and Chap. ix. § 3).
f_c . . . = centrifugal force per unit arc of cylinder.
f_{sc} . . . = centrifugal force per sector of commutator.
f_s . . . = safe permissible shearing stress.
f_t . . . = safe permissible tensile stress.
$f(t)'_k$ . . . = value of reversing field to give straight-line commutation (Chap. xviii. § 17).

$f(t)_z$ . . . = part of commutating field which causes the divergence, i_z (Chap. xviii. § 17).
f_u . . . = form factor of current curve under brushes during short-circuit (Chap. xviii. § 15).
= controlling force per unit of displacement (Chap. xxvi. § 12).

g . . . . = acceleration due to gravity.
= angular pitch of slots in terms of a bipolar machine (Chap. xxiii. § 8).
γ . . . . = number of slots per pole-pitch in a toothed armature.
γ_1 . . . = number of slots per pole and per phase in alternator armature.

H . . . = magnetic difference of potential in C.G.S. units.
= hysteresis loss per revolution per minute in a given armature.
H_w . . . = loss by hysteresis in watts (eq. 26*a*).
h . . . = specific loss by hysteresis in joules per cycle per c.cm. (eq. 26).
h_a . . . = thickness of arm of armature hub (eq. 40).
h_s . . . = depth of slot in toothed armature.
h_3 . . . = depth of wooden wedge in slot.

I . . . = moment of inertia.
i, I, **I** . . = instantaneous, effective, and maximum values of alternating current in amperes.
I_e . . . = R.M.S. value of external current from alternator.
I_o . . . = short-circuit current of alternator per phase.
I_n . . . = normal armature current of alternator per phase.
I_c . . . = corrective current of alternator per phase (Chap. xxvi. § 13).
I_s . . . = synchronising current of alternator per phase (Chap. xxvi. § 17).
i_1 . . . = current in leading commutator connector of short-circuited section (Chap. xviii. § 17).
i_2 . . . = current in trailing commutator connector of short-circuited section (Chap. xviii. § 17)
i_k . . . = instantaneous value of straight-line short-circuit current (Chap. xviii. § 17).
i_z . . . = instantaneous value of additional short-circuit current (Chap. xviii. § 17).

J . . . = amperes in any one wire on continuous current armature.
j (with index) = number of coil-sides simultaneously short-circuited and acting on the same flux-path or permeance (Chap. xviii. § 21).

K . . . = voltage factor in E.M.F. equation (18) of alternator (Chap. xxii.).
= capacity in farads.
KW . . = kilowatts.
KVA . . = kilovolt-amperes.

k . . . = heating coefficient $\frac{t^\circ . S_c}{W}$ (Chap. xix. §§ 9, 19).

k . . . = ratio of excitation of magnetic circuit to magnetising ampere-turns of alternator armature on short-circuit, *i.e.*, $\frac{X}{X_a}$ (Chap. xxiii. § 22).

k' . . . = width factor of armature winding of alternator (Chap. viii. § 11) or continuous-current dynamo (Chap. xi. § 9).

k'' . . . = form factor, *i.e.*, ratio of R.M.S. to average value of a varying quantity.

k' . . . = permeance factor of slot in inductance of short-circuited section (Chap. xviii. § 21).

k'' . . . = permeance factor of slot-opening inductance of short-circuited section (Chap. xviii. § 21).

k''' . . . = permeance factor of surface of core inductance of short-circuited section (Chap. xviii. § 21).

k'''' . . . = permeance factor under commutating pole-face (eq. 134) (Chap. xviii. § 32).

k_c . . . = coefficient of cross-magnetising armature ampere-turns of alternator (eq. 172).

k_d . . . = coefficient of direct-magnetising armature ampere-turns of alternator (eq. 168).

κ . . . = specific electrical resistivity between opposite faces of a cm. cube in absolute units, *i.e.* = $\rho \times 10^9$.

L . . . = gross length of armature core.
= length of bobbin.
= self-inductance in henrys.

L_a . . . = inductance in henrys of alternator armature per phase from secondary leakage in local circuits (Chap. xxiii.).

L_c . . . = inductance in henrys of alternator armature per phase, equivalent to cross field (Chap. xxiii. § 11).

L_d . . . = inductance in henrys of alternator armature per phase, equivalent to direct magnetising ampere-turns (Chap. xxiii. § 12).

L_f . . . = width of pole-face along axis of armature core.

L_{sz} . . . = apparent inductance of short-circuited coil in relation to additional current, i_z (Chap. xviii. § 17).

l_a . . . = length of magnetic path in armature core in cm.

l_g . . . = length of magnetic path through single air-gap.

l_t . . . = length of magnetic path through one armature tooth.

l_m . . . = length of magnetic path through a pair of magnet-cores.

l_y . . . = length of magnetic path in yoke.

l_r . . . = length of pole-face of commutating pole.

l_{gr} . . . = length of air-gap of commutating pole.

l_{gx} . . . = equivalent length of air-gap within the interpolar gap.

l_x . . . = mean length of exciting turn.

l_{mx} . . . = mean length of series exciting turn.

l_{sx} . . . = mean length of shunt exciting turn.

l_1 . . . = distance between opposite ends of alternator coil (Chap. xxiii. § 24).

l' . . . = length of journal in inches.

l' . . . = length of end-connector in barrel winding of armature.
l_e . . . = axial projection of winding at one end of barrel-wound drum (eq. 52).
λ . . . = angle of lead of brushes.
= ratio of excitation on one magnetic circuit to short-circuit ampere-turns of alternator coil (Chap. xxiii. § 22).
= permeance factor in calculation of inductance of short-circuited section of armature winding, *i.e.* $4\pi \times$ equivalent permeance per cm. length.
λ_1 . . . = $4\pi \times$ equivalent permeance of the one side of coil per cm. length of armature iron (Chap. xviii. § 21).
λ_2 . . . = $4\pi \times$ equivalent permeance of the opposite side of coil per cm. length of armature iron (Chap. xviii. § 21).
λ' . . . = $4\pi \times$ equivalent permeance of the end-connector per cm. length (Chap. xviii. § 23).
λ_a . . . = $4\pi \times$ equivalent permeance of the two embedded sides of coil on alternator (Chap. xviii. § 24).
λ_b . . . = $4\pi \times$ equivalent permeance of the straight projections of coil on alternator (Chap. xviii. § 24).
λ_k . . . = $4\pi \times$ equivalent permeance of the surface of core on alternator (Chap. xviii. § 24).

M . . . = coefficient of mutual inductance.
Mk^2 . . = moment of inertia.
m . . . = number of phases of alternator.
= half-pitch of armature winding measured on circumference.
= air-gap coefficient of toothed armature.
= number of multiplex windings on continuous-current armature.
= thickness of a mica strip in commutator.
μ . . . = permeability.
= coefficient of friction.

N . . . = number of revolutions per minute.
N_1 . . . = number of elements in continuous-current armature winding (Chap. xi. § 11).
N_2 . . . = number of commutator sectors.
N_c . . . = total number of field-magnet coils.
N_o . . . = critical speed of self-excitation.
N_k . . . = critical speed of whirling of shaft (Chap. xiii. § 10).
N_s . . . = number of linkages of self-induced lines with circuit.
n . . . = number of revolutions per second.
n_a . . . = number of arms in armature hub.
n_c . . . = number of impulses on prime mover per revolution (Chap. xviii. § 18).
n_d . . . = number of air-ducts in armature core.
n_s . . . = number of slots or teeth in armature.

P . . . = number of poles.
= load on bearing in pounds.

P_m . . . = total uniform magnetic pull summed up all round armature.

$P_m^{P'}$ . . . = resultant magnetic pull on one-half of armature when its value is non-uniform.

P_m'' . . . = magnetic pull all round armature if supposed to be uniformly at its maximum value.

P_t . . . = pull due to transmitted torque.

P_u . . . = unbalanced magnetic pull.

$\mathcal{P}$ . . . = permeance.

$\mathcal{P}_l$ . . . = leakage permeance.

p . . . = number of pairs of poles in heteropolar dynamo, or number of polar projections in homopolar alternator (Chap. ix. § 3).

= intensity of pressure in pounds in square inch.

p_1 . . . = number of rows of brushes of one polarity.

p_m'' . . . = maximum abnormal pull due to deflection of yoke-ring per radian.

q, Q, **Q** . = instantaneous, effective, and maximum value of charge of condenser in coulombs.

Q . . . = ratio of E.M.F. consumed by synchronous impedance at full-load to open-circuit volts in alternator design (Chap. xxv. § 1).

q . . . = number of parallel paths through armature from $-$ to $+$ brush, $=2a$

q_1 . . . = number of slots between which one side of an alternator armature coil is divided.

R . . . = resistance in ohms.

$= r + 2r_c \dfrac{\beta}{b_1}$ (eq. 97) in relation to short-circuited section of armature.

R_a . . . = resistance of armature in ohms.

R_b . . . = resistance of brushes in ohms.

R_e . . . = resistance of external circuit in ohms.

R_k . . . = specific contact-resistance of brush in ohms per unit area (Chap. xviii. § 16).

R_m . . . = resistance of series winding in ohms.

R_r . . . = resistance of rheostat in ohms.

R_s . . . = resistance of shunt winding in ohms.

R_o, R_m, R_i = outer, mean, and inner radius of hollow cylinder.

$\mathcal{R}$ . . . = magnetic reluctance.

$\mathcal{R}_c$ . . . = magnetic reluctance of cross-circuit of alternator (Chap. xxiii. § 11).

$\mathcal{R}_d$ . . . = magnetic reluctance of main-circuit of alternator (Chap. xxiii. § 12).

$\mathcal{R}_l$ . . . = magnetic reluctance of leakage paths.

$\mathcal{R}_m$ . . . = magnetic reluctance of field magnet.

$\mathcal{R}_a$ . . . = magnetic reluctance of armature.

r . . . = resistance of short-circuited coil of armatures (Chap. xviii. § 18).

r_c . . . = resistance of one commutator connector (Chap. xviii. § 18).

r_g . . . = radius to centre of gravity.
r_s . . . = distance between centres of adjacent bars in slot of toothed armature (Chap. xviii. § 40).
r_q . . . = radius of circle having the same perimeter as a sheaf of active wires on alternator (Chap. xxiii. § 24).
r_τ . . . = radius of circle having the same perimeter as a sheaf of end-connections on alternator (Chap. xxiii. § 24).
r_a . . . = radius of arm of hub (eq. 40).
= resistance of alternator armature per phase (Chap. xxiii. § 13).
r_a' . . . = resistance of alternator armature per phase, increased to allow for eddy current (Chap. xxiii. § 13).
r_n . . . = radius to nave of hub (eq. 40).

ρ . . . = specific electrical resistivity, *i.e.* resistance between opposite faces of a cm. cube in ohms.

S_c . . . = cooling surface of field-magnet bobbins or armature (Chap. xix. §§ 9, 19).
S_k . . . = number of coils simultaneously short-circuited by a brush (eq. 123).
s . . . = sectional area.
s_a . . . = sectional area of armature core in square centimetres.
s_g . . . = sectional area of air-gap core in square centimetres.
s_m . . . = sectional area of magnet-core in square centimetres.
s_y . . . = sectional area of yoke-ring in square centimetres.
s_a . . . = stress on material of armature core due to centrifugal force and magnetic pull.
s_a' . . . = stress on material of armature core at junction with arm.
s_c . . . = stress on material due to centrifugal force.
= compressive stress on commutator copper and mica.
s_b . . . = bending stress on arm of hub.
s_u . . . = normal current-density over brush contact-area.
s_u' . . . = current-density at contact between brush and leading sector.
s_u'' . . . = current-density at contact between brush and trailing sector.
s_{ueff} . . . = R.M.S. value of non-uniform current-density at brush contact surface.

σ . . . = space factor, ratio of copper volume to total volume.
= ratio of final to initial displacement of alternator in parallel (Chap. xxvi. § 20).

T . . . = number of turns.
= torque.
= driving tension in belt.
= time of short-circuit of an armature section (eq. 121) in seconds.
T′ . . . = remainder of time of short-circuit when leading edge of leading sector coincides with tip of brush (Chap. xviii.).
T_c . . . = heating time constant (Chap. xix. § 8).

T_{eu} . . . = synchronising torque in alternator per radian of electrical displacement (Chap. xxvi. § 9).
T_u . . . = synchronising torque in alternator per radian of mechanical displacement (Chap. xxvi. § 9).
T_p . . . = periodic time in seconds.
T_e . . . = periodic time of forced oscillation from prime mover in seconds (Chap. xxvi. § 18).
T_o . . . = maximum value of variable part of driving torque (Chap xxvi. § 18).
T_m . . . = twisting moment of shaft.
= number of series turns per pair of poles.
T_s . . . = number of shunt turns per pair of poles.
= synchronising torque.
T_{so} . . . = initial synchronising torque without effect of variable driving torque.
T° . . . = temperature.
t . . . = number of turns in coil, or of active wires in one sheaf corresponding to one pole and phase on alternator armature.
= time in seconds.
= depth of winding of bobbin.
t° . . . = rise of temperature.
t_1 . . . = tooth-pitch.
t_v . . . = instantaneous value of variable part of driving torque (Chap. xxvi. § 18).
τ . . . = total number of active conductors on armature.

u_n . . . = number of coil-sides per slot in toothed armature.

V . . . = velocity in cm. per second.
V_c . . . = volume of iron in c.cm.
v . . . = peripheral speed of armature.
= peripheral speed of journal in feet per minute (Chap. xiii. § 12).
v_k . . . = peripheral speed of commutator.
v_k' . . . = peripheral speed of commutator in feet per minute.
v_g . . . = linear velocity of centre of gravity.

W . . . = watts.
= weight.
W_a . . . = weight of armature.
W_c . . . = weight concentrated at one point.
W_d . . . = weight distributed along shaft.
W_s . . . = synchronising watts (Chap. xxvi. § 8).
w_s . . . = width of slot.
w_t . . . = width of tooth.
w_{t1} . . . = width of tooth at top.
w_{t2} . . . = width of tooth at bottom.
w_d . . . = width of ventilating air-duct in armature core.
w_3 . . . = width of slot-opening.

X . . . = ampere-turns of excitation on one magnet circuit or per pair of poles.
X_a . . . = ampere-turns of excitation over armature core.
X^g . . . = ampere-turns of excitation over air-gaps.
X_m . . . = ampere-turns of excitation over field-magnet cores.
X_t . . . = ampere-turns of excitation over teeth.
X_y . . . = ampere-turns of excitation over yoke.
X_P . . . = ampere-turns of excitation over interpolar gap.
X_m . . . = ampere-turns of series winding.
X_s . . . = ampere-turns of shunt winding.
X_b . . . = demagnetising ampere-turns of armature.
X_{DM} . . . = equivalent direct-magnetising ampere-turns from all phases of alternator in relation to one magnetic circuit (eq. 167).
X_{CM} . . . = equivalent cross-magnetising ampere-turns from all phases of alternator in relation to one magnetic circuit (eq. 171).
X . . . = total magnetising ampere-turns on alternator armature from all phases in relation to one magnetic circuit (Chap. xxiii. § 16).
x . . . = reactance (Chap. vii. § 20).
x_a . . . = reactance of alternator armature per phase from secondary leakage.
x_a' . . . = apparent synchronous reactance of alternator armature per phase.

y_F . . . = front pitch of armature winding at commutator end in elements (Chap. xi. § 22).
y_R . . . = rear pitch of armature winding in elements (Chap. xi. § 22).
y_R' . . . = rear pitch of armature winding in slots (Chap. xi. § 22).
y_k . . . = average pitch, or pitch in commutator sectors (Chap. xi. § 22).
y . . . . = resultant pitch (Chap. xi. § 22).

Z . . . = number of C.G.S. lines.
Z_a . . . = number of useful lines passing through one polar air-gap into armature core.
Z'_a . . . = total number of lines entering armature of inductor alternator within the pole-pitch (Chap. ix. § 4).
Z_m . . . = total number of lines passing through a magnet-core.
Z_y . . . = total number of lines passing through a yoke.
Z_c . . . = number of lines in one cross-field of alternator (eq. 175).
z_a' . . . = useless lines entering armature of inductor alternator beyond the pole-pitch (Chap. ix. § 4).
= apparent synchronous impedance of alternator armature per phase in henrys (Chap. xxiii. § 19).
z_a . . . = impedance of alternator armature per phase due to secondary leakage.
ζ . . . . = number of leakage lines in one magnet circuit.

θ . . . . = rise of temperature.
= mechanical angle of displacement in radians (Chap. xxvi. § 9).
θ_e . . . = electrical angle of displacement in radians (Chap. xxvi. § 8).
θ_o . . . = initial maximum displacement of phase due to variable driving torque without effect of synchronising torque (Chap. xxvi. § 20).
θ_{mf} . . . = final maximum mechanical angle of displacement (eq. 216).
ξ . . . . = reluctivity (Chap. iii. § 9).
ϕ . . . . = polar arc.
= angle of lag or lead of alternating current or E.M.F.
ϕ_o . . . = angle of lag or lead of current vector in relation to centre of pole (Chap. xxiii. § 5).
ϕ_e . . . = angle of lag of external current vector behind terminal E.M.F.
ϕ_a . . . = angle of lag of armature current behind E.M.F. consumed by armature impedance.
ϕ_a' . . . = angle of lag of armature current on short-circuit (Chap. xxiii. § 17).

Ω . . . = mean angular velocity of alternator.
ω . . . . = angular velocity in radians per sec., or $=2\pi f$.
= resistance of unit length of copper of given section.
ω' . . . = resistance of 100 yards of copper of given section at 60° F.
ω_o . . . = maximum value of variable part of velocity of alternator.
ω_v . . . = instantaneous value of variable part of velocity of alternator.
ω_k . . . = critical angular velocity for whirling of shaft (eq. 44).

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